

# Contents

<b>List of Figures</b>	<b>xv</b>
<b>List of Tables</b>	<b>xvii</b>
<b>Symbols</b>	<b>xix</b>
<b>Preface</b>	<b>xx</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Fuzzy space geometry . . . . .	2
1.2 Application of fuzzy geometry on fuzzy image processing . . . . .	4
1.2.1 Fuzzy image processing . . . . .	4
1.2.2 Fuzzy Hough transform . . . . .	7
1.3 Preliminaries . . . . .	8
1.3.1 Fuzzy sets . . . . .	8
1.3.2 Classical Hough transform . . . . .	13
1.4 Literature survey . . . . .	19
1.4.1 Literature on fuzzy geometry . . . . .	19
1.4.2 Literature on fuzzy Hough transform . . . . .	24
1.4.3 Literature on applications of fuzzy geometry . . . . .	25
1.5 Objective of the Thesis . . . . .	27
1.6 Outline of the Thesis . . . . .	28
<b>2 Analytical fuzzy space geometry I</b>	<b>31</b>
2.1 Introduction . . . . .	31
2.2 Motivation and Contributions . . . . .	32
2.3 Space fuzzy point . . . . .	33
2.3.1 A reference function of three variables . . . . .	34
2.3.2 Representation of a space fuzzy point by a reference function . . . . .	36
2.3.3 Same and inverse points . . . . .	39

2.3.3.1	Fuzzy numbers along a line . . . . .	39
2.3.3.2	Addition operation of two $S$ -type space fuzzy points . . . . .	45
2.3.3.3	Separation of effective combinations for the addition of two $S$ -type space fuzzy points . . . . .	46
2.3.3.4	General expression of same points with respect to two continuous $S$ -type space fuzzy points . . . . .	56
2.3.3.5	General expression of inverse points with respect to two continuous $S$ -type space fuzzy points . . . . .	63
2.4	Fuzzy distance . . . . .	64
2.5	Space fuzzy line segments . . . . .	75
2.6	Comparison . . . . .	80
2.7	Conclusion . . . . .	86
<b>3</b>	<b>Analytical fuzzy space geometry II</b>	<b>87</b>
3.1	Introduction . . . . .	87
3.1.1	Motivation and novelty . . . . .	87
3.2	Space fuzzy line . . . . .	90
3.2.1	Symmetric fuzzy line . . . . .	95
3.3	Shortest distance . . . . .	101
3.3.1	A brief discussion of the shortest distance between non-symmetric skew fuzzy lines . . . . .	112
3.4	Fuzzy plane . . . . .	113
3.4.1	Fuzzy plane passing through three $S$ -type space fuzzy points ( $\tilde{\Pi}_{3P}$ ) . . . . .	114
3.4.2	Intercept form ( $\tilde{\Pi}_I$ ) . . . . .	120
3.4.3	Fuzzy plane passing through an $S$ -type space fuzzy point and perpendicular to a given crisp direction ( $\tilde{\Pi}_{P_n}$ ) . . . . .	129
3.4.4	Symmetric fuzzy plane ( $\tilde{\Pi}_S$ ) . . . . .	134
3.5	Discussion and comparison . . . . .	142
3.6	Conclusion . . . . .	149
<b>4</b>	<b>Analytical fuzzy space geometry III</b>	<b>151</b>
4.1	Introduction . . . . .	151
4.1.1	Motivation and novelty . . . . .	151
4.2	Fuzzy sphere . . . . .	153
4.3	Fuzzy cone . . . . .	192
4.4	Discussion and comparison . . . . .	200
4.5	Conclusion . . . . .	205
<b>5</b>	<b>Hough transform generalization for detecting fuzzy lines and fuzzy circles</b>	<b>207</b>
5.1	Introduction . . . . .	207

---

5.2	Fuzzy Hough transform . . . . .	208
5.2.1	Generalized version of fuzzy Hough transform . . . . .	208
5.2.2	Fuzzy line detection using FHT . . . . .	210
5.2.3	Fuzzy circle detection using FHT . . . . .	220
5.3	Similarity measure between two fuzzy shapes . . . . .	225
5.3.1	Fuzzy shape descriptor . . . . .	226
5.3.2	Distance measure between two fuzzy shapes in the fuzzy image	227
5.3.3	Similarity measure between a crisp line and a symmetric fuzzy line . . . . .	229
5.4	Experimental results . . . . .	230
5.5	Conclusion . . . . .	232
<b>6</b>	<b>Conclusion and future work</b>	<b>235</b>
6.1	General conclusions . . . . .	235
6.2	Contributions of the thesis . . . . .	236
6.3	Future work . . . . .	240

<b>Bibliography</b>	<b>243</b>
---------------------	------------