To

The

Supreme Personality

Lord Shiva

and

my beloved family

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DECLARATION

I, **Sandeep Kumar Soni**, certify that the work embodied in this thesis is my bonafide work and carried out by me under the supervision of **Dr. Sandip Ghosh** and **Dr. Shyam Kamal** from July-2017 to March-2021, at the Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted to award any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not deliberately copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, thesis, etc., or available at websites and have not included them in this thesis and have not cited as my work.

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Abstract

A system is said to have a delay when the rate of variation in the system state depends on past states. Such a system is called a time-delay system. Delays frequently appear in real-world engineering systems. They are often a source of instability, poor performance of systems and significantly increase the difficulty of stability analysis and control design. On the other hand, it has a stabilizing effect. In other words, the employment of small time-delay in the control law stabilizes some unstable dynamical systems. This thesis work is on utilizing such stabilizing impact of time-delay.

Another major concern deal with in this work is the unmeasurable states. In many practical applications, we do not have complete state information, only partial state information, or, in more limited cases, just measured state information is available. Therefore, we have to estimate the unmeasurable states and use the observer-based or dynamic output feedback controller in these circumstances. But due to the involvement of the observer dynamics, the closed-loop system complexity is increased. An alternative approach that relaxes the complexity is defined as artificial delay-based feedback stabilization.

First, a multiple delayed partial state feedback sliding mode control for uncertain nonlinear systems is proposed in this work. An output-based delayed sliding surface is designed. Subsequently, a novel Lyapunov-Krasovskii functional is constructed, leading to the Linear Matrix Inequalities (LMIs) criterion, which yields a feasible solution for small delays. The efficacy of the proposed theory is demonstrated on a ball and wheel system.

Next, delayed output feedback sliding mode control is proposed for uncertain nonlinear systems. A delayed sliding surface with a decreasing exponential term is presented based on the output. The decreasing exponential term is used to achieve a sliding surface from the very initial time. A novel Lyapunov-Krasovskii functional is constructed, leading to the Linear Matrix Inequalities (LMIs) criterion, which yields a feasible solu-

tion for small delays. Finally, the proposed method's efficacy is demonstrated through a Translation Oscillator Rotating Actuator (TORA) system.

Finally, a distributed delayed output feedback control to the consensus problem of uncertain multi-agent systems (MASs) is developed, where only the agent output is used for feedback. The consensus problem of the leader-follower and leaderless systems is presented on the directed communication graph. Using distributed delayed output feedback control, the consensus is achieved for both problems. It has been shown that in the absence of external disturbances, the tracking errors are asymptotically converging to zero, and in the presence of disturbances, tracking errors are uniformly bounded. Numerical results demonstrate the effectiveness of the proposed techniques.

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Nomenclature

List of Greek and Roman Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_{\geq 0}$	Set of positive real numbers
$\mathbb{N}$	Set of positive numbers
$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$	Symmetric matrix $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$
$A > 0 (< 0)$	Symmetric positive (negative) definite matrix
$A \geq 0 (\leq 0)$	Symmetric positive (negative) semi-definite matrix
$\lambda(A)$	Eigenvalue of matrix A
$\lambda_{\max}(A)$	Largest eigenvalue of matrix A
$\lambda_{\min}(A)$	Smallest eigenvalue of matrix A
h	Time-delay
$\otimes$	Kronecker product
$\mathcal{L}$	Laplacian matrix
$\mathbf{1}_N$	Unity column vector
$\mathcal{C}([a,b],\mathbb{R}^n)$	family of continuous functions ϕ from $[a,b]$ to $\mathbb{R}^n$
$\lvert \cdot \rvert$	Absolute value (or modulus)
$\lVert \cdot \rVert$	Euclidean norm of a vector or spectral norm of a vector

$\|\cdot\|_\infty$	Induced l_∞-norm
$\|\phi\|_c$	Continuous norm $\sup_{a\le t\le b}\|\phi(t)\|$ for $\phi\in\mathcal{C}([a,b],\mathbb{R}^n)$
$\forall$	For all
$\in$	Belongs to
$\exists$	There exists
$\subseteq$	Is a subset of
$\cup$	Union
$\Rightarrow$	Implies
$:=$	Is defined as
$\square$	End of proof

Abbreviations

SMC	Sliding mode control
SOF	Static output feedback
MAS	Multi-agent systems
SISO	Single-input single-output
MIMO	Multi-input multi-output
STC	Super-twisting control
ISS	Input-to-state stability
LMIs	Linear matrix inequalities
LTI	Linear time-invariant
PDE	Partial differential equation
TORA	Tanslation oscillator rotating actuator
NCS	Networked control system