

Chapter 2

Review of the literature

2.1 Review of the literature

For a long time, the investigation of the peristalsis mechanism in mechanical and physiological situations has been the main motive of scientific research. Investigations on peristaltic flows have been largely carried out for physiological and industrial applications under various considerations such as types of fluids, types of waves, and geometries of vessels.

Since the first attempt made by Latham (1966), numerous research papers published on theoretical and experimental investigations have enriched the literature on peristaltic transport. This large number of scientific contributions has consequently revolutionized this area of research. A few of them are focused on the physical and medical problems the mankind is facing, but many challenges are yet unsolved. Hence it is required to throw light on them. The relevant research and the milestones left behind during the long journey of investigation are presented here.

2.1.1 Short study for Newtonian fluids

For Newtonian fluid, a lot of investigations on the peristaltic transport via channels or cylindrical tubes have been carried out by the researchers. In the earlier analyses on peristalsis, mostly theoretical investigations were taken by assuming the Newtonian fluid in various channels (Burns and Parkes, 1967; Fung and Yih, 1968; Barton and Raynor, 1968).

Investigations took time to gain tempo in the perspective of diagnostic observations. The study of Shapiro *et al.* (1969) may be considered as a pioneering work on peristaltic transport for two-dimensional and axisymmetric cases under the assumptions of low Reynolds number and long wavelength. They discovered the notions of reflux, trapping and mechanical efficiency. Chow (1970) presented a model that dealt with peristaltic transport through an axisymmetric tube by using perturbation technique. Some important contributions in the literature on peristaltic pumping of Newtonian fluids were made by Lykoudis and Roos (1970), Yin & Fung (1971), Jaffrin and Shapiro (1971) and Jaffrin (1973). Lykoudis and Roos (1970) observed that the shape of ureter during peristalsis is not sinusoidal and it may be best described by a polynomial expression corresponding to an arbitrary wall shape. Yin & Fung (1971) studied experimental peristaltic transport in a two-dimensional channel and compared experimental and theoretical results. Takabatake and Ayukawa (1982) and Takabatake *et al.* (1987) employed finite difference techniques to study creeping flows and discussed reflux, trapping and mechanical efficiency in detail. They (1987) also corrected pumping efficiency for the axisymmetric case given by Shapiro *et al.* (1969).

2.1.2 Non-Newtonian flows

It has been acknowledged that the bio-fluids do not behave like Newtonian fluids with flourish of physical and medical sciences and fail short to provide an appropriate understanding when peristaltic mechanism is involved in small blood vessels, intestine, transport of spermatozoa in the cervical canal etc. It has now been accepted that most of the physiological fluids behave like non-Newtonian fluids. These considerations of non-Newtonian nature of fluids enriched the literature on peristalsis. Raju and Devanathan (1972) presented a theoretical investigation for blood flow by considering blood as a non-Newtonian power-law fluid and it was further extended for viscoelastic fluid by the same authors (1974) and observation were that the rate of discharge does not depend on the elasticity of the fluid. Bohme and Friedrich (1983) studied peristaltic flow of viscoelastic fluids and reported that the pumping efficiency decreases for fixed pressure with the memory parameter that represents viscoelastic characteristic. Srivastava and Srivastava (1984) modelled peristaltic flows of Casson fluids (blood) flowing inside small capillaries and blood vessels. Srivastava and Srivastava (1985) examined peristaltic flows of power law fluids in the vas deference by considering it a non-uniform diverging tube. Many more research papers taking non-Newtonian fluid are detailed in subsequent paragraphs.

Stokes (1966) introduced a mathematical model for the flow of couple stress fluids. Srivastava (1986) studied the problem of peristaltic transport of couple stress fluids under the long wavelength and low Reynolds number approximations. Some other mathematical and computational methods for couple stress fluid was carried out by Elshehawey and Mekheimer (1994), Elshehawey and Sebaei (2001) and Mekheimer (2004). Pandey and Tripathi (2011d) published a paper for swallowing of couple-stress fluids in oesophagus. Pandey and

Chaube (2011a) investigated wall properties on peristaltic transport of couple stress fluids.

Various types of fluid models for peristaltic transport have been studied by researchers. Such as peristaltic transport of magneto-fluid models was studied by Elshehawey and Husseny (2002). The moving boundary with a non-Newtonian fluid was discussed by Siddiqui *et al.* (2002). Propagation of peristaltic waves in a magnetic field was discussed by Katiyar and Basavarajappa (2002) which can be applied to studies pertaining to flow of physiological fluids. Sinha *et al.* (2015) introduced a model for peristaltic flow of a MHD fluid and found that the velocity at the central region diminishes with increasing values of the velocity-slip parameter. Peristaltic transport of a MHD dusty Casson fluid in a rectangular duct has been examined by Zeeshan *et al.* (2017). They revealed that the Casson fluid parameter increases the fluid velocity near the walls of the duct and decreases the flow in the middle of the duct.

Peristaltic transport of micro-polar fluids in a two-dimensional channel was studied by Muthu *et al.* (2003) and reported the effects of viscoelastic wall properties and micro-polar fluid parameters on the flow. Srinivasacharya *et al.* (2003) also investigated some more aspects of peristaltic flow of micro-polar fluids. Micro-polar flows in oesophagus were studied by Pandey and Tripathi (2011b).

Vajravelu *et al.* (2005a, 2005b) studied peristaltic pumping of Herschel-Bulkley fluids in channels and inclined tubes respectively. The effects of wave amplitude and yield stress on the flow characteristics were obtained and discussed. The results obtained for the flow characteristics revealed many interesting behaviours. Sanyal and Biswas (2010) presented a mathematical analysis for peristaltic flow of blood.

There were reports (Dean, 1928; Zalosh and Nelson, 1973, etc.) on flows through curved tubes without peristaltic considerations. Sato *et al.* (2000) investigated two-dimensional peristaltic flow of a viscous fluid in a curved channel. They also discussed reflux and trapping. Peterson (2010) extended the work of Zalosh and Nelson (1973) for wavy walls. Hayat *et al.* (2010) discussed the peristaltic transport in a curved flow configuration. Tripathi *et al.* (2014) investigated peristaltic propulsion with exponential variable viscosity. The peristaltic phenomenon for third order and Carreau-Yasuda materials in curved geometry has also been investigated by Abbasi *et al.* (2015). They observed that the size of trapped bolus decreases and finally vanishes for large values of magnetic parameter. Hayat *et al.* (2017a) studied numerically MHD peristaltic transport of Sisko nano-fluid in a curved channel.

2.1.3 Flows with particulate suspensions: application to diseased ureter

The hypothesis of particle-fluid blend is gigantically valuable for understanding a number of physical incidents including transportation of solid particles by liquids, blending operations, particulate suspension theory of blood, flow of food suspension through oesophagus and digestion tracts, urine flow through the ureters, transportation of liquid slurries in chemical and nuclear processing etc. A few industrial food processes include stream of food suspension in which the information of flow properties is fundamental for evaluating pumping prerequisites.

Hung and Brown (1976) investigated various geometric and dynamic effects on peristaltic transport of suspended solid particles in a fluid in a two-dimensional channel. Drew (1979, 1983) displayed a two-phase stream demonstrate that accounts for a blend of scattered small particles in a liquid. Srivastava and Srivastava (1989) used Drew's model (1979) to a particle-fluid mixture flowing in a channel and acquired perturbation solution for small amplitude ratio. The flow of particle-fluid suspension through the ureters was examined by Misra and Pandey (1994) and concluded that the mean flow caused by peristaltic transport is proportional to the square of the amplitude ratio and depends on the mean pressure gradient. Ureters are muscular ducts that propel urine from the kidneys to the bladder by peristalsis. Jimenez-Lozano *et al.* (2011) also presented a model for peristaltic flow in ureters due to a solitary wave with the objective of explaining the flow mechanics of a particle-fluid mixture. Mekheimer and Abdelmaboud (2008) theoretically analysed peristaltic flow through uniform and non-uniform annuli filled with particle-fluid suspension by long wavelength approximation. Popularity of the Drew's model is revealed through a series of recent publications in biomechanics involving peristalsis (Bhatti and Zeeshan, 2016; Zeeshan *et al.*, 2017) and rheological flow of blood (Zeeshan *et al.*, 2018). A model for heat transfer on solid particle motion of dusty Jeffrey fluid through a planar channel has been investigated by Bhatti and Zeeshan (2016). Pandey and Singh (2018a) investigated influence of suspended particles in oesophageal swallowing.

2.1.4 Multi-layered flows: application to intestinal flows

The analysis of Shapiro *et al.* (1969) was extended to include a Newtonian peripheral layer close to the wall to simulate the effect of a coating in physiological flows by Brasseur *et al.* (1987) who corrected a similar investigation for peripheral layer by Shukla *et al.* (1980). Similar corrections were then required in Shukla and Gupta (1982) and Srivastava and Srivastava (1984) whose models were meant for power-law and Casson fluids respectively. Rao and Usha (1995) presented a peripheral layer model for axisymmetric flows. Misra and Pandey (1999, 2001a) later reported corrected versions of power-law peripheral models for channel and tubular flows respectively. They further presented a peripheral Casson fluid model for blood flows. A three layered peripheral model was published by Elshehawey and Gharsseidien (2004) while porosity in the peripheral layer was introduced by Mishra and Rao (2005). Pandey *et al.* (2011) worked for intestinal flows and reported that an intermediate layer of viscosity lower than that of the mucus layer may overcome constipation. They (2015) further published similar reports for axisymmetric flows.

2.2 Flows in vessels of finite length: application to swallowing in oesophagus

Li and Brasseur (1993) theoretically developed a model for Newtonian fluids through oesophagus and focused the study on both the local and global dynamics. Unlike the consideration that the wall oscillates about the stationary boundary, they pointed out that the oesophageal wall undergoes contraction followed

by relaxation. They also discussed the effect of integral and non-integral number of peristaltic waves. Nguyen *et al.* (1997) studied dynamics of oesophageal bolus transport in healthy subjects using multiple intraluminal impedancometry. Misra and Pandey (2001) modified the wall equation for oesophageal swallowing and considered the food bolus to follow the power law. Later on, several mathematical models (Pandey and Tripathi, 2010a, 2010b, 2010c, 2010d; 2011a, 2011b, 2011c, 2011d, 2012; Tripathi *et al.* 2011, 2013) were presented based on the modified model of oesophageal wall motion under the consideration of different fluids resembling the properties of different edible foods. Misra and Maiti (2012) developed a mathematical model with the specific aim of exploring some important information concerning the movement of food bolus through the oesophagus. The model was formulated and analysed for peristaltic transport of Ostwald-de Waele power law fluid for arbitrary wave shapes and tube lengths. Dhar *et al.* (2020) studied a model and evaluate the acute effects of pyridostigmine on high-resolution manometry parameters in patients suffering from dysphagia with evidence of esophageal dysmotility.

2.2.1 Flows with dilating wave amplitude: application to sliding hiatus hernia

Kahrilas *et al.* (1995) made an experimental investigation and found a high pressure zone in the distal part of the oesophagus. The findings contradicted uniform pressure distribution along the entire length of the oesophagus reported by several researches. In the meantime, Xia *et al.* (2009) published the measurement of oesophageal wall thickness in contracted and dilated states. This paved the way for modification in the popular models describing oesophageal flow. Pandey

et al. (2017) modified the oesophageal wall equations accordingly by considering exponentially increasing wave amplitude. The main inference drawn were that the pressure along the length of the oesophagus rises as the wave amplitude increases slightly down the oesophagus during the swallowing process. The pressure is largest where the wave propagation culminates. The cause of this natural rising of pressure ensures natural surety of delivery of food bolus into the stomach through the cardiac sphincter. This discovered knowledge was implemented to investigate peristaltic transport of Casson fluid through oesophagus by Pandey and Tiwari (2017). Pandey and Singh (2018b) further used this to investigate oesophageal flow affected by sliding hiatus hernia.

Ribolsi *et al.* (2019) studied and reveals the adjunctive value of HRM in assessing reflux evidence and confirms the value of MNBI in the evaluation of endoscopy-negative patients. And results show that both on univariate and multivariate analysis, a normal EGJ morphology on HRM is associated with a lack of abnormal reflux burden in PPI responders and is compatible with PPI response. In contrast, the presence of a hiatus hernia resulted in lower MNBI values compared with the absence of a hiatus hernia. Su *et al.* (2020) studied and evaluated to the practice of performing manometry after endoscopy with conscious sedation by evaluating its impact on oesophageal motility findings.

Oesophageal chemical clearance has been evaluated with the post-reflux swallow-induced peristaltic wave (PSPW) index. The factors triggering PSPW in Gastro-oesophageal reflux disease (GERD) have not yet been investigated. Therefore further Ribolsi *et al.* (2021) studied this multicenter study and aimed at evaluating the characteristics of reflux episodes associated with PSPW occurrence in patients with typical GERD symptoms.

2.2.2 Heat transfer in peristaltic transport: application to cryosurgery

Radhakrishnamacharya and Murty (1993) presented a model for heat transfer and obtained closed form solutions for temperature, coefficient of heat transfer and velocity up to the second order by perturbation techniques. Vajravelu *et al.* (2007) discussed the interaction of peristalsis with heat transfer in a vertical porous annular region and formulated heat transfer at the wall and the pressure–flow relationship. Observations were that the heat transfer at the wall was affected significantly by the amplitude of the peristaltic wave but effect of pressure drop on flux was almost negligible for peristaltic waves of large amplitude. Mekheimer and Abdelmaboud (2008b) investigated heat transfer and magnetic field on peristaltic transport of Newtonian fluids in a vertical annulus with application to endoscope. Srinivas *et al.* (2009) studied the effects of wall slip conditions and heat transfer both on peristaltic flow of MHD Newtonian fluids in a porous channel with elastic wall properties under the assumptions of long wavelength and low-Reynolds number approximations. Makinde and Chinyoka (2010) presented a model for transient heat transfer in channel flow and solved the governing nonlinear equations of momentum and energy transport numerically by finite difference methods. Sreenadh *et al.* (2012) presented a model for the effect of heat transfer and wall properties of flexible walls in oesophageal swallowing. Tripathi (2012) and Tripathi *et al.* (2013) investigated mathematical model for swallowing of food bolus under the influence of heat transfer and derived the expressions for temperature field, axial velocity, volume flow rate, pressure gradient, local wall shear stress, stream function and reflux limit under the assumptions of long wavelength and low Reynolds number approximations. Hayat *et al.* (2014) presented a study in order to show the effects of convective

boundary conditions on peristaltic transport of a micropolar fluid in an asymmetric channel with heat source/sink. A model for heat transfer on solid particle motion of dusty Jeffrey fluid through a planar channel was constructed by Bhatti and Zeeshan (2016). Prakash *et al.* (2018) discussed a numerical simulation to study the heat and flow characteristics of blood flow altered by electro-osmosis through the tapered micro-vessels assuming blood as non-Newtonian (micropolar) nano-fluids. This study explored the nano-fluid dynamics in peristaltic transport as symbolized by heat transport in biological flows and also in gastrointestinal motility enhancement. Misra *et al.* (2018) formulated a mathematical model to analyse the peristaltic transport of magneto hydrodynamic fluid associated with heat and mass transfer in an asymmetric channel. On the basis of this study, the authors reported that fluid velocity and the distributions of concentration and temperature are considerably influenced by Grashof number.

Waini *et al.* (2021) studied a model and examined the radiative and magneto hydrodynamic micropolar hybrid nanofluid flow over a shrinking sheet with Joule heating and viscous dissipation effects. Mohanty *et al.* (2021) introduced a mathematical model for the peristaltic flow of conducting micropolar nanofluid within a wavy channel for the behavior of radiative heat energy and the heat source/sink. Nadeem *et al.* (2021) introduced a mathematical model that interprets the peristaltic flow in the elliptic duct with heat and mass transfer and provides exact analytical solutions. Saba *et al.* (2021) explored and study the combined influences of applied electric and magnetic fields on the two-phase peristaltic motion of nanofluid through a curved channel.

2.2.3 Flows in asymmetric channels

Mishra and Rao (2003) investigated steady peristaltic transport of a Newtonian fluid through an asymmetric channel. It was followed by investigations of peristaltic transport through an asymmetric channel of Casson fluid by Rani and Sarojmma (2004), of a couple stress fluids by Ali *et al.* (2007) and of a power-law fluid by Reddy *et al.* (2007). Hayat *et al.* (2008) investigated peristaltic mechanism of in an asymmetric channel using a perturbation method in terms of the wave number. They (2008) explored the effects of various parameters on pressure rise per wavelength and the axial pressure gradient through numerical integration. Mekheimer and Ablelmaboud (2008b) investigated the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus with application to endoscope. Peristaltic flow of Sisko fluid in a symmetric or asymmetric channel was studied by Wang *et al.* (2009). They studied the shear-thickening and shear-thinning effects of the non-Newtonian fluid. Akbar *et al.* (2014) presented a numerical simulation of peristaltic flow of a Carreau nano-fluid in an asymmetric channel. The study of heat and mass transfer of an MHD fluid having temperature dependent properties in an asymmetric channel was explored by Misra *et al.* (2018). Khan *et al.* (2021) designed a mathematical model for the GNF model and studied the impact of GNF models in the field of biotechnology and chemical engineering. Feigl and Tanner (2022) introduced a mathematical model to investigate the behavior of liquid drops in the repulsive flow that is produced by peristaltic motion.

2.2.4 Flows in non-uniform vessels: application to vas deferens and uterine cavity

The mechanism of peristaltic transport in non-uniform tubes or channels that may exist in physiological conduits like uterus, vas deferens and blood vessels has been the objective of scientific research for long. Several researchers have made attempts to investigate peristaltic motion in non-uniform tubes or channels. Manton (1975) presented a model for peristaltic flow in an axisymmetric tube of varying radius whose wall was subjected to arbitrary wave propagation under Stokes approximation. Peristalsis in male reproductive system was observed experimentally and numerically by Batra (1974), Guha *et al.* (1975), Gupta and Seshadri (1976) and Srivastava and Srivastava (1985). Guha *et al.* (1975) studied the transport of the spermatic fluid in the vas deferens of monkey and reported that the transportation during ejaculation is mainly due to contraction of the ampulla and filling during the non-ejaculatory phase is due to peristalsis and epididymal pressure. Srivastava and Srivastava investigated peristaltic flows in the vas deferens by considering it a diverging tube and channel (1982) for Newtonian fluids and later (1985) for non-Newtonian fluids. The models were more realistic model investigating power-law fluid flow in a non-uniform tube and blood as a Casson fluid flowing inside small capillaries and blood vessels. Misra and Pandey (1995) modeled axisymmetric peristaltic motion of a Newtonian viscous incompressible fluid through a flexible tube of changing cross section, by retaining nonlinear convective acceleration terms. Their reports were more ascribable than the previous reports for spermatic flows reported by Guha *et al.* (1975) as their results were closer to experimental observations. Eytan and Elad (1999) and Eytan *et al.* (2001) investigated the effect of peristalsis in embryo transport within the uterine cavity. They discussed in detail the phenomenon

of trapping and how the particle reflux impedes the embryo implantation at the fundus. Hariharan *et al.* (2008) studied peristaltic transport of non-Newtonian fluid in a diverging tube with different waveforms and concluded that square wave has the best pumping characteristics of all the wave forms and triangular wave has the worst characteristics. Pandey and Chaube (2010) modified the results of Misra and Pandey (1995) for Maxwell viscoelastic fluids. The same authors (2011b) investigated the peristaltic transport of Maxwell viscoelastic fluid in a channel of varying cross sections.

2.2.5 Flows of nano-fluids

Nowadays nano-science and nano-technology have emerged as a substantial development due to the vital role of non-Newtonian nano-fluids in bio rheology, medical nano-scale electro-osmotic devices and some other engineering applications. Nano fluids are produced by suspended nano-particles which are generally 1–100 nm in dimension, in the base fluids. Kumar *et al.* (2010) analysed nano-fluid flow using a single phase thermal dispersion model and explained that the nano-fluid is a two-phase mixture in which the solid phase consists of nano-sized particles. Chakraborty and Roy (2008) developed electroosmotic transport of nano-fluids in micro-channels. Abbasi *et al.* (2014) simulated peristaltic transport of copper–water nano-fluid in an inclined channel. The presence of Grashoff number and inclination of the channel enhance the temperature of nano-fluid. Akbar *et al.* (2014) presented a numerical simulation of peristaltic flow of a Carreau nano fluid in an asymmetric channel. A theoretical analysis of the slip and joule heating effects in mixed convection peristaltic transport of nano-fluid was presented by Hayat *et al.* (2014a). Shehzad *et al.* (2015) incorporated an analytical model of peristaltic transport of water based nano-fluids in their comparative

study. The influence of magnetic field on peristaltic transport of nano Eyring-Powell fluid in an asymmetric channel was investigated by Akbar (2015). Abbasi *et al.* (2016) analysed peristaltic transport of copper-water nano-fluid with temperature-dependent effective viscosity. Mathematical analyses for peristaltic flow of nano fluids were presented recently by several researchers (Kothandapani and Prakash, 2015; Reddy and Makinde, 2016; Hayat *et al.* 2016; Ding *et al.* 2017; Bhatti *et al.* 2017a). Tripathi *et al.* (2017) studied theoretically the electrokinetic pumping of nano fluids with heat and mass transfer in a micro-channel under peristaltic waves. They incorporated Soret and Dufour cross-diffusion effects and also thermal and species buoyancy effects. A numerical simulation was presented for heat and flow characteristics of micro polar nano fluids altered by electro-osmosis through the tapered micro-vessels by Prakash *et al.* (2018). They examined thermal radiation effects on nano-particle volume fraction and thermal characteristics. Prakash and Tripathi (2018) analysed the electroosmotic flow of non-Newtonian Williamson's fluid model for nano-liquids in presence of peristaltic propulsion through tapered channel. They employed perturbation method to find solutions for axial velocity, pressure rise, volumetric flow rate, temperature field and nano particle volume fraction.

Chandrawat *et al.* (2021) designed a mathematical model for the unsteady flow of two immiscible Eringen micropolar and Newtonian fluids with a moving interface in the horizontal channel. Akram *et al.* (2021) studied and observed that the nano-solid particle concentration drops when large values of Brownian motion parameter and Soret number are considered. Also, random collisions transfer the molecular kinetic energy into thermal energy during the micro-mixing process of solid nano-particles within the nano-liquid, resulting in a rise in fluid temperature.

2.2.6 Flows in elastic tubes

Study of viscous flow in elastic tubes is a requirement for learning the true mechanisms of various phenomena (e.g., atherosclerosis, artery replacement by a graft, bolus transport in oesophagus, etc.) in medicine, biology, biomedical technology and also in industry. The information on the mechanisms of oesophageal and intestinal transport of food and liquids is of immense importance for the treatment of patients suffering from transport disorders. Initial investigation on wall properties on peristaltic transport of Newtonian fluids in a channel with flexible (elastic or viscoelastic) walls was carried out by Mitra and Prasad (1973). Srivastava and Srivastava (1997) extended their work from single-phase Newtonian fluid analysis to a two-phase flow. Carew and Pedley (1997) studied pumping phenomenon of peristaltic flow in the ureter by using lubrication theory by taking into account wall deformation. Rubinow and Keller (1972) and Fung (1997) considered the Poiseuille's flow locally, and found that the radius of a tube can be determined by the balance between the transmural pressure (i.e. the difference between the inside and outside pressure) and the tension in the tube wall. Takagi and Balmforth (2011) modelled tube wall deformation using linear elasticity and the internal flow assuming the lubrication approximation.

Waters and Guiot (2001) investigated flow in an elastic tube subject to a prescribed force. They considered blood as homogeneous Newtonian fluid and the vein an isotropic thin walled elastic tube. Muthu *et al.* (2001, 2003) respectively developed mathematical models for peristaltic flow of Newtonian fluid through an axisymmetric tube and that of micro-polar fluid through a channel with flexible walls. They observed that the mean axial velocity decreases with micro-polar parameter and flow reversal takes place at the wall of the tube for non-zero viscous damping. Sreenadh *et al.* (2012) presented a model to study the

effect of heat transfer and wall properties of flexible walls in oesophageal swallowing. Sochi (2014) investigated the flow of Newtonian and power law fluids in elastic tubes considering the lubrication approximation theory. He demonstrated qualitative similarity in general and asymptotic reduction of the flow equations to limiting cases. Uddin *et al.* (2018) investigated peristaltic transport of a nano-fluid via elastic sheets. Selvi *et al.* (2018) analysed the flow of blood in elastic arteries by considering blood as a power-law fluid. Pulsatile flow of a conducting Jeffrey fluid through a porous elastic tube with variable cross section was investigated by Selvi and Srinivas (2018). An analysis was carried out to study magneto-hydrodynamics peristaltic flow of Prandtl fluid in a channel with flexible walls by Hayat *et al.* (2018). Wolf *et al.* (2021) investigated the effects of the vessel contraction wave speed, contraction amplitude, adverse pressure gradient and valve elastic properties on the pumping performance.

2.2.7 Flows in porous media

Peristaltic transport in a porous medium has attracted researchers due to its pathological applications. It plays a key role to explore the transport process in bio-fluid mechanics, industrial mechanics and engineering fields. Best biological examples of porous medium are the intestine and pathological situation of gallstones falling into the bile ducts and closing them completely or partially. Miyamoto *et al.* (1983) investigated the laminar flow in a porous tube and considered a small water absorption or secretion in the intestinal perfusion experiment. Elshehawey and Husseny (2002) developed a model for peristaltic transport of a magneto-fluid with porous boundaries. The flow fields generated by peristaltic reflex in isolated guinea pig ileum were discussed by Jeffrey *et al.* (2003). Mishra and Rao (2004) investigated peristaltic transport of power-law fluids in a porous

tube. Elshehawey *et al.* (2006) presented a mathematical model for peristaltic transport through an asymmetric porous channel and focused the application to intra uterine fluid motion in uterus. Vajravelu *et al.* (2007) discussed the interaction of peristalsis with heat transfer in a vertical porous annular region and derived expressions for pressure–flow relationship and heat transfer at the wall. Observations were that the heat transfer at the wall was affected significantly by the amplitude of the peristaltic wave but effect of pressure drop on flux was almost negligible for peristaltic waves of large amplitude. Srinivas *et al.* (2009) studied the effects of both of wall slip conditions and heat transfer on peristaltic flow of MHD Newtonian fluid in a porous channel with elastic wall properties under the lubrication approximations. Maiti and Misra (2011) investigated peristaltic flow of a fluid in a porous channel for studying flow of bile within ducts in pathological state. They concluded that due to the presence of gallstones, the critical pressure for reflux decreases as porosity increases. Maiti and Misra (2012) also developed theoretically a model to study the peristaltic transport of couple stress fluids in a porous channel. This study was motivated towards investigating physiological flow of blood in the micro-circulatory system. Tripathi *et al.* (2015) investigated peristaltic viscoelastic bio-fluid flow in symmetric porous media. Magneto-hydrodynamic peristaltic transport of couple stress fluid through porous medium in an inclined asymmetric channel with heat transfer was studied by Ramesh and Devakar (2015). Sheikholeslami *et al.* (2017) simulated transport of magneto hydrodynamic nano-fluid in a porous media.

Ilyas *et al.* (2021) formulated the mathematical model for periodic mixed convection flow across the surface of electrically and thermally magnetic cone embedded in the porous medium. Imankulov *et al.* (2021) analyzed a mathematical model for the one-dimensional problem of multicomponent fluid flow

in a porous medium and solved the system of the algebraic equation with the Newton-GMRES method. Dinariev et al. (2021) developed a modified version of Boltzmann's equation for micro-scale particulate flow with capture and diffusion that describes the colloidal-suspension-nano transport in porous media. Maryshev and Klimenko (2022) studied a problem of porous media cleaning by a pulsating external flow numerically and observed the effect of variation of pulsation amplitude and frequency on the cleaning process.

2.2.8 Electro-osmosis/electric double layer/electro-kinetic flows

Siddiqui and Lakhtakia (2009) analysed the electro-osmotic flow of micro-polar fluids in micro-channels and noted that the decay time of the fluid velocity is lesser for micro-polar fluid than that for Newtonian fluid under applied uniform electric fields. Tang *et al.* (2010) presented the Herschel-Bulkley model to analyse electro-osmotic flow in porous media. This model simulates actual biological fluids for which a critical value of yield stress must be achieved before flow is initiated. Casson model for electro-osmotic flow in a micro-channel was discussed by Ng (2013). He noted that the yield surface describes the regime into sheared and unsheared zones and that the opposing effect of yield stress on time averaged-volume flow rate may be compensated via pressure gradient. Schit *et al.* (2016) presented the electro-osmotic flow of power-law bio-fluids in a micro-channel. They considered the scenario wherein the channel depth is substantially greater than the thickness of electrical double layer. Tripathi *et al.* (2017) studied theoretically the electro-kinetic pumping of nano fluids with heat and mass transfer in a micro-channel under peristaltic waves. They incorporated Soret and Dufour cross-diffusion effects and also thermal and species buoyancy effects.

Chaube *et al.* (2018) carried out a theoretical model to analyse the effects of electric double layer and applied external electric field on peristaltic flow of non-Newtonian aqueous solution in a micro channel. Prakash *et al.* (2018) discussed a numerical simulation to study the heat and flow characteristics of blood flow altered by electro-osmosis through tapered micro-vessels assuming blood as a non-Newtonian (micro-polar) nano-fluid. This study explored the nano-fluid dynamics in peristaltic transport as symbolized by heat transport in biological flows and also in gastro-intestinal motility enhancement. Tripathi *et al.* (2018) presented a model to study the electro-kinetic flow in a finite length micro-channel. They observed that cooling occurs in the micro-channel electro-kinetic flow with positive joule heating effect and heating occurs with negative joule heating effect.

Alharbi *et al.* (2021) worked on a mathematical problem for isotropic, micro-polar thermo-elastic medium with voids subjected to the Thomson effect, and the solution to the problem is presented in the context of the refined multiphase-lags theory of thermo-elasticity. Asghar *et al.* (2021) studied a model for the theoretical analysis of electro-osmosis peristaltic flow of generalized Newtonian fluid bounded in a complex wavy divergent channel.
