Chapter 1

Introduction

Time-delay is encountered in many engineering applications, such as aircraft systems, chemical control systems, laser models, and biological systems [2, 10]. The appearance of time-delay in the system can be of different forms, such as, transport, communication, or measurement delays. Time-delay systems are also called systems with aftereffect or dead-time, or differential-difference equations. It belongs to the class of functional differential equations that are infinite-dimensional compared to ordinary differential equations (ODE). An example of a time-delay system is

$$\dot{z}(t) = -z(t-h), \quad z(t) \in \mathbb{R}$$

where $h > 0$ is the time-delay.

Control systems always act in the presence of delays, initially due to the time it takes to obtain the information required for decision-making, create control decisions, and execute them. Actuators, sensors, and field networks that are involved in feedback loops usually introduce delays. Therefore, delays are fully employed in communication and information technologies challenging areas, such as the stability of networked control systems (NCSs) or high-speed communication networks [4].

In many systems, time-delay is the primary source of instability [1, 5, 101]. On the other hand, for some systems, the presence of time-delay can have stabilizing effect [3, 4, 14]. For example, consider the double integrator systems:

$$\ddot{y}(t) = u(t), \quad y(t) \in \mathbb{R},$$

which is not stabilizable by the non-delayed static output feedback $u(t) = K_0 y(t)$. However, this system is stabilizable using a delayed static-output feedback

$$u(t) = K_1 y(t) + K_2 y(t-h), \quad h > 0$$

because it is stabilizable by $u(t) = K_3 y(t) + K_4 \dot{y}(t)$ and

$$\dot{y}(t) \approx \frac{y(t) - y(t-h)}{h}, \quad h > 0.$$

Therefore, the study of stabilizability effect of time-delay is of theoretical and practical significance.

Lyapunov's direct method is an effective approach to study the stability of delay-free systems. Also, it is an efficient way for the stability analysis of time-delay systems. For time-delay systems, two types of Lyapunov methods are used (a) the Lyapunov-Krasovskii functional method and (b) the Lyapunov-Razumikhin function method. The Lyapunov-Krasovskii functional method handles a broader class of systems and provides less conservative results than the Lyapunov-Razumikhin function method.

The Lyapunov stability method can also be formulated in terms of linear matrix inequality (LMI) for linear systems. The realization of LMI is a convex optimization problem, and the advancement of the efficient interior point method, which commenced to formulate many control problems and their solutions in the form of LMIs [11]. The LMI approach to the analysis and control design of time-delay systems provides finite-dimensional conditions despite significant disturbances/uncertainties.

Representing continuous-time systems with digital control in the form of continuous-time systems with time-varying delay [157] and extending the Lyapunov-Krasovskii method to time-delay systems without any constraints on the delay derivative [155] and discontinuous delays [156] have enabled the extension of the time-delay strategy to sampled-data and network-based control.

The stabilization problem of linear/nonlinear systems becomes more complex and challenging when the control laws require partial state information or output information. These issues are widespread in the practical scenario; therefore, many researchers provided observer-based control. Still, this technique becomes complicated due to model uncertainties in the system matrices and input/output delays. The decoupling of the estimation error equation from the state equation is not allowed due to unknown delays [166]. Therefore, simple static output feedback has been implemented, which has advantages over observer-based control in the presence of perturbations and input/output delays. However, some systems (chain of integrators, oscillators, inverted pendulums) are not stabilizable with simple static output feedback control. To cope with these critical issues, many researchers proposed delayed control laws. But the available results are valid only for nominal (disturbance-free) linear systems. Another critical issue is the presence of uncertainty in the modeling of physical systems. The control laws must be designed in such a way that the closed-loop system becomes robust against uncertainty. In literature, we have different types of robust nonlinear controllers, such as backstepping, high gain controller, passivity, feedback linearization, sliding mode control, etc. [21]. Among them, sliding mode control is best due to its inherent features, i.e., finite-time stability, reduced-order dynamics, uncertainties, or perturbations rejection.

The sliding mode controller has immense potential, insensitivity to uncertainties and tackling nonlinear features attracting many researchers to explore their applicability in electrical, electrohydraulic, and electromechanical systems. The sliding mode control has two tasks: the first is to design the sliding surface, and the second is to develop the control laws, which force the sliding surface to zero. The sliding surface design consists of two phases; the first phase is the reaching phase, which aims to drive the state trajectory from any initial state to the sliding surface in finite-time. The second phase is a sliding-mode phase, which seeks to maintain the state trajectory on the sliding surface itself for all future time. Initially, the sliding mode control requires upper bound information of the perturbations. Because of this, we have to assume that the upper bound of the perturbation is known. But due to the development of adaptive sliding mode control, this assumption can be removed [169–171]. Further, higher-order sliding mode control [25,29], sliding mode observer [143–147], and sliding mode differentiator [172–174], is extensively investigated in the literature.

1.1 Literature Review

The stability problem in control theory is significant and challenging, and also widely examined [5, 7, 19]. The time-delay systems stability research starts with the frequency-domain approach and later with the time-domain approach. Frequency domain methods determine the stability of the system from the distribution of the roots of the characteristic equations [8] or the solutions of a complex Lyapunov matrix function equation [9]. The time-delay system renders an infinite number of roots of the characteristic equation, making it challenging to study with classical methods, particularly in examining the stability and designing stabilizing controllers. Therefore, such problems are usually resolved indirectly by employing approximation. The most common approximation method is Padé approximation [165], which is a rational approximation and results in a reduced fraction as a substitute for the exponential time-delay term in the characteristic equation. Such an approximation has accuracy limitations and can cause the instability of the original system, non-minimum phase, and high-gain problems [165]. Prediction-based methods [159], finite spectrum assignment [161], and adaptive Smith predictor [160] have been used for time-delay system by transforming the problem into a delay-free system. Moreover, straightforward implementation is still an open problem due to computational concerns. These frequency-domain methods provide complex formulation and can cause conservative results.

The time-domain techniques are Lyapunov-Krasovskii functional and Lyapunov-Razumikhin function methods [10]. These techniques are very frequently used in the stability analysis of time-delay systems. It is challenging to construct the Lyapunov-Krasovskii functionals that lead to sufficient conditions. This approach is theoretically important, but there was no efficient way to implement it at the starting (around the 1990s). After that, the MATLAB toolboxes were developed, making things easier and providing a straightforward way to construct the Lyapunov-Krasovskii functionals. Thus, significant improvement, development, and applications of these methods are widely studied in [162, 163].

The control method for the stabilization of the dynamic system requires output and its derivative information. In more restricted cases, the output derivatives are not available but can be approximated by providing the delayed feedback. An artificial time-delay approach for stabilizing inverted pendulum using a model transformation approach is discussed in [116]. In [18], a delay-induced stabilization of the second-order systems is presented for both constant and time-varying delay. It also provides two approaches to achieve the goals (i) Lyapunov-Krasovskii stability analysis using simple functionals (ii) extensions of model transformation approach proposed by [116, 127] to second-order systems using comprehensive techniques. The same work is extended for the general linear systems in [6] with less conservative results. Also, the choice of the Lyapunov functional is different from [18]. This method employs the Lyapunov functional dependent on the state derivative that does not apply to the stochastic case.

The delay-induced stability using frequency domain is presented in [3, 14, 37] and references therein. The Lyapunov-based method for the delay-induced stability is widely studied [38,101]. In [117], a discretized Lyapunov function is used for linear systems with both discrete and distributed delay to design $\mathcal{H}_\infty$ controller. The discretized Lyapunov-Krasovskii functional discretizes the Lyapunov-Krasovskii functional and LMI conditions are derived for delay-induced stability analysis [13, 101]. In some works, augmented Lyapunov functional is utilized as a certain choice of the general Lyapunov functional for a delay-induced stability [38, 43]. Both approaches begin to high-order LMIs with a large number of decision variables. Further, the same strategy is employed for the design of control laws under continuous-time and sampled measurements in [125]. The derivative-dependent control based on the sampled-data implementation is proposed for nominal LTI system in [126] and for stochastic systems in [85].

Next, we present further essential works related to time-delay systems. In [100], an output feedback controller is designed for MIMO systems with relative degree one or two, and derivative feedback is approximated as delayed feedback by Euler approximation. For the nonlinear state-delayed systems, a convex approach based on robust regional stability analysis is presented in [103]. Delay-dependent/delay-independent stability analysis is described in [104] for linear retarded and neutral type systems with discrete and distributed delay. In [105], a descriptor form defines the neutral system, and Lyapunov-Krasovskii functional is proposed. For linear systems, stability, $\mathcal{L}_2$ gain analysis and $\mathcal{H}_\infty$ state feedback control are designed with uncertain time-varying delays [106]. A new time-delay representation is given, which involves multiple successive delay components in the state for the network-based control [107]. Robust stability and control of uncertain time-delay has been proposed in [108–111]. In [112], stability analysis of a class of systems with uncertain time-delay is studied. New stability and $\mathcal{L}_2$ gain analysis is proposed for the networked control system using discontinuous Lyapunov functions [113].

An artificial delay-based static output feedback sliding mode control is applied to an uncertain linear system in [12]. This paper introduced the delayed sliding surface depending on the output information only. In [128], a robust sliding mode control is designed for time-delay systems with unmatched parametric uncertainty. A robust stabilization problem of linear time-varying delay system with both matched and unmatched disturbances is discussed in [129] employing sliding mode control. In [130], a sliding mode control technique is implemented to Markovian jump singular time-delay systems. Passivity-based sliding mode control for uncertain singular time-delay systems is studied in [131]. A robust adaptive controller is presented in [132] for robot manipulators using adaptive integral sliding mode control and time-delay estimation.

Other important works on the sliding mode control are demonstrated next. A sliding mode control strategy and the precise elimination of chattering phenomenon is presented in [30, 133–136]. The sliding mode control theory and its implementation on the electric drive are examined in [137]. The discrete-time sliding mode control is offered that provides improved robustness, faster transients response, and better steady-state accuracy of the closed-loop system compared to similar existing works [138]. An application of sliding mode control strategy for electro-mechanical systems is studied in [139] and references therein. In [140, 141], an integral sliding mode control technique is presented for uncertain systems. Further, the higher-order sliding mode control strategy is also widely investigated in [25, 29]. The state estimation of uncertain systems based on sliding mode observer is studied in [143–147]. A terminal sliding mode control is frequently examined for the finite-time convergence of the system states to the origin [148–151].

Further, the artificial delay-based approach is implemented on multi-agent systems. In [20], artificial delay-based feedback control is designed for the consensus and quasi consensus problem of the second-order multi-agent dynamical systems, and the frequency domain method is utilized for the stability analysis. Again, the same approach is utilized for the consensus problem of second-order multi-agent systems, and stability analysis is guaranteed using a simple Lyapunov functional in [86]. Both of the papers [20, 86] are under undirected communication graph. In [87], under a directed communication graph, a delay-induced consensus control is designed based on the sampled position data and with the help of time-delay for the second-order multi-agent systems. A delay-induced consensus tracking problem is discussed under intermittent communication and non-intermittent communication in [88] for the general linear multi-agent systems on a directed graph.

Other essential works on the cooperative control of the multi-agent system are discussed next. In [89], various Lyapunov functions are constructed under different communication topology. A summary work on recent advancements of the distributed multi-agent coordination is presented in [93]. In [91], cooperative control and information flow of the vehicle formation has been suggested. Information consensus in multivehicle cooperative control is recommended in [92] under time-invariant and dynamically changing communication topologies. A consensus problem for the second-order multi-agent systems with time-delay and jointly-connected topologies is presented in [94]. In [95], consensusability of Linear multi-agent systems is summarized. Distributed consensus control for linear multi-agent systems with intermittent communication over a time-invariant undirected graph is carried out in [96]. A distributed adaptive consensus controller is designed under a directed graph for the general linear systems [90]. Distributed coordination tracking problems are investigated in [97] for the general linear multi-agent systems over a directed fixed communication topology. In [98], a distributed consensus problem of multi-agent chaotic systems with time-varying delay under a switching topology and directed intermittent communication is reported. The three design methods for cooperative control, namely, Lyapunov design, the adaptive neural design, and the linear-quadratic-regulator (LQR), are designed for multi-agent systems in [99]. The work in [114] offers a consensus problem of linear systems with a distributed delay in modeling the traffic flow dynamics.

1.2 Motivation

The stabilization problem is often solved using the complete state information of dynamical systems. However, all state information is not available in practice due to the sensor's economic infeasibility and inaccessibility of states. Sometimes, only partial state or, in more limited cases, only the system's output (a certain combination of the states) is available. Many results on observer-based control have been studied in the literature to overcome the unavailability of unknown states. However, the observer-based controller becomes complicated when uncertainty and uncertain input/output delays are present in the system. The partial state feedback and static output feedback control are opted to serve the purpose.

The previous results indicate that some systems (inverted pendulum, oscillators, double integrator) cannot be stabilized by only partial state feedback and static output feedback, which may be stabilized by introducing artificial time-delay in control. Thus, delayed partial state feedback and delayed static output feedback controller design are fundamental problems to be studied. The sliding mode control is a well-known technique in the presence of parametric uncertainty. Its interior features like robustness against parametric uncertainty, finite-time convergence, and reduced-order dynamics make it one of the best methods in the presence of disturbances. Therefore, sliding mode control with delayed partial state feedback and delayed output feedback may improve certain robustness.

1.3 Objectives

The objectives of the work is stated as follows:

- To relax the estimation of unknown states.
- To obtain robustness against parametric uncertainty and perturbations.
- To relax the reaching phase of the nonlinear system.
- To form an LMI based on the Lyapunov-Krasovskii functional which is feasible for the small time-delay.
- To achieve the consensus for both leaderless and leader-follower problem based on directed graph in the presence of uncertainty and perturbations.

1.4 Organization of the Thesis

This thesis is comprised of six chapters. This chapter introduces the fundamental time-delay systems and delayed static output feedback. After that, the motivation behind the work is explained. Further, the main objectives of the proposed work are outlined.

In continuation, the second chapter outlines the preliminaries. In this, we have described notations, norms, different fundamental topics like LMIs, graph theory. Also, various definitions and necessary lemmas are presented.

In the third chapter, an uncertain nonlinear system stabilization is presented when only partial state information is available. A sliding mode control technique is given without estimating the unknown states, using partial and delayed states. To obtain robustness against disturbances, a sliding mode strategy is used. Further, the Lyapunov-Krasovskii functional is constructed, which leads to an LMI and feasible for small time-delay h. At last, its application to the ball and wheel system is presented through simulation.

The fourth chapter deals with the stabilization of uncertain nonlinear systems under the availability of the output. Using only output and its delayed state information, a sliding mode control technique is presented. A reaching phase-free phenomenon is reported, and robustness is ensured at the beginning of the evolution of the states. The Lyapunov-Krasovskii functional leads to an LMI, which is feasible for the small value of time-delay h. Simulation results of the TORA system illustrate the efficacy of the proposed theory.

In the fifth chapter, the leaderless and leader-follower multi-agent systems consensus problem is described when only the output information is available. The distributed delayed output feedback control is proposed for both leaderless and leader-follower consensus. The Lyapunov-Krasovskii theorem formulates the LMI, which is feasible for a small delay. A numerical example illustrates the effectiveness of the proposed method.

Finally, the sixth chapter concludes the overall work carried out in this thesis and provides the future work direction.