Chapter 6

Conclusion and future directions

In the previous chapters, successful attempts to solve GNEPs were presented. In this chapter, the significant contributions of this thesis are summarized. The possible directions for future work are also discussed.

6.1 General conclusions

The principal conclusions of this thesis are as follows.

- We have solved GNEP using proposed algorithms and have compared their numerical performances in each chapter.
- The algorithms of each proposed method have been provided and the convergence analysis for each algorithm have been discussed.
- We have provided the numerical results for solving GNEPs using the proposed algorithms in Chapter 2, 3, and 4.

6.2 Contribution of the thesis

This thesis mainly focused on solving GNEPs by different optimization methods. In this thesis, we have proposed algorithms in Chapter 2, 3, 4. Chapter 5 proposes an extended

Karush-Kuhn-Tucker condition to characterize efficient solutions to constrained interval optimization problems and shows its application to support vector machines.

In the first contribution, reported in Chapter 2, we have solved GNEPs by an improved BFGS using two line search techniques: The Armijo-Goldstein line search technique and the MWWP line search technique. We have reformulated GNEPs into a smooth system of equations, and with the help of the merit function, we have solved GNEPs by improved BFGS method using two-line search techniques. The BFGS method using the MWWP line search technique converges globally and works well compared to other quasi-Newton methods. However, we have used the Armijo-Goldstein line search technique to minimize computation costs. The improved BFGS method with the Armijo-Goldstein line search technique takes lesser computation costs than the MWWP-line search technique. We have solved five numerical problems using the proposed algorithms, and have given a numerical comparison of both algorithms.

In the next contribution, reported in Chapter 3, an inexact Newton method to solve generalized Nash equilibrium problems is proposed for both the cases of player convex GNEP and jointly convex GNEP. In the proposed approach, we have reformulated GNEP into a nonsmooth system of equations and then solved it by the inexact Newton method. Under some mild conditions, the numerical Algorithms globally converge Q-quadratically, which is a faster rate of convergence for such equilibrium problems. The numerical Algorithms have been tested on various problems found in the specialized literature on GNEPs (see [41, 63–65]). Previously, GNEP was solved by other conventional methods, such as the smoothing Newton method [77], a feasible direction interior point method [78], etc., but it has been reported that the proposed numerical scheme converges faster than semismooth Newton method II and hence than all the existing method (see [1]).

In another contribution, reported in Chapter 4, we have proposed an INATR method for constrained optimization problems and have shown its application to solve GNEPs.

In this method, we have computed an adaptive trust region radius ((4.5)-(4.9)) using gradient and Hessian matrix information. It affects the convergence of the algorithm as well as the number of iterations. Also, we have used a new nonmonotone technique (4.11), a convex combination of the functional value obtained in the current iteration and the maximum of the functional values obtained from some prior successful iterations. Subsequently, we have given a global convergence of the proposed algorithm. Further, we have solved a dataset of 35 GNEPs using the INATR method and have provided a comparison of the performance profiles of the INATR method with the existing two methods: the nonmonotone trust region (NTR) method and monotone trust region (MTR) method.

In Chapter 5, we have considered the problem of interval optimization for constrained IOPs with the aim of characterizing the efficient solution from a geometrical viewpoint. We have proposed extensions to Gordon's Theorems of the alternatives for an interval-valued system of inequalities and used it to derive the Fritz John conditions for IOPs. We also derived an extension to KKT conditions for IOPs and thereby proposed the optimality conditions for both constrained and unconstrained IOPs. These proposed optimality conditions have been applied to binary classification problems using SVMs for interval-valued dataset and a comparison has been drawn with existing methods.

Next, the possible future scope of the above-discussed work is discussed.

6.3 Future directions

The above-discussed contributions in the thesis may lead to several future research directions:

(i) We have proposed an extended Karush-Kuhn-Tucker condition to characterize efficient solutions to constrained interval optimization problems. Therefore, we can compute the extended Karush-Kuhn-Tucker condition for the inter-valued generalized Nash equilibrium problems, so that we can reformulate the inter-valued generalized Nash equilibrium problems into an inter-valued system of equations. In the future, we will try to solve inter-valued GNEPs.

(ii) In this thesis, we have solved a smooth version of the constrained optimization problem using the proposed INATR method. In the future, we will try to develop an INATR method for nonsmooth-constrained optimization problems so that we can apply the nonsmooth version of the INATR method to the nonsmooth reformulation of GNEPs.