Chapter 2
Literature Review

2.1. Introduction

Carbon nanotubes (CNTs) represent a cutting-edge material in the field of composite materials, with unique combinations of outstanding mechanical and thermal properties that make them ideal for a wide range of applications. From aerospace engineering to automotive design, CNTs have already proven themselves to be a promising and increasingly popular choice. By combining CNTs with polymer matrices, researchers have been able to create composite materials that exhibit even more impressive properties. This chapter presents an in-depth literature review on the development of CNT-reinforced composite plates resting on Pasternak elastic foundations, non-polynomial shear deformation theories based on secant function and inverse hyperbolic sine function. Specifically, this literature review explores the latest research on the structural responses of carbon nanotube-reinforced composite plates and sandwich structures, and discusses the different modelling approaches and solution methods that have been employed to solve the governing equations.

2.2. Development of CNTs

In recent years, carbon nanotubes (CNTs) have captured the attention of scientists and engineers due to their remarkable properties, including high stiffness, strength, and low density. First discovered by Iijima (Iijima, 1991), CNTs consist of hexagonal cells and possess an impressive tensile strength of up to 63 GPa and Young's modulus of approximately 1 TPa. Researchers have employed a variety of analytical and numerical methods to better understand and analyse the unique properties of CNTs. In material science, for instance, the weighted average has emerged as a popular method for predicting the properties of CNT-reinforced composites, such as modulus of elasticity, bulk density, tensile strength, thermal conductivity, and electrical conductivity. Micromechanical models, such as the modified Halpin-Tsai model, rule of mixture, modified rule of mixture, extended rule of mixture, Mori-Tanaka model, Eshelby–Mori–Tanaka approach, and refined rule of mixtures, have also been utilized to gain insights into the mechanics of CNT-reinforced composites. With these approaches, researchers have been able to analyse the mechanical behaviour of CNT-reinforced composites with greater accuracy. Overall, CNTs offer numerous advantages and are poised to revolutionize various industries in the near future. By unravelling the secrets of CNTs, researchers are opening up exciting new possibilities for the design and development of materials with unprecedented levels of strength, durability, and performance.

2.2.1. The mechanical behavior of carbon nanotube reinforced composites plates

Researchers have also conducted various tests to examine the mechanical behavior of carbon nanotube reinforced composites plates. For instance, CNTs were mentioned by Liu and Chen (2003) as one of the ideal reinforcement materials for the development of a novel nanocomposite due to their extremely high strength, resilience, and stiffness. In this study, a three-dimensional nanoscale RVE based on continuum mechanism and the FEM were used to evaluate the effective mechanical characteristics of CNTs-based composites. Making advantage of the extended rule of mixing, the effective Young's moduli in the axial direction of the RVE have been discovered. Furthermore, ultrasonic tests have been used to predict the mechanical behavior of carbon nanotube reinforced composites plates by Ning et al. (2003). Kreupl et al. (2004)

talked about how carbon nanotube applications are interconnected. Yeetsorn (2004) researched carbon nanotubes as a cutting-edge composite material in the form of zigzag and armchair-shaped CNTs. They discovered that the mechanical, thermal, and electrical properties of CNTs are quite good. CNT was created using a variety of techniques, including chemical vapour deposition, arc discharge, and laser ablation. Bakshi et al. (2007) conducted tensile tests on multi-walled carbon nanotube reinforced composites manufactured with electrostatic spraying, while Kulkarni et al. (2010) studied tensile properties of CNT fiber reinforced polymer composites. The vibration property of CNTRC was examined by Formica et al. (2010) using an analogous continuous model based on the Eshelby-Mori-Tanaka approach. Ma et al. (2010) showed the mechanical properties of the CNTs based composite impacted by CNTs functionalization as well as the dispersion, surface, and interfacial properties of carbon nanotubes. Yu et al. (2011) examined the characteristics of a composite made of carbon nanotubes using the precursor infiltration and pyrolysis procedure (PIP). Methyl trichlorosilane was used in the chemical vapor deposition (CVD) procedure to organize the fibre and matrix interface coating (MTS). The three-point single edge notched beam test, the bending test, and the laser flash method have all been used to quantify the impact of the CNTs on the mechanical and thermal properties of the composite. The physical and mechanical characteristics of CNTs depend on their atomic arrangement hence; computational simulation was investigated for predicting the mechanical properties of carbon nanotubes by Lu and Hu (2012). It has been used as a potent tool in light of the difficulty of the experiment. An enhanced 3D finite element model for armchair, zigzag, and chiral SWCNTs has been created using molecular mechanics. Representative volume element (RVE) based on continuous mechanism was explored by Odegard et al. (2012) for developing the relationship between structural properties of nanostructured materials. In mesoscopic and atomistic simulations, Volkov et al. (2012) examined the impact of bending and buckling analysis on the thermal conductivity of carbon nanotube materials. Grace (2013) explored many varieties of CNTs such as SWCNTs and MWCNTs as well as the geometrical arrangement of carbon atoms as armchair, chiral, and zig-zag. Zhu et al. (2013) performed uniaxial tensile tests to investigate their mechanical properties, and Laine et al. (2014) conducted 3-point bending tests to assess the transverse shear stiffness of composites sandwich polymer foam core.

2.2.2. Structural responses of CNTRC composite and sandwich plate

Various authors have also conducted static, free vibration and buckling analysis of carbon nanotube reinforced composites (CNTRC) plates. By employing the Airy stress function approach, Vodenitcharova and Zhang (2006) provided a buckling and bending study of a nano composite beam reinforced by SWCNTs. Also, it was discovered that increasing the amount of CNT reinforcement in the matrix boosted the structure's capacity to support loads. Using a higher order gradient continuum and a mesh less technique, Sun and Liew (2008) investigated the bending buckling behavior test of SWCNTs. Using the creation of a finite element formulation used for the computation of mechanical elastic response of zigzag and armchair SWCNT over a wide range of values for nanotubes radius, Giannopoulos et al. (2008) explored the calculation for shear and Young's modulus of SWCNTs. Guo et al. (2008) analysis of the bending and buckling behavior of SWCNTs using the atomic scale finite element method (FEM). Using FEM, Zhang et al. investigated the buckling reactions of CNTs. Wang and Shen (2012) conducted the nonlinear bending and free vibration analysis of CNTR sandwich plates. Lie et al. (2013) conducted the buckling analysis of the CNTR sandwich plate with FOSDT, utilizing Hamilton's Principle and the kp-Ritz Method to derive the governing differential equation and its corresponding solution, respectively. The bending, buckling, and free vibration responses of the CNTs reinforced composite (CNTRC) plate with the elastic foundation using analytical solution technique was then carried out by Wattanasakulpong et al. (2013). Zhu et al. (2013) conducted the analysis for uniformly distributed and functionally graded CNTRC plate using First Order Shear Deformation Theory and solved the problem by Finite Element Method, while Alibeigloo (2013) used Differential Quadrature Method for the similar analysis. Alibeigloo and Liew (2013) investigated thermoelastic behavior of functionally graded CNTRC plates subjected to thermal and mechanical load, while Alibeigloo (2013) and Phung et al. (2014) used Higher Order Shear Deformation Theory for isogeometrical analysis of FG CNTRC plates. Natarajan et al. (2014) conducted static and free vibration analysis for functionally graded CNTRC sandwich plates. Finally, Malekzadeh and Zarei (2014) undertook three-dimensional static and free vibration analysis for functionally graded CNTRC plates. Zhang et al. (2014) carried out nonlinear bending and dynamic analysis using First Order Shear Deformation Theory. Zhang et al. (2015) adopted Hamilton's Principle and the IMLS-Ritz Method to evaluate the buckling behavior of the CNTR plate, deriving the governing differential equation and its corresponding solution. Lie et al. (2015), Zhang et al. (2015), Zhang et al. (2015), Zhang et al. (2016), and Zhang and Xiao (2017), and Kiani et al. (2017) have conducted numerous studies that involve the analysis of the natural frequency of CNT reinforced composite plates. Through the use of Hamilton's principle, the researchers derived the governing differential equation and corresponding solution of the governing differential equation. Mirzaei and Kiani (2016) studied the buckling analysis of the plate and conical shell using the Hebyshev function method. Lie et al. (2016) employed the same FOSDT to analyze the buckling response of a CNTR sandwich plate, utilizing

Hamilton's Principle and Meshless Ritz Method to derive the governing differential equation and the corresponding solution, respectively. Kiani et al. (2017) performed a buckling analysis of a CNTR composite plate using the First Order Shear Deformation Theory (FOSDT) in conjunction with the Ritz Method, providing a corresponding solution. Kiani et al. (2017) evaluated the buckling load of the CNTR sandwich plate using the FOSDT, and employed the Gram-Schmidt Method to solve the governing differential equation. The authors concluded that the FG-X distribution of CNTs produces maximum stiffness for the buckling load. Fantuzzi et al. (2017) analyzed the free vibration of arbitrarily shaped FG CNTRC. The static and dynamic responses of the Fiber Glass (FG) Carbon Nano Tube Reinforced (CNTR) sandwich plate were studied in depth by Sciuva and Sorrenti (2019), who implemented the rule of mixture to calculate the mechanical properties of four different types of CNTR reinforcement distributions. the bending analysis of CNTRC plates carried out by Soni et al. (2020) using inverse hyperbolic shear deformation theory (IHSDT) Grover et al. (2013).

2.2.3. Theories used for the analysis of carbon nanotube reinforced composite structure

Various authors uses different theories for the analysis of the carbon nanotube reinforced composite structures. Using a micromechanical model, Shen and Zhang (2010) investigated the thermal buckling and post buckling behavior of FG-CNTRC. The authors assessed the effect of material characteristics of SWCNT using MD, the Eshelby-Mori-Tanaka technique and the extended rule of mixture. Timoshenko beam theory was used by Yas and Samadi (2012) to analyze the free vibrations and buckling behavior of FG-SWCNT resting on an elastic base. In order to predict the mechanical reaction of SWCNTs, Yan et al. (2012) investigated the buckling test of SWCNTs using a moving Kriging interpolation and the higher order Cauchy-Born rule. Ansari et al.

(2012) examined the bending behavior of single-walled silicon carbide nanotubes using density functional theory. The vibration and bending evaluations of FG-CNTRCs were given by Zhu et al. (2012) using a finite element approach based on FSDT. The buckling response of composite plate assemblies utilizing HSDT and dynamic stiffness technique was presented by Fazzolari et al. (2013). In the context of HSDT, Neves et al. (2013) investigated the stability behavior of isotropic and functionally graded sandwich plates employing a messless method. For the determination of the bending responses of a rectangular FG-CNTRCs plate that was simply supported and subjected to thermo-mechanical loads Alibeigloo and Liew (2013) used the three-dimensional theory of elasticity. Shen and Xiang (2013) looked into the post buckling of nanocomposite cylindrical shells reinforced with SWCNTs when subjected to thermo mechanical pressure. The model was created using von Karman type nonlinearity kinematics and HSDT shell theories. Lei et al. (2013) used the element-free kp-Ritz method to explore the vibration analysis of FG-SWCNT. SWCNT was strengthened into a matrix using different distribution types. According to various linear distributions of the volume percentage of carbon nanotubes, it was assumed that the material properties of FG-CNTRCs were graded across the thickness direction, and FSDT was employed as the governing equation. It also investigated the buckling process and several CNT kinds. By using finite element methodology for mechanical modelling of a SWCNT, Rangel et al. (2013) developed an analytical method to determine the elastic characteristics of SWCNTs of the armchair type. It was discovered that the mechanical properties of CNTs were exceptional. Using FSDT midplane kinematics, Mehrabadi et al. (2013) investigated the mechanical buckling behavior of a rectangular plate made of FG-CNTRCs. First order shear deformation theory (FSDT) was used by Lei et al. (2013) to analyze the buckling behavior of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates under varied in-plane mechanical stresses. The variation in the buckling strength on composite plates with volume fraction, aspect ratio, loading circumstances, width-to-thickness ratio and environment temperature were obtained for composite using rule of mixture or Eshelby-Mori-Tanaka technique. Wu and Chang (2014) studied the stability analysis of the FG-CNTR plate bonded with PZT layers under bi-axial compression loading. Malekzadeh et al. (2015) utilized the finite element–based on the strain gradient theory (FOSGT) to analyze the natural frequency of the CNTR composite plate. Hamilton's principle and Newmark's time integration method were employed to derive the corresponding governing differential equation and solution, respectively. Zhang et al. (2015) and Lie et al. (2016) employed the Föppl–von Kármán strain displacement transformation field. Additionally, Zhang et al. (2015) and Lie et al. (2016) utilized the Pasternak and Reddy's displacement fields, respectively. Zhang et al. (2015), Lie et al. (2016), Kiani et al. (2017), and Zhang and Xiao (2017) concluded that the natural frequency of the CNTR composite plate is directly proportional to the volume fraction of CNTs. Furthermore, Zhang et al. (2015) and Lie et al. (2016) employed the improved IMLS-Ritz method and Levy solution method, respectively, to derive the governing differential equation and corresponding solution of the governing differential equation. Numerous studies have been conducted on the vibration and buckling behaviors of functionally graded carbon nanotube reinforced (FG-CNTR) composite plates over the years. Zhang et al. (2016) concluded that the non-dimensional natural frequency is maximal for the FG-X distribution among all other distributions selected for the analysis. Wu and Li (2016) further examined the free vibration and stability analysis of the FG-CNTR plate sandwiched between surfaces bonded with piezoelectric transducers (PZT) for open and closed-circuit conditions via the Quasi-3D approach. Mohammadimehr and Mostafavifar (2016) investigated the behavior of the FG-CNTR sandwich plate with homogenous core under magnetic intensity and temperature variation, while Zhang and Xiao (2017) used the Mori-Tanaka approach. Sciuva and Sorrenti (2019) employed the extended refined zigzag theory (eRTZ) to analyze the bending, buckling and free vibration of the FG-CNTR sandwich plate with various stacking sequences. Civalek et al. (2020) evaluated the buckling and free vibration response of the laminated composite plate using the FOSGT in conjunction with discrete singular convolution (DSC). Civalek and Avcar (2020) studied the free vibration and buckling response of the non-rectangular CNTR laminated plate. Finally, Adhikari and Singh (2020) conducted buckling analysis of the FG-CNTR laminated plate using polynomial based high order shear deformation theory (HOSDT) and finite element method (FEM) as the solution technique. Singh and Sahoo (2020), Singh and Sahoo (2021), and Singh and Sahoo (2021) investigated the static and free vibration analysis of a carbon nanotube (CNT) reinforced sandwich plate by leveraging inverse hyperbolic shear deformation theory (IHSDT).

2.2.4. Computational modelling techniques for the analysis of carbon nanotube reinforced composite structure

Over the years, researchers have employed a range of computational modelling techniques to investigate the impact of carbon nanotube (CNT) reinforcement on the elasticity, plasticity, and buckling response of nanocomposites. For example, Additionally, Chen and Liu (2003) used finite element modelling of CNT composites based on 3D approaches to calculate their effective mechanical response Georgantzinos et al. (2008) used finite element modeling to analyze rubber-like materials reinforced with single-walled CNTs and predict the macroscopic mechanical behavior of the CNT-reinforced structures. Montazeri and Naghdabadi (2010) employed molecular dynamics to study the composition of carbon nanotube composites. Farsadi et al. (2012) modeled carbon nanotube composites reinforced with corrugated CNTs using a 3D approach. Malekzadeh and Shojaee (2013) used first-order shear deformation theory to analyze quadrilateral laminated thin or moderately thick sheets to investigate the buckling response. Shahrbabaki and Alibeigloo (2014) conducted a finite element analysis of carbon nanotubes modeled as 1D, 2D and 3D structures, exploring the effects of nanotube distribution and volume fraction on the in-plane frequency of rectangular plates. Mayandi and Jeyerad (2015) used finite element simulation to model the vibrational, bending, and buckling behavior of an FG-CNT reinforced polymer composite beam under thermal conditions.

2.2.5. Carbon nanotube reinforced composite structure resting on the elastic foundation

The research into the use of CNTs as a structural component was pioneered by Ajayan et al. (1994), who found them to produce outstanding results. This sparked further research into the practical applications of CNTs Odegard et al. (2003), Griebal and hamaekers (2004), and Mokashi et al. (2007). Further work in the Timoshenko beam theory by Yas and Samadi (2012) focused on the vibration analysis of CNTRCs with Winkler-Pasternak elastic foundations. Again, different reinforcement distribution patterns, volume fractions and side-to-thickness ratios were tested, and the same FG-X reinforcement distribution pattern provided the highest frequencies. Kutlu and Omurtag (2012) also conducted studies on the vibration response of the angular CNTRC plate with elastic foundation sandwich and the bending response of the elliptical CNTRC plate with elastic foundation. Wattanasakulpong and Ungbhakorn (2013) conducted static analysis on carbon nanotube reinforced composites (CNTRC) beams using Winkler-Pasternak elastic foundations, exploring the effects of different reinforcement distributions, volume fractions, and side-to-thickness ratios. Results suggested that a functionally graded (FG)-X reinforcement distribution pattern provided the greatest stiffness. Shen and Xiang (2013) then explored the non-linear behavior of CNTRCs with Winkler elastic foundations in the context of high order shear deformation theory (HSDT). The use of carbon nanotube reinforced composite (CNTRC) materials in engineering structures has been increasing in recent years due to their superior mechanical and physical properties. However, their complex microstructure and reinforcement distribution patterns pose a challenge to the analysis and design of such structures. Researchers have conducted various types of analyses to study the effect of reinforcement distribution patterns of CNTs, their volume fraction, Winkler spring constant factor, and side-to-thickness ratios on the static and dynamic behavior of CNTRC structures. For instance, Shen and Xiang (2014) studied the non-linear behavior of CNTRC beams with Winkler elastic foundation for different reinforcement distribution patterns across the beam's thickness. They found that the FG-A reinforcement distribution pattern produced the highest transverse deflection, whereas the FG-X reinforcement distribution pattern produced the lowest transverse deflection. Similarly, Shen and Xiang (2014) studied the effect of temperature on the dynamic analysis of CNTRC shells with Winkler elastic foundation. They found that increasing the volume fraction of CNTs in the CNTRC shell and Winkler spring constant factor had a positive effect on their dynamic response, whereas increasing the temperature of the system had a negative effect. In the framework of the element-free IMLS-Ritz formulation, Zhang et al. (2015) studied the nonlinear buckling behavior of CNTRC plates with Winkler elastic foundation. Their analysis revealed that the volume fraction of CNTs, Winkler spring constant factor, and side-to-thickness ratio had a significant effect on the buckling behavior of the CNTRC plates. Overall, these studies demonstrate that the reinforcement distribution pattern of CNTs, their volume fraction, and Winkler spring constant factor have a significant effect on the static and dynamic responses of CNTRC structures. Zhang et al. (2015) investigated the static response of CNTRC plates with Winkler elastic foundations using the element-free IMLS-Ritz method. Wattanasakulpong and Chaikittiratana (2015) examined the bending, buckling, and free vibration response of the CNTRC plate with Pasternak elastic foundations, which consist of Winkler springs and shear layers. The analysis was conducted for different reinforcement distribution patterns of CNTs, volume fractions of CNTs, Winkler spring constant factors, shear layer constant factors, and side-to-thickness ratios. The results showed that the FG-X reinforcement distribution of CNTs in the composite plate with Pasternak elastic foundation was the most effective at countering bending reactions and producing the minimum transverse deflection. Meanwhile, Zhang and Liew (2015) investigation of the non-linear bending analysis of the CNTRC skew plate with Pasternak elastic foundation revealed that the skew angle, volume fraction of CNTs in the CNTRC plate, Winkler spring constant factor, and shear layer constant factor all had an influence on the plate's bending response. Lei et al. (2015) and Lei et al. (2016) then studied the buckling and vibration response of the CNTRC plate with Pasternak elastic foundations, in the framework of the first order shear deformation theory (FSDT). The analysis was also conducted for various reinforcement distribution patterns, volume fractions, Winkler spring constant factors, shear layer constant factors, and side-to-thickness ratios. Their results showed that the different parameters had an effect on buckling and free vibration response of the plate. The study of the CNTRC plate has yielded some interesting outcomes about its buckling and free vibration response. It was found that the volume fraction of CNTs in the CNTRC plate, Winkler spring constant factor, and shear layer constant factor all had an impact on the buckling and free vibration response. Keleshteri et al. (2017) conducted studies on the vibration response of the angular CNTRC plate with elastic foundation sandwich and the bending response of the elliptical CNTRC plate with elastic foundation. Duc et al. (2017) focused on the buckling response of the conical shell with elastic foundation.

The studies discussed suggest that the distribution of CNTs in the composite plate according to the FG-X reinforcement pattern can reduce transverse deflection when countering bending reactions. Additionally, the IHSDT is deemed effective in predicting both static and dynamic responses of the composite plate, as per the authors' findings.

2.3. Development of plate theory

Plate theories are the sophisticated tools to analyse and build thin and flat structures, which has revolutionised the discipline of structural engineering. Plate theory has advanced significantly from its early stages to the current state of the art, allowing for numerous deformation modes and loading situations. Engineers may push the bounds of innovation and build safer, more effective structures that form our modern world by continually developing and improving plate theory. The study and design of thin, flat structures were revolutionised by the plate theory, a basic idea in structural engineering. Plate theory has been crucial in the growth of several engineering fields, including civil, mechanical, and aerospace engineering, from the late 19th century to the present. The purpose of this article is to examine the historical development and significant advances of plate theory while stressing its significance for structural design. The early activities of mathematicians and engineers who wished to comprehend the behaviour of flat structures can be linked to the development of plate theory. The development of plate theory was made possible by the pioneering work of Augustin-Louis Cauchy, Leonhard Euler, and Daniel Bernoulli. These researchers created the first conceptual basis for further developments by deriving mathematical equations that explain the bending and deformation of plates under applied stresses. Plate theory is continually changing as academics work to improve current models and create fresh approaches. The development of cutting-edge computer methods like artificial intelligence and machine learning provide prospects to improve plate analysis and design even more. Plate theory has the ability to significantly progress by integration with other fields, such as materials science and optimisation methods, allowing engineers to more successfully address challenging engineering problems.

Structural analysis is a crucial stage in the design of beams, plates, and shell structures. The first step in this process is deriving the governing equation for physical phenomena such as bending, vibration, and buckling. This equation is typically in the form of partial differential equations (PDEs) or ordinary differential equations (ODEs). The obtained equation can be exact or approximate, depending on the level of numerical error. An exact analytical formulation involves no assumptions and is free from numerical error, while an approximate formulation involves approximations and assumptions to model the structure's displacement and stress variations. These assumptions are made to simplify the mathematical model and make it more economical. Once the governing equation is obtained, the next step is to find solutions. This can be done through closed-form analytical solutions or numerical solutions. Closed-form analytical solutions do not add additional error to the results since the solution is assumed to satisfy support boundary conditions and be valid at all points in the domain. The results obtained through numerical solutions methodology introduce additional errors because the solutions are obtained at discrete points as opposed to the continuous solution obtained through analytical solutions methodology. The discrete values of the field variables are then interpolated using interpolation functions to obtain the magnitude of the field variables between the selected points. Numerical solutions methodology is preferred to make the solution scheme mathematically economical, especially in cases where it is difficult to find closed-form analytical solutions for problems. A wide range of literature exists that deals with the deformation of composites beams, plates, and shell structures, as well as CNTRC beams, plates, and shell structures, along with different solution methods for solving governing equations. The literature review is divided into two parts. The first part deals with the derivation of governing equations for physical phenomena such as bending, vibration, and buckling, in the form of PDEs or ODEs. The second part deals with the solution methodology, beginning with elasticity solutions followed by modelling of plates using plate theories.

2.3.1. Elasticity Solutions (3 D)

The exact analytical formulations of the elasticity solution are free from numerical errors, as no assumptions are made in the modelling of the deformation responses of beams, plates, and shell structures. In the elasticity solutions3D equilibrium equations (EE) of elasticity is adopted for the deformation modelling of responses for beams, plates, and shell structures. The three equilibrium equations (EE) of elasticity are deemed ineffective as they are linked with six unknown stresses. In order to obtain a determinate analysis, the three equilibrium equations (EE) of elasticity are transformed into strains using the constitutive relationship, and then the resulting equations are converted into displacements using the strain-displacement relationship. Now we have 15 unknowns which are associated with 15 equations which make the problem determinate. The resulting partial differential equations for unknown displacements are then solved using mathematical solutions for the given set of boundary conditions. The exact analytical formulations based on elasticity solutions for bending and vibration analysis of laminated composites and sandwich plates is proposed by Pagano (1970), Pagano and Hatfield (1972), Srinivas and Rao (1970), Noor (1973) and Bhaskar*et al*. (1996).The absence of assumptions in the exact analytical formulations ensures that they are free from numerical errors. The use of closed-form analytical solutions methodology further eliminates the possibility of additional errors in the results. This is because the solution is assumed in a manner that satisfies the support boundary conditions and remains valid at all points in the domain. The references which are mentioned above, make use of the trigonometric functions in the spatial domain (x, y) as closed-form analytical solutions which are assumed for the primary variables and exactly satisfy the diaphragm-supported boundary conditions. A part from the elasticity solutions, there are semi-analytical solutions or pseudo 3D solutions for the bending responses of simply-supported traditional laminated composites. The semi-analytical solutions or pseudo 3 D solutions are proposed by Kant *et al*. (2007, 2008), Pendhari*et al*. (2012), Sawarkar*et al*. (2016, 2020) and Lomte Patil*et al*. (2018).As discussed above the formulation of analytical solutions consist 15 displacement equations, which make the problem a determinant equation. The resulting PDEs are converted to a system of six first-order ODEs in the thickness direction associated with the primary variables. In semi-analytical solutions or pseudo 3D solutions, three displacements and the three transverse stresses are considered as the primary variables.

2.3.2. Modelling of plates using Plate Theories

The development of various plate theories was inspired by the complex theoretical formulations and solution methods associated with the 3D approach. In different plate theories, the primary unknowns are the deformation modes defined at the midplane. The 3D displacement components (U, V and W) are expressed as known mathematical functions of the thickness coordinate and unknown deformation modes defined at the midplane. Once the deformation modes are determined, the stresses and strains at any point in the thickness domain can be calculated.

2.3.2.1. Plate theories

Plate theories rely on the crucial assumption that the plate's thickness is significantly smaller compared to its in-plane dimensions. This assumption allows for the derivation of plate theory based on the study of thin plates. The following assumptions are typically employed in the development of plate theory.

- Internal 3D displacements (*U*, *V*) vary linearly with the plate thickness. This assumption allows the 3D displacements of any arbitrary point *P* to be expressed in terms of a constant stress mode at depth *z* of the midplane and a rotation mode specified in the midplane, which is related to a linear function of the thickness coordinate (*z*).
- Transverse displacement (*W*) is constant through the plate thickness. This simply means that the length of the normal cross does not change in the thickness direction.

With these assumptions, we can relate the 3D displacement components in the following way.

$$\begin{bmatrix} U(x,y,z,t) \\ V(x,y,z,t) \\ W(x,y,z,t) \end{bmatrix} = \begin{bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_0(x,y,t) \end{bmatrix} + z \begin{bmatrix} \theta_x(x,y,t) \\ \theta_y(x,y,t) \\ 0 \end{bmatrix} \qquad 2.1$$

where u_0, v_0, w_0, and θ_x, θ_y are the displacement modes and rotational deformations at the mid plane. The equation given in Eq. (1.1) represents the deformation profile of the first-order plate theory, which is discussed in detail in the following sections.

2.3.2.2. Classical Plate theory

The deformation responses of thin plates under the action of forces can be predicted using the Classical plate theory, which is based on 2D analysis. The Classical plate theory is an extension of the Euler-Bernoulli beam theory in conjunction with the Kirchhoff hypothesis. The Kirchhoff hypothesis is based on the following assumptions.

- Straight lines perpendicular to the mid-surface (transverse normal) shall remain straight under the deformation of the plate.
- The transverse normal does not undergo any change in length under the deformation of the plate.
- The transverse normal rotate in a fashion such that it shall remain perpendicular to the midplane under the deformation of the plate.

The first two assumptions mentioned above have already been introduced earlier in the Eq. (1.1). As per the first assumption the 3D displacements of any arbitrary point *P* to be expressed in terms of a constant stress mode at depth (z) of the midplane and a rotation mode specified in the midplane, which is related to a linear function of the thickness coordinate (z) and to the second assumption discusses the transverse displacement (W) is constant through the plate thickness. This simply means that the length of the normal cross does not change in the thickness direction. The third assumption establishes a relationship between the transverse deformation and the rotational modes. The third assumption has a mechanistic interpretation that the transverse shear strains are zero because the angle between the transverse normal and the midplane remains constant during deformation.

The transverse shear strains based on Eq. 1.1 are shown below.

$$\gamma_{xz} = \left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \right) \qquad 2.2a$$

$$\gamma_{xz} = \theta_x + \frac{\partial w_0}{\partial x} \tag{2.2b}$$

$$\gamma_{YZ} = \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) \tag{2.2c}$$

$$\gamma_{YZ} = \theta_y + \frac{\partial w_0}{\partial y} \tag{2.2d}$$

According to the last assumption, $\gamma_{YZ} = \gamma_{YZ} = 0$ so, the Eq. (2.2a - 22.d) is modified as

$$\theta_x = -\frac{\partial w_0}{\partial x} \tag{2.3a}$$

$$\theta_y = -\frac{\partial w_0}{\partial y} \tag{2.3b}$$

Now, using Eq. (2.3a - 2.3b), the modified form of Eq. 2.1 is presented below

$$\begin{bmatrix} U(x,y,z,t) \\ V(x,y,z,t) \\ W(x,y,z,t) \end{bmatrix} = \begin{bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_0(x,y,t) \end{bmatrix} + z \begin{bmatrix} \dfrac{\partial w_0(x,y,t)}{\partial x} \\ \dfrac{\partial w_0(x,y,t)}{\partial y} \\ 0 \end{bmatrix} \tag{2.4}$$

The Eq. (1.4) represents the displacement field for the Classical Plate Theory. Ashton and Whitney (1970), Reissner and Stavsky (1961), Stavsky (1961), Dong *et al*. (1962) and Yang *et al*. (1966)used the Classical Plate Theory for modelling and analysis of the deformation responses of laminated composite plates. The mathematical development by classical plate theory as mentioned above is valid only for behaviour analysis of thin plates only. In the case of thick isotropic plate structures, the transverse shear strains are significant and hence discarding the effects of the transverse strains will lead to erroneous responses.

In the case of traditional laminated composites and sandwich plates, the ratio of Young’s modulus to the transverse shear modulus is very high. Therefore, it becomes essential to include the transverse shear strains for understanding the real behaviour of composite plate structures.

The first-order shear deformation theory (FSDT) is the refinement of CPT which considers the effects of transverse shear strains without many theoretical complexities. (Whitney and Pagano (1970), and Librescu and Reddy (1987).

2.3.2.3. First Order Shear Deformation theory

The assumptions of CPT stated above are also valid for the First Order Shear Deformation theory except the third assumption which constraints the transverse normal to rotate in a fashion so that it remains perpendicular to the midplane during the deformation of the plate. Whitney (1973), Noor and Burton (1989), and Meunier and Shenoi (1999) used First Order Shear Deformation theory for the analysis of the plate structure.

$$\begin{aligned} U(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) \\ V(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) \\ W(x,y,z,t) &= w_0(x,y,t) \end{aligned} \qquad 2.5$$

2.3.2.4. Higher-Order Shear Deformation theories

The HSDTs are refinements of the CPT and FSDT which try to bridge the gap between the mathematical complexities in obtaining the exact responses from the 3D formulations and the unsatisfactory performances of the CPT and FSDT for isotropic and traditional composite plate structures.

In the literature, the HSDTs are available in the form of polynomial-based higher-order theories (PHSDTs) and non-polynomial-based higher-order theories (NHSDTs). The foremost difference among the two classes of theories is the use of polynomial and non-polynomial mathematical functions for expressing the displacement components in terms of the deformation modes.Touratier (1991), Soldatos (1992), Aydogdu (2009), Mieche et al. (2011), and Mahi et al. (2015) adopted Higher-Order Shear Deformation theories for the structural analysis of the plate structure. The non-polynomial-based higher-order shear deformation theory contains various non-polynomial shear strain functions such as secant function, sine function, tangent function, etc. to introduce the non-linearity of transverse shear stresses through thickness at the cost of a smaller number of field variables with respect to the HSDT's available in the literature which are generally of polynomial nature. FSDT does not have the required deformation modes to model thick CNT reinforced sandwich plates and is usually preferred to study the thin ones where shear deformation is not dominant. While the higher-order deformation modes (membrane and bending) are present in the polynomial based HSDT's, yet their inclusion is only possible with a large number of higher-order terms which increases computational costs. In trigonometric shear deformation theory, the non-linearity of shear deformation is accommodated with the aid of a single non-polynomial function 'secant function' in the kinematic field. Hence, efficient results are obtained at the cost of lesser computational efforts. Next, the non-polynomial-based higher-order shear deformation theory is inherently satisfying the traction free conditions of transverse shear stresses at the top and bottom surfaces of the plate while in most of the polynomial based HSDT's, these conditions are generally not considered and at some cases they are artificially enforced.

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y,t) \\
w(x,y,z,t) &= w_0(x,y,t) \\
f(z) &= g(z) + \Omega\, z
\end{aligned}
\tag{2.6}
$$

The first order shear deformation theory (FSDT) retain the following demerits which is originates from the assumptions taken while developing the plate model

- Straight lines perpendicular to the mid-surface (transverse normal) shall remain straight under the deformation of the plate
- The transverse normal does not undergo any change in length under the deformation of the plate.

FSDT model considers transverse shear deformation to be constant along the thickness direction. As a result, the responses of thick and moderately thick plates are not efficient. Also, FSDT requires a shear correction factor (SCF) to be multiplied with the transverse shear stiffness coefficients to improve the responses. The values of SCF are found out using an in-direct post-processing technique by comparing the responses obtained from FSDT with elasticity results. Therefore, in problems where elasticity solutions are not available, then such an approach would be difficult to follow,

The next plate models in the hierarchy are the higher-order shear deformation theories (HSDTs). There are numerous HSDITs in the literature that completely or partially discard the assumptions of the classical laminated plate theory. Koiter (1960) made a recommendation popularly known as the Koiter's recommendation (KR) in his article of 2 D modelling of traditional isotropic elastic shells, which is stated as “a refinement of Love's first approximation theory is indeed meaningless in general unless the effects of the transverse shear and normal strains (stresses) are taken into account at the same time”. There are numerous HSDTs to date which account for the effects of transverse shear strains only or both transverse shear and normal strains in the kinematic model that partially or completely follow the KR. respectively. The models which do not take into consideration the effects of transverse normal strains are not able to accurately model the problems of thermoelasticity. The thermal loads act like body forces on the plates and. produce stretching or contraction of normal. In such problems, the transverse normal strains must be accommodated in the models to model the deformations.

However, problems of plates that are not exposed to any thermal effects can be efficiently modelled using HSDTs which do not consider transverse stretching of normal Initially, much of the works have been done on HSDTs which are polynomial theories based on Taylor's series expansion. Contributions are made by Nelson and Lorch (1974), Lo et al. (1977), Levinson (1980), Kant (1982), Reddy (1984), to name a few for developing HSDTs of various orders. The models in Kant (1982), Lo et al. (1977), and Nelson and Lorch (1974) consider the effects of both transverse shear and normal deformations. However, the traction-free boundary conditions of transverse shear are not satisfied at the top and bottom surfaces of the plates. The displacement field in the models of Levinson (1980) and Reddy (1984) satisfies the traction-free conditions, however, the models do not account for transverse normal deformation. The displacement fields in Levinson (1980) and Reddy (1984) are the same, however, the governing equations are different in both works. The governing equation in Levinson (1980) is based on the FSDT which is variationally inconsistent Reddy (1984) produced the correct governing equations and boundary conditions based on the energy principle. The mathematical function (z-($4z^3/3h^2$)) in Reddy (1984) obtained after incorporating the transverse shear stress-free conditions, is considered as a basis for developing more rich functions. Such studies are presented in Touratier (1991), Soldatos (1992), Aydogdu (2009), Karama et al. (2009), Mieche et al. (2011), Mantari et al. (2012), and Mahi et al. (2015) to name a few, in which new mathematical functions of non-polynomial type have been proposed which implicitly accommodate the higher order expansions of polynomial-based HSDTS. For instance, if we compare the functions of Reddy (1984) and Soldatos (1992), we would get the following expansions:

Reddy (1984): $\left(z-\frac{4z^3}{3h^2}\right)=z-\frac{4z^3}{3h^2}$

Soldatos (1992): $\frac{h}{\pi} sin\left(\frac{\pi z}{h}\right) = z - 1.645\frac{z^3}{h^3} + 0.812\frac{z^5}{h^4} - 0.191\frac{z^7}{h^6}$

The above expansion reveals that the non-polynomial function in Soldatos (1992) is inherently considering all the odd-powered terms of "z" which refines the bending deformations of the plate. Additionally the traction-free conditions of transverse shear are also inherently satisfied with the function. The advantages of using non- polynomial models are mentioned below:

1. Computational costs are not high as most of the non-polynomial models have the same degrees of freedom like the FSDT.

2. The shear-strain functions are richer and implicitly consider the higher-order expansions of in the model.

3. Inherently satisfies the traction-free conditions of transverse shear.

2.3.3. Extension of the plate theories for the modelling of multi-layered structures

The following section provides a detailed discussion of the literature pertaining to the modelling of the behaviour of multi-layered composite plates.

2.3.3.1 Equivalent Single Layer (ESL) Approach

The Equivalent Single Layer approach provides the simplest way of extension single-layered plate theories to multi-layered structures without many computational complexities. In this approach, the total number of primary variables required to model the multi-layered systems are kept constant and are equal to the total number of field variables in the original kinematic model. Chandrashekhara and Agarwal (1993), Ray and Mallik (2004), Ray and Sachade (2006), and Shivakumar and Ray (2008) had used the Equivalent Single Layer approach to analyse the plate structure.

2.3.3.2. Layer wise (LW) approach

In the LW approach, separate kinematic field expansions are assumed foreach individual layer of the multi-layered laminated composite plate. In various kinematic models can be CLPT, FSDT and HSDTs of both polynomial and non-polynomial form assumed in each discrete layer. Kapuria et al. (2003), Garção et al. (2004), Lage et al. (2004), Kapuria and Hagedorn (2007), Beheshti-Aval et al. (2013), Naji et al. (2016, 2018) and Li and Shen (2018) used the Layer wise (LW) approach to analysis the plate structure.

2.3.3.3. Zigzag (ZZ) Approach

The ZZ approach is agreeable to ESL which considers same procedure of fixing the total number of primary variables but it is contrary to LW approach. In this approach to generate the slope discontinuities of the displacement components at the interfaces, some unknowns are assumed at the interfaces of the multi-layered laminated composite pate structures in addition to the usual kinematic representation followed in the ESL approaches. The unknowns are related to the 3D displacement components with a piecewise linear interpolation function of the thickness coordinate and are useful to create discontinuities of transverse shear strains. Tessler et al. (2010), Gherlone (2013), Iurlaro et al. (2013), Sahoo and Singh (2013), Sahoo and Singh (2014) and Flores (2014), Nath and Kapuria (2009, 2012), Mishra et al. (2019) and Das and Nath (2021) used the ZZ approach to analysis the plate structure.

2.4. Solution Schemes

In this section, various solution techniques are discussed which are used to determine the solutions of the governing equations of plate analysis. The Navier-based analytical method is very popular in the research community as it gives exact solutions of the governing equations of beams/plates and shell structures with diaphragm supported boundary conditions. There are several articles in the literature like Kulkarni*et al*. (2015), Punera and Kant (2017), Singh and Sahoo (2020) and Soni*et al*. (2020) in which the Navier-based solutions are obtained for the static and dynamic analysis of multilayered composites, FG and CNT-reinforced plate and shell structures. Navier's solution is restricted to diaphragm supported boundary conditions only. Other analytical solution methods like the Galerkin method (Singh and Harsha(2019) and Daikh and Zenkour (2020), power series solution method Shariyat and Alipour (2013), and Ritz method (Aydogdu (2005) and Nguyen *et al*. (2017)) are available in the literature in which solutions for different boundary condition are obtained analytically. A levy-type boundary condition is assumed in one direction and the assumed solutions for the primary variables reduce the system of PDEs to a system of higher-order ODEs in the other direction. Then the ODEs are further converted to a system of 1st order ODEs using the state-space approach and then solved to get the responses. There are many popular numerical approaches adopted in the literature for solving the governing equations of beams, plates, and shell structures. The main principle of any numerical approach is to reduce the governing PDEs and ODEs to a system of algebraic equations by making some approximations. This reduction helps to replace a continuous differential equation having a solution space that is infinite-dimensional with a finite system of algebraic equations whose solution space is now finite-dimensional. To begin with, the very popular and commonly used method in almost all disciplines of science and engineering, the finite element method (FEM). In the FEM, the field variables are assumed over an element as a linear combination of the polynomial shape functions and the nodal coordinates. The strong form of the governing equations is converted to an equivalent weak form and the assumed solutions are plugged in the weak form to get the elemental level equations. The elemental equations are then assembled to get the global discretized equations of the problem and then solved for the primary variables. Research works on structural analysis of homogeneous and non-homogeneous structures are available in Talha and Singh (2010), Bhar*et al*. (2010), and Sarangi*et al*. (2014). The

Extended Finite Element Method (XFEM) is another numerical method based on the FEM and it is especially used to treat crack discontinuities. The background of XFEM is the partition of the unity concept. By taking the advantage of the partition of unity concept, the FE approximation space isenriched with some enrichment functions and extra degrees of freedom in the nodes near the cracks. Natarajan*et al*. (2011), Nguyen-Vinh (2012), and Nasirmanesh and Mohammadi (2015) employed the XFEM to predict the dynamic responses and buckling of cracked composite and FG plates. The conventional FEM requires that the mesh is frequently refined as the crack propagates in a crack propagation problem along with a conforming mesh. This is a limitation that is posed by the FEM when crack propagation problems are studied. As discussed above, the enrichment technique such as the XFEM proves to be very useful because the method does not require conforming mesh and mesh adaptation when the discontinuities, *i.e*, crack propagates. However, the enrichment functions near the cracks should be known a priorly. Like any other numerical approach, the governing differential equations are transformed to a set of algebraic equations in terms of the discrete values of the field variables in DQM. This is achieved by expressing the derivatives of the primary variables at each grid point in a particular direction as the weighted linear sum of the values of the primary variables at all the discrete points in the same direction. The Discrete Singular Convolution (DSC) approach is employed by Civalek (2007, 2017) for the free vibration analysis of laminated composites and FG plates and shells. The method is somewhat identical to the DQM, and in DSC also, the derivatives of the primary variables at a grid point are approximated by a linear sum of the discrete values of the primary variables and approximation kernels in a narrow bandwidth.

2.5 Critical Observation from Literature Review

This review of the literature assesses the present knowledge of carbon nanotube (CNT) reinforced composite plates critically and offers information on their manufacturing processes, mechanical characteristics, interface characterisation, and possible applications. By examining the body of research that has already been done, it is clear that CNT-reinforced composite plates offer notable improvements in mechanical strength, stiffness, thermal conductivity, and electrical conductivity, making them appropriate for a variety of applications. To fully use the promise of these materials, however, issues including obtaining uniform nanotube dispersion, assuring strong interfacial bonding, scalability, and cost-effectiveness must be resolved. Future work should concentrate on standardising production processes, examining the effects on the environment and human health, evaluating the integration with other nanomaterials, and determining the commercial feasibility of CNT-reinforced composite plates. Overall, the critical findings from this literature analysis support the creation of novel, high-performance composite materials and serve as an invaluable basis for future research in the subject.

2.6. Motivation and Literature Gap

After reviewing the literature extensively, it is observed that no research has been conducted on the structural analysis of functionally graded carbon nanotube reinforced composite and sandwich plates using the non-polynomial shear deformation theory, based on secant function and inverse hyperbolic sine functions, as of yet. The trigonometric shear deformation theory, based on secant function and inverse hyperbolic sine functions employed in this study adopts a non-polynomial shear strain function, specifically the secant and inverse hyperbolic sine functions, to account for the nonlinearity of transverse shear stresses through the plate thickness. This approach utilizes fewer field variables compared to other higher-order shear deformation theories (HSDTs) available in the literature, which are generally of polynomial nature. While the first-order shear deformation theory (FSDT) is commonly used for thin plates where shear deformation is negligible, it lacks the necessary deformation modes to model thick carbon nanotube reinforced composite and sandwich plates. HSDTs, on the other hand, require a large number of higher-order terms to incorporate the membrane and bending deformation modes, resulting in increased computational costs. In contrast, the trigonometric shear deformation theory incorporates the nonlinearity of shear deformation with a single non-polynomial function, the secant function, in the kinematic field, resulting in more efficient results at a lower computational cost. Additionally, the trigonometric shear deformation theory inherently satisfies the traction-free conditions of transverse shear stresses at the top and bottom surfaces of the plate, unlike most polynomial-based HSDTs that often do not account for these conditions or artificially enforce them. Therefore, in this study, the trigonometric shear deformation theory is utilized to model functionally graded carbon nanotube reinforced composite and sandwich plates, enabling the evaluation of structural responses using both analytical and finite element modelling approaches.Carbon nanotube reinforced distributions are selected such as uniformly distribution (UD) and three types of functionally graded (FG) i.e. FG-O, FG-X and FG-V which are considered for the analysis. In the analytical approach Hamilton’s principle is used to develop governing differential equations and then solved using Navier’s solution technique. The analytical approach is used to find the deflections, stresses, buckling load, natural frequency and corresponding mode shapes of functionally graded carbon nanotube reinforced composite and sandwich plates for different span thickness ratios, distributions of carbon nanotube and loading conditions.The finite element method is the numerical solution-based approach which discretized the functionally graded carbon nanotube reinforced composite and sandwich plate in the "*n*" number of parts using an eight noded isoparametric serendipity biquadratic quadrilateral element. Trigonometric shear deformation theory based on secant function introduces two new degrees of freedom, resulting in a total of seven degrees of freedom. This means that each node is associated with seven degrees of freedom. In the case of an eight-noded isoparametric serendipity biquadratic quadrilateral element, the degree of freedom is considered as 56. The primary variables associated with the problem are transformed in the terms of shape functions and generalized nodal coordinates. The ESL models exhibit discontinuous stress values at the interfaces due to the through-thickness variations of transverse shear stresses, which are inconsistent with the 3D variations of transverse shear stresses. The reason behind this is that the ESL models assume global functions of thickness coordinate for the entire thickness of the laminated composite plates in the kinematic expansions. The constitutive relations are used to predict the transverse shear stresses.

Structural analysis of advanced composite plates supported on an elastic foundation is an important area to understand the complex soil–structure interaction behavior. Analysis of structures like the raft foundations, swimming pools and storage tanks requires the analysis of plates resting on elastic foundations. The responses of these structures under various loading conditions are dependent on the elastic medium supporting the structures. Therefore, it is important to understand the interaction between the soil and structures to enhance structural safety and to provide a reliable design.On the basis of the literature review, the bending, buckling and free vibration responses of the functionally graded carbon nanotube reinforced composite plate and sandwich structure resting on Pasternak's elastic foundation is not modeled in the framework of the trigonometric shear deformation theory based on secant and inverse hyperbolic sine functions till now. The present study includes the bending, buckling and free vibration responses of the functionally graded carbon nanotube reinforced composite plate and sandwich structure resting on Pasternak's elastic foundation in the framework of trigonometric shear deformation theory based on the secant function and inverse hyperbolic sine function. The analysis is conducted for the different reinforcement distribution pattern of CNTs across the thickness of the CNTRC plate, volume fraction of CNTs in functionally graded carbon nanotube reinforced composite plate and sandwich structure resting on Pasternak's elastic foundation, Winkler spring constant factor, shear layer constant factor, and side to thickness ratio. Further, the buckling analysis of the composite plate is investigated for the uniaxial and bi-axial compressive loading condition. The presented mathematical model and results are verified numerically by comparing with published results in the literature.

2.7. Objective and Scopes of the Present Work

Based on the observations from the previous section, the present work aims to accomplish the following key objective:

Objective

To develop an analytical and efficient finite element model for the structural analysis of functionally graded carbon nanotube reinforced composite and sandwich plates resting on Pasternak's elastic foundation in the framework of different non-polynomial shear deformation theories.

Scopes

- Development of an analytical model for the structural analysis of the functionally graded carbon nanotube reinforced composite and sandwich plates using different non-polynomial shear deformation theories with different trigonometric functions which include inverse hyperbolic sine function and secant function.

- Investigation of the bending, free vibration, and buckling analysis of carbon nanotube reinforced composite and sandwich plates using analytical approach based on above non-polynomial mathematical models.
- An efficient C^0 finite element implementation for the structural analysis of functionally graded reinforced composite and sandwich plates in the framework of different non-polynomial shear deformation theories which incorporate the inverse hyperbolic sine function and secant function for modelling the non-linearity of transverse shear stress through the thickness of the plate.
- Investigation of the bending, free vibration, and buckling analysis of carbon nanotube reinforced composite and sandwich plates based on the developed FE model using different theories.
- Development of an analytical and FE model for the structural analysis of the functionally graded carbon nanotube reinforced composite plates resting on a Pasternak's elastic foundation using above non-polynomial shear deformation theories. To develop a model for the elastic foundation that accounts for both the horizontal and vertical stiffness of the foundation.
- Investigation of the bending, free vibration, and buckling responses of functionally graded carbon nanotube reinforced composite plates resting on a Pasternak's elastic foundation based on the developed analytical and FE model for different non-polynomial shear deformation theories which incorporate the inverse hyperbolic sine function and secant function.

2.8. Summary

This chapter reviews the literature on the use of trigonometric shear deformation theories based on secant and inverse hyperbolic sine function to develop CNT reinforced composite plates resting on a Pasternak's elastic foundation. The above literature review encompasses a comprehensive overview of CNTs and the diverse types of composite plates developed. It delves into the mechanical properties of CNTs and their interactions with polymer matrices and other composite materials. Various methods of developing CNT reinforced composite plates and their advantages and disadvantages are discussed in detail, with a particular focus on the development based on higher-order shear deformation theory. Additionally, the review explores the various types of Pasternak elastic foundations and their effects on the structural behaviour of CNT reinforced composite plates. Analytical, and numerical approaches utilized for characterizing the behaviour of such plates are also examined. The review highlights the wide-ranging applications of CNT reinforced composite plates and briefly evaluates their potential for future applications, along with the challenges that needs to be addressed.The review concludes by highlighting several areas for future research related to the development and application of CNT reinforced composite plates.