


To  
the  
Supreme Personality  
Lord Shiva  
and  
My Beloved Family



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Sunil Kumar

# Abstract

The thesis focuses on the rated convergence for the dynamical systems. The rated convergence is an essential attribute for a given control strategy, for attaining finite-time, fixed-time or predefined/desired-time convergence to the equilibrium point. Most of the recent researches are focused on obtaining finite-time and fixed-time convergence. This thesis presents the results pertaining to observation and stabilization of dynamical system in predefined time. This type of convergence aims to reach the desired state in a user-defined or predefined time. It allows the control designer to set a specific time frame within which the system should achieve the equilibrium point.

First, a switched high-gain observer is designed with the desired convergence time for nonlinear systems. The proposed approach is based on a switching structure that plays an essential role, resulting in desired convergence and avoiding the observer's states' singularity. The observer's state estimates to the actual state within a desired convergence time, where the convergence time can be chosen at the will of the designer. Further, the system's gain varies linearly with the order of the system. Using the Lyapunov theorem, the stability analysis of the proposed approach is investigated. The simulation results of two practical systems: (1) Van der Pol oscillator circuit, and (2) Genesio-Tesi chaotic system, demonstrate the effectiveness of the proposed method.

Based on the aforementioned discussions, we have explored the problem of state estimation. Successful system identification often requires both state and parameter estimation. Therefore, our next task is to address the issue of parameter estimation for a class of uncertain systems.

The parameter estimation problem for a class of uncertain nonlinear systems is developed. The unknown parameters are estimated using an adaptive super-twisting algorithm in the presence of uncertainties. The estimated parameters are shown to converge to the actual parameters in finite time. The sufficient condition for the Lyapunov stability is

discussed. An explanatory example is presented for which simulation results establish the proposed estimator's satisfactory performance even under the presence of uncertainties.

The problem of designing controls for uncertain systems has garnered considerable attention in the control community. When there is uncertainty in the control affine model, it is often possible to represent the system in a polytopic form. In the upcoming problem, we will investigate the utilization of polytopic systems.

A predefined time controller is developed for nonlinear polytopic systems. With such a control, the settling time function is uniform with respect to initial conditions of the system and can be chosen by the designer. By using the control Lyapunov function, a sufficient condition is investigated for the existence of a continuous and predefined time stable state feedback controller. The obtained sufficient condition is also necessary, such that the closed-loop nonlinear polytopic system has a robust control Lyapunov function (RCLF) for all possible parametric uncertainties. Finally, the simulation results for continuous stirred tank reactor shows the efficacy of the proposed approach.

The proposed approach has potential in the area of aerospace and defense related technologies. To this end, we will now look at how rated convergence is used with respect to these application areas such as when creating missile guidance systems in situations when Line-of-Sight (LOS) rate is to be made zero within a constraints time framework.

The guidance law is developed using various approaches for planar motion of missile-target system. Firstly, a guidance law based on super-twisting algorithm with an adaptive gains is designed by using an extended state observer. With the proposed scheme, the LOS rate converges to zero within a finite time. Further, the unknown target acceleration is estimated by using an extended state observer. Secondly, an event-triggered adaptive super-twisting algorithm (ETASTA) based guidance law is proposed, where the LOS rate converges to zero. Further, a triggering condition is provided using an event-based approach that uses the minimum amount of control while meeting the stability requirements. Moreover, it is shown that the proposed theory does not exhibit the Zeno phenomenon. Lastly, a predefined guidance law is introduced, where the LOS rate to zero within a predefined time. Moreover, the convergence time of the LOS rate can be chosen by the designer in advance. The sufficient condition for Lyapunov stability analysis is established for both the cases. Finally, a practical example validates the efficacy of the proposed guidance laws for both the maneuvering and non-maneuvering target.

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# Nomenclature

## List of Greek and Roman Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_{\geq 0}$	Set of positive real numbers
$\mathbb{N}$	Set of positive numbers
$\lambda(A)$	Eigenvalue of matrix $A$
$\lambda_{\max}(A)$	Largest eigenvalue of matrix $A$
$\lambda_{\min}(A)$	Smallest eigenvalue of matrix $A$
$ \cdot $	Absolute value ( or modulus)
$\ \cdot\ $	Euclidean norm of a vector or spectral norm of a vector
$\ \cdot\ _{\infty}$	Induced $l_{\infty}$ -norm
$\forall$	For all
$\in$	Belongs to
$\Rightarrow$	Implies
$:=$	Is defined as
■	End of proof

# Abbreviations

AETSTC	Adaptive event-triggered super-twisting control
AST	Adaptive super-twisting
CLF	Control Lyapunov Function
ESO	Extended state observer
ETASTA	Event-triggered adaptive super-twisting algorithm
LMI	Linear Matrix Inequality
LOS	Line-of-Sight
MIMO	Multi-input multi-output
PE	Persistent Excitation
PN	Proportional Navigation
RCLF	Robust Control Lyapunov Function
SISO	Single-input single-output
SMC	Sliding mode control
STA	Super-twisting algorithm