Structural Analysis of Functionally Graded Carbon Nanotube Reinforced Composite Plates

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Structural Analysis of Functionally Graded Carbon Nanotube Reinforced Composite Plates

A thesis submitted in partial fulfillment

for the Award of Degree

Doctor of Philosophy

By
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"I would like to dedicate
this thesis to
my Parents, who instilled
in me the virtue
of perseverance and commitment,
and relentlessly encouraged me
to strive for excellence…"

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DECLARATION BY THE CANDIDATE

I, **Surya Dev Singh**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Rosalin Sahoo** from January 2019 to June 2023 at the Department of Civil Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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Abstract

Carbon nanotubes (CNTs) are cutting-edge materials that offer great mechanical features like high strength, high stiffness, and high durability. As a result of these exceptional qualities, CNTs have been widely used as a reinforcing material. The mechanical characteristics of carbon nanotube reinforced composites (CNTRC) depend on a number of factors, including volume fraction of CNTs, orientation, matrix's characteristics, loading conditions, and side-to-thickness ratio. It is necessary to understand the complex behaviour of the interaction between the soil and the structure, which goes hand in hand with the structural investigation of these materials. Hence, in this work, an effective analytical and finite element (FE) model is developed in this work to investigate the structural behaviour of a CNTRC plate resting on Pasternak's elastic foundation, which includes bending, free vibration, and buckling analysis, within the context of various non-polynomial shear deformation theories based on secant function and inverse hyperbolic sine function. Further, in this work, different types of carbon nanotube reinforced distributions and stacking sequences are also considered. Here, an optimal configuration for the functionally graded CNTRC plate is sought out in order to achieve precise static, buckling, and free vibration responses. The analytical and FE techniques are used in order to carry out a detailed parametric study of functionally graded CNTRC plates with a wide range of material characteristics, stacking configurations, span thickness ratios, core to face sheet thickness ratios, and loading conditions. The FE based results in the form of deflection, stresses, natural frequency and buckling loads are obtained using in house generalized MATLAB code. In order to develop an improved comprehension of carbon nanotubes as a structural material, some new results are also been obtained.

Keywords: Non-polynomial shear deformation theory; Analytical method; Finite Element method; Composites plate; Sandwich structure; Elastic foundation

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4. Result and Discussion

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List of Symbols

x, y, z	Cartesian coordinate system
U, V, W	3D displacements in the global x, y and z- direction
u, v, w	Mid-plane displacement components in the$x$, y and z- direction
β_1, β_2	Rotation of the transverse normal to the mid-plane about$y$ and x-direction
$\{\sigma\}$	Stress vector in the global coordinate system
$\{\varepsilon\}$	Strain vector in the global coordinate system
$[Q]$	Transformed reduced stiffness matrix
$\{\bar{\sigma}\}$	Stress vector in the material coordinate axis
$\{\bar{\varepsilon}\}$	Strain vector in the material coordinate axis
E_{11}, E_{22}	Young's Modulus in the longitudinal and transversedirection to the CNTs direction
G_{12}	In-plane shear modulus
G_{13}, G_{23}	Transverse shear modulus
β_w	Winkler stiffness of the foundation
β_s	Shear stiffness of the foundation
U	Strain energy of the carbon nanotube reinforced composite plate
U_F	Strain energy of the elastic foundation
W	Work potential of the applied loads
K	Kinetic energy of the carbon nanotube reinforced composite plate
E^m	Young's modulus of matrix
G^m	shear modulus of matrix
υ^m	Poisson's ratio of matrix
V^m	volume fraction of matrix
υ_{12}^{CNT}	Poisson's ratio of carbon nanotubes
V_{CNT}^*	volume fraction of carbon nanotubes added
E_{11}^{CNT}	Young's modulus of carbon nanotube in longitudinal direction
E_{22}^{CNT}	Young's modulus of carbon nanotube in lateral direction
G_{12}^{CNT}	shear modulus of carbon nanotube
ρ^m	Density of matrix
ρ^{CNT}	Density of carbon nanotube
ρ	Density of carbon nanotube reinforced composite plate
V_{CNT}	volume fraction of carbon nanotubes
η	Efficiency paremeter
Ω	Ohm a constant
N_{11}, N_{12}, N_{22}	In-plane stress resultants
M_{11}, M_{12}, M_{22}	Moment stress resultants
Q_1, Q_2	Transverse shear stress resultants
h	Overall thickness of the carbon nanotube reinforced composite plate
q	Mechanical pressure
$\bar{I}_0, \bar{I}_1, \bar{I}_2, \bar{I}_3, \bar{I}_4, \bar{I}_5, \bar{I}_6, \bar{I}_7$	Inertia components

[A], [B], [C], [D], [G], [H], [I], [L],[M], [P]	Rigidity sub matrices relating the stress-resultants andderivatives of the primary variables
$\{\Delta\}$	Displacement vector
$\{\bar{F}_M\}$	Mechanical force vector
ω	Natural frequency of the plate
$[N_{\bar{i}}]$	Shape function matrix
[m]	Mass matrix
ξ, η	Natural coordinate system used in finite element
[H]	Matrix relating the strains and derivatives of the primaryvariables
[B]	Matrix relating the derivatives of the primary variablesand the nodal coordinates
Pe	Penalty function
[K]	Elemental stiffness matrix of the carbon nanotube reinforced composite plate
$[K_{pe}]$	Elemental penalty stiffness matrix
$[K^{(F)}]$	Elemental stiffness matrix of the foundation
$\{F_M\}$	Elemental force vector
[M]	Elemental mass matrix
L	Lagrangian
$\{\bar{F}_M\}$	Global mechanical force vector
$[\bar{M}]$	Global mass matrix
$[\bar{K}]$	Global stiffness matrix of the carbon nanotube reinforced composite plate
$[\bar{K}^{(F)}]$	Global stiffness matrix of the foundation
$[\bar{K}^{(pe)}]$	Global stiffness matrix containing the penalty terms

Abbreviations

3 D	Three Dimensional
2 D	Two Dimensional
CNTs	Carbon nanotubes
SWCNTs	Single walled carbon nanotubes
MWCNTs	Multi-walled Carbon nanotubes
CNTR	Carbon nanotube reinforced
CNTRC	Carbon nanotube reinforced composite
CLPT	Classical Laminated Plate Theory
CPT	Classical Plate Theory
FSDT	First Order Shear Deformation Theory
HSDT	Higher-Order Shear Deformation Theory
TSDT	Trigonometric Shear Deformation Theory
FRP	Fiber Reinforced Polymers
FGM	Functionally Graded Material
ESL	Equivalent Single Layer
LW	Layerwise
CVD	Chemical vapor deposition
HiPCO	High-pressure carbon monoxide
PECVD	Plasma enhanced chemical vapour deposition method
MPECVD	Microwave plasma chemical vapour deposition method
RF-CVD	Radiofrequency chemical vapour deposition method
HFCVD	Hot-filament chemical vapour deposition method
FCCVD	Floatingcatalyst chemical vapour deposition method
PmVA	{(mphenylenevinylene)-co-[(2, 5-dioctaxy-p-phenylene) viny-lene]}

PHSDT	Polynomial Higher-Order Shear Deformation Theory
NPHSDT	Non-Polynomial Higher-Order Shear Deformation Theory
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
ROM	Rule of Mixture
FEM	Finite Element Method
XFEM	Extended Finite Element Method
FDM	Finite Difference Method
DSC	Discrete Singular Convolution
DQM	Differential Quadrature Method
EE	Equilibrium Equations
EKM	Extended Kantorovich method
SCF	Shear-Correction Factor
RHZZT	Refined Higher-Order Zigzag Theory
RFSDT	Refined First Order Shear Deformation Theory
BEM	Boundary Element Method
UDL	Uniformly Distributed Load
SSL	Sinusoidal Load
ND	Non-Dimensional Parameter