

# WAVELET TRANSFORMS AND SOME APPLICATIONS OF MATCHED WAVELETS



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# Chapter 5

## Application of Matched Wavelets in the Compression of ECG Signals

### 5.1 Introduction

The wavelet transform has emerged over recent years as a powerful time-frequency analysis and signal processing tool used for the analysis of complex non stationary signals. Its application to bio-signal processing has been at the forefront of these developments where it has been found particularly useful in the study of these, often problematic, signals: none more so than the Electrocardiogram (ECG)(Addison (2005)).

The ECG is a measure of the electrical activities of due to re-polarization and depolarization of atrial and ventricular chambers of the heart. The ECG is recorded at the body surface and results from electrical changes associated with activation first of the two small heart chambers, the atria, and then of the two larger heart chambers, the ventricles. The contraction of the atria manifests itself as the P wave in the ECG and contraction of the ventricles produces the feature known as the QRS complex. The subsequent return of the ventricular mass to a rest state repolarization produces the T wave. Repolarization of the atria is, however, hidden within the dominant QRS complex (Addison (2005)). The study of slope, variations and location of these waveforms helps in diagnosing any abnormality present in heart. A typical ECG signal is shown in Figure 5.1 (Image source: bme.elektro.dtu.dk).

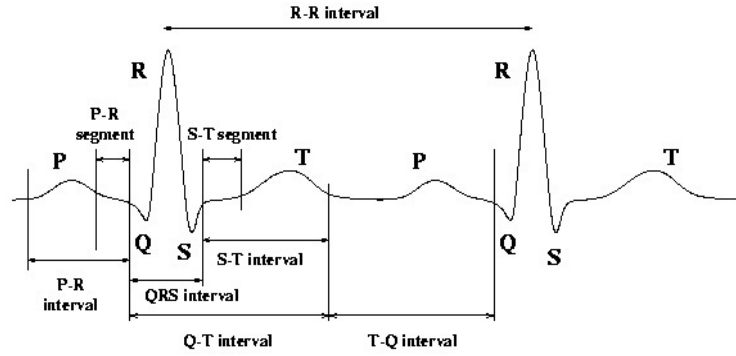


Figure 5.1: A typical ECG signal

### 5.1.1 ECG Signal Compression

ECG signal play important role in diagnosis and survival analysis of heart diseases. These signals are recorded from patients for both diagnostic and monitoring purposes. With advancement of communication technology and remote medical monitoring, long continuous ECG record-keeping and its real-time transmission is required. The recording of ECG signals produce substantial volumes of data for storage and transmission. Due to limited capacity of storage devices, the compression of ECG data is needed. Also, for fast and economical transmission of ECG signal high efficiency compression is required. This had led ECG signal compression as an important issue of research in biomedical signal processing (Addison (2005); Ranjeet *et al.* (2011)).

The main goal of any compression technique is to achieve maximum data reduction while preserving the significant signal morphology features upon reconstruction in order to achieve a correct clinical diagnosis. Conceptually, data compression is a process of detecting and eliminating redundancies in a given set of information.

Compression techniques can be broadly classified in two categories, namely- Lossy and Lossless. In lossless techniques, the reconstructed ECG signal is exact replica of original signal whereas reconstructed signal in lossy compression is obtained with some error. For ECG compression mostly lossy techniques are used.

### 5.1.2 Some ECG Signal Compression Techniques

The existing ECG data compression algorithms can be divided into three categories:

1. Time domain methods (Direct methods): In time domain methods, the samples of the signal are directly handled to provide the compression. The compression in these methods is based on the idea of extracting a subset of significant signal samples to represent the signal. The examples of the methods belonging to this category are: Amplitude-Zone-Time Epoch Coding method (AZTEC), Coordinate Reduction Time Encoding System (CORTES), Turing Point technique (TP), Scan-Along Polygonal Approximation (SAPA), Entropy Coding, FAN algorithm and improvements to time-domain algorithms such as SLOPE and AZTDIS. long-term prediction (LTP), analysis by synthesis ECG compressor (ASEC), and the cardinality constrained shortest path technique (Rajoub (2002)). Although the implementations of these algorithms are easy, the compressed signal distortion is often a serious drawback. The key to the successful algorithm is a good rule to determine the most significant samples.
2. Transform domain methods: In these methods, the original samples are subjected to a transformation and the compression is performed in the new domain. They are based on redundancy of transformed data. Examples belonging to this group are Wavelet Transform (WT), Wavelet PacketBased Transform (WPT), Discrete Cosine Transform (DCT), Fourier Transform, and the karhunen-Loeve Transform (KLT). Most of these transforms compact the signal's energy into a few transform coefficients. This implies that many of the transform coefficients will have little energy and may be discarded.
3. Parametric methods: in such methods, the extraction of a set of useful parameters from the original signal is carried out and the same are used in the reconstruction process. Some examples of this type of methods are Linear prediction methods, peak picking method etc.

### 5.1.3 Wavelet based ECG Compression

Transform based ECG compression using the wavelet transform (WT) is an efficient and flexible scheme. This is due to the ability of wavelet transform to provide simultaneous local spectral and temporal information by employing a window of variable width which other Fourier based transforms (DCT, FT, STFT) lack. There is a great number of wavelet

compression techniques available in the literature. However, the search for new methods and algorithms continues to achieve higher compression ratio while preserving the clinical information content in the reconstructed signal.

Since WT results large runs or zeros in the transformed signal, it can be efficiently used for compression. Moreover, the nonzero small coefficients can be thresholded using appropriate techniques with a further increase in the number of zeros. Hence, improvement in the compression ratio is expected. In technical literature there exist a large number of thresholding techniques. Among them the universal thresholding (Donoho *et al.* (1995)), and thresholding methods based on energy packing efficiency (Yip *et al.* (1978)) are the most efficient methods. The process of thresholding results in compromise between compression ratio and the quality of the reconstructed signal (Ahmed *et al.* (2000)).

There is a need to select the optimal matched wavelet bases to analyse the signal and the signal needs to be expressed with the fewest coefficients, i.e. sparse coefficients. The signal compression with wavelet is a procedure in which the input signal is expressed with a sum of a few of power terms for wavelet function. The more similar the bases function is to input signal, the higher the compression ratio is. In other words, the design of matched wavelet is needed. This is a reason that match wavelets are finding applications in diverse fields and is a topic of current research.

In this chapter, we have constructed a matched wavelet for the ECG signal and used it for ECG signal compression. This chapter is organized as follows: Section 2 presents a brief introduction to the wavelet transform and its implementation. Section 3 presents the proposed method of compression. The performance of method is presented in Section 4. Finally, Section 5 concludes this chapter.

## 5.2 Wavelet Transform and its implementation

Wavelets transform is a method to analyze a signal in time and frequency domain. DWT gives the multiresolution decomposition of a signal. DWT decompose a signal at several levels in different frequency bands. At each level signal is decomposed into approximation coefficients (low frequency band of processing signal) and detailed coefficients (high

frequency band of processing signal). At level  $j$  these coefficients are given by

$$x^j(n) = \sum_{k \in \mathbb{Z}} h(k - 2n)x^{j+1}(n) \quad (5.2.1)$$

$$y^j(n) = \sum_{k \in \mathbb{Z}} g(k - 2n)x^{j+1}(n) \quad (5.2.2)$$

where  $x^{j+1}(n)$  and  $y^{j+1}(n)$  denote the approximation coefficients and the wavelet coefficients respectively. With these relations DWT decomposition can be interpreted as: at each level the convolution of approximation coefficients is taken with filter coefficient  $h$  (and  $g$ ), followed by downsampling by two. The process of DWT decomposition is shown in Figure 5.2.

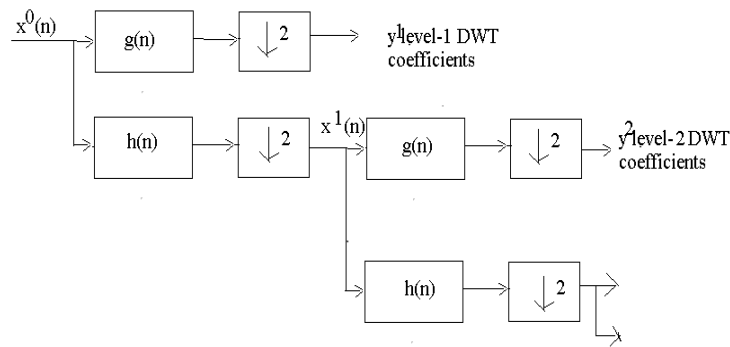


Figure 5.2: Block diagram of DWT decomposition

The reverse process of decomposition is called reconstruction. In reconstruction, upsampling by two is applied on the approximation and detail coefficients at each level. These coefficients are fed into the low pass and high pass synthesis filters and added afterward. This process continues until the number of levels become same as the number of levels in decomposition process. Reconstruction process is necessary to achieve the original signal.

### 5.3 The Proposed Compression Methodology

Our method of ECG signal compression is a wavelet transform based compression. We have used matched wavelet constructed for MIT-BIH ECG database record no. 108 as mother wavelet for compression. The steps involved in our method are following:

**Preprocessing:** The ECG signal is first preprocessed by normalization, mean removal, and zero padding. Normalization is done by dividing the original signal by its maximum value  $A_m$ . This guarantees that all DWT coefficients will be less than one. Mean removal is done by subtracting from the normalized ECG signal its mean  $m_x$ . Thus process reduces the number of significant wavelet coefficients. Zero padding reduces the reconstruction errors at both ends of the compressed signal (Rajoub (2002); Abo-Zahhad *et al.* (2013)). The above process can be represented by following relation:

$$y(n) = \frac{x(n)}{A_m} - m_x, \quad n = 1, 2, \dots, N \quad (5.3.1)$$

where  $y(n)$  and  $x(n)$  are the normalized and original signal samples respectively.  $A_m$  and  $m_x$  are the maximum value of the original ECG signal and the average value of normalized ECG signal respectively.

**DWT decomposition:** Mother wavelet is chosen, and then DWT decomposition is performed on the ECG signal. The selection of mother wavelet is based on the energy conservation properties in the approximation part of the wavelet coefficients. Then, decomposition level for DWT is selected which usually depends on the type of signal being analyzed or some suitable criteria such as entropy (Kumar *et al.* (2013)).

**Thresholding:** After computing the wavelet transform of the ECG signal, compression involves truncating wavelet coefficients below a threshold value which make a fixed percentage of coefficients equal to zero. The selection of the threshold influences the effect of data compression directly. With a large threshold we can have high data reduction but poor quality of the reconstructed signal. On the other hand, a small threshold produces low data reduction but high signal fidelity. So, a threshold must be optimally chosen for ECG compression. For thresholding purpose, we used Global Thresholding which involves taking the wavelet decomposition of the signal and keeping the largest absolute value coefficients. The threshold value which is set manually, is chosen from DWT coefficient  $(0 \dots x_{max}^j)$ , where  $x_{max}^j$  is the maximum value of coefficients.

**Entropy encoding:** In this step, signal compression is further achieved by efficiently encoding the truncated small-valued coefficients. We used run-length encoding on the redundant data to overcome redundancy problem without any loss of signal data. Run length coding is a simple form of data compression in which runs of data are stored as a single data value and count, rather than as the original run.

In our approach, we have performed the DWT decomposition using the wavelet

matched to the ECG signal obtained from MIT-BIH Arrhythmia Database. The construction of matched wavelet for ECG signal is presented in section 5.3.1.

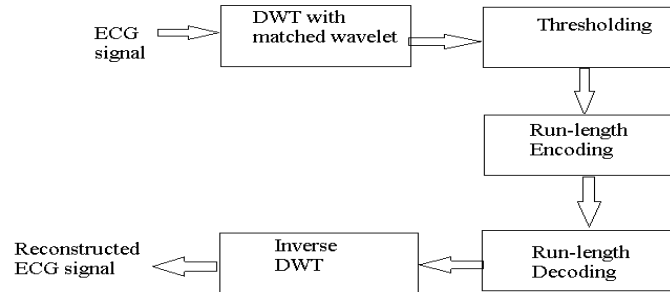


Figure 5.3: Block diagram of methodology for ECG signal compression

### 5.3.1 Construction of Wavelet Matched to the ECG Signal

We have followed the algorithm proposed by Chapa *et al.* (2000) (discussed in Chapter 3) for designing the wavelet matched to ECG signal. The record no. 108 is taken from the MIT-BIH arrhythmia database. We have taken one cycle of the record no. 108, as the desired signal for which the matching wavelet is to be constructed. The first step of the matching algorithm is to dilate the signal such that there is a maximum amount of energy in the wavelet passband. We have also used suitable zero-padding to the signal to get its spectrum in the passband. The wavelet passband taken in this construction was  $2\pi/3 \leq |\omega| \leq 8\pi/3$  for getting orthonormal wavelets. We have taken  $N = 512$  and  $\Delta\omega = 2\pi/16$ . With these values the algorithm of designing wavelet matched to a specified signal (presented in chapter 3) used to obtain matched wavelet for ECG signal.

The matched wavelet function for the ECG signal (record no. 108) is shown in Figure 5.4.

The values of  $h(k)$  are shown in Table 5.1.

## 5.4 Results and Discussions

In this section, wavelet based methodology has been used for the ECG signal compression. Different ECG records have been obtained from MIT-BIH Arrhythmia Database



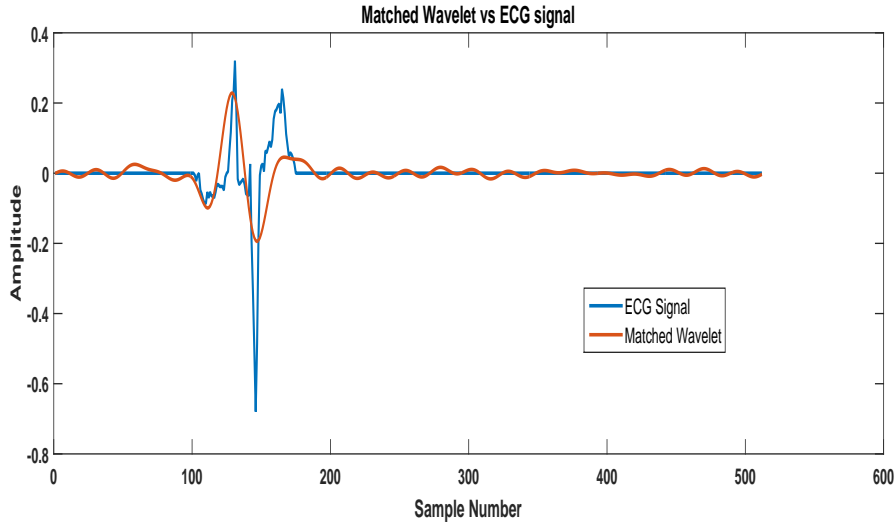


Figure 5.4: Wavelet Matched to MIT-BIH database ECG record no. 108

k	h(k)	k	h(k)
1	0.0056	9	-0.0225
2	-0.0118	10	0.0165
3	0.0001	11	0.0068
4	0.0104	12	-0.0683
5	-0.0080	13	0.0272
6	-0.0016	14	0.3281
7	0.0229	15	0.4604
8	-0.0009	16	0.2352

Table 5.1:  $h(k)$  for the matched wavelet for MIT-BIH database ECG record no. 108

and the matched wavelet constructed for record no. 108 was used for ECG signal compression. Several examples are included to illustrate the effectiveness of the matched wavelet in the field of data compression. The performance of the matched wavelet can be evaluated by considering the fidelity of the reconstructed signal to the original signal. For this, the following fidelity assessment parameters were used:

1. Compression Ratio (CR):

$$CR = \frac{\text{Length of Original Signal}}{\text{Length of Compressed signal}} \quad (5.4.1)$$

2. Percent root mean square difference (PRD):

$$PRD = \left( \frac{\text{Reconstructed noise energy}}{\text{Original Signal Energy}} \right)^{1/2} \times 100 \quad (5.4.2)$$

ECG records have been obtained from MIT-BIH Arrhythmia Database [MIT-BIH REF]. The number of samples for each record was taken 3600. We applied DWT upto level 3 for ECG signal records no. 108, 100,101,102 using matched wavelet constructed for signal record no. 108 as mother wavelet, and compared it to DWT results with db8 wavelet. The decomposition by the two wavelets are shown in Figures 5.5 and 5.6. It was observed that the detail coefficients with matched wavelet at level 1 itself are comparable to that at level 3 with db8 wavelet. The global thresholding was used for thresholding. The results with thresholding value of 0.001 is shown in Tables 2. The high values of coefficients with matched wavelet results in high compression ratio at the cost of high PRD.

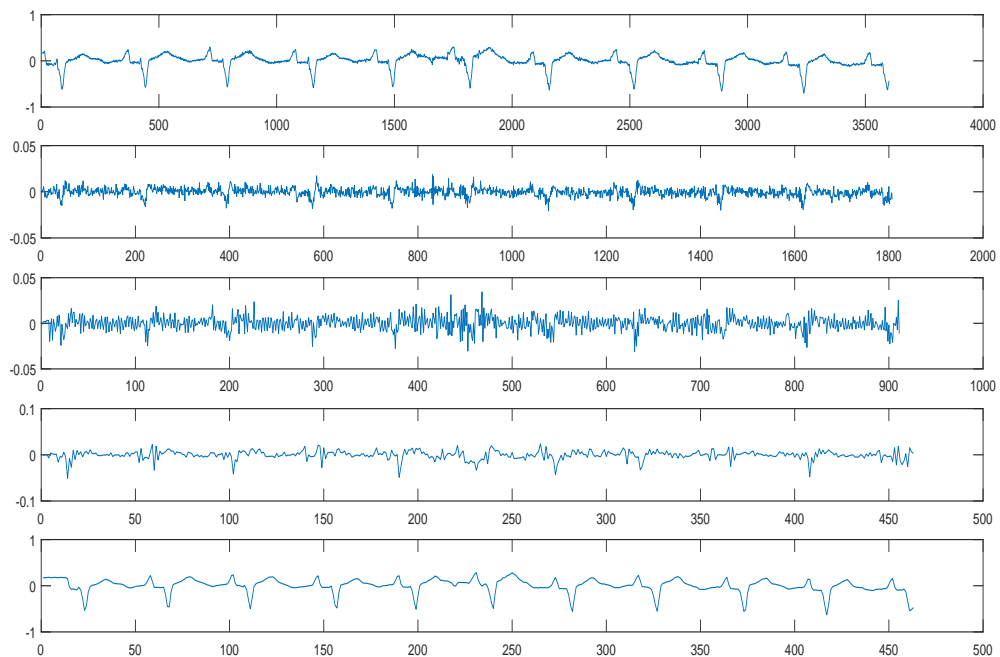


Figure 5.5: DWT decomposition of ECG record 108 using matched wavelet

The original ECG signal (record no. 108), the reconstructed signal and error signal with the matched wavelet and with db8 wavelet are shown in Figures 5.7 and 5.8, respectively.

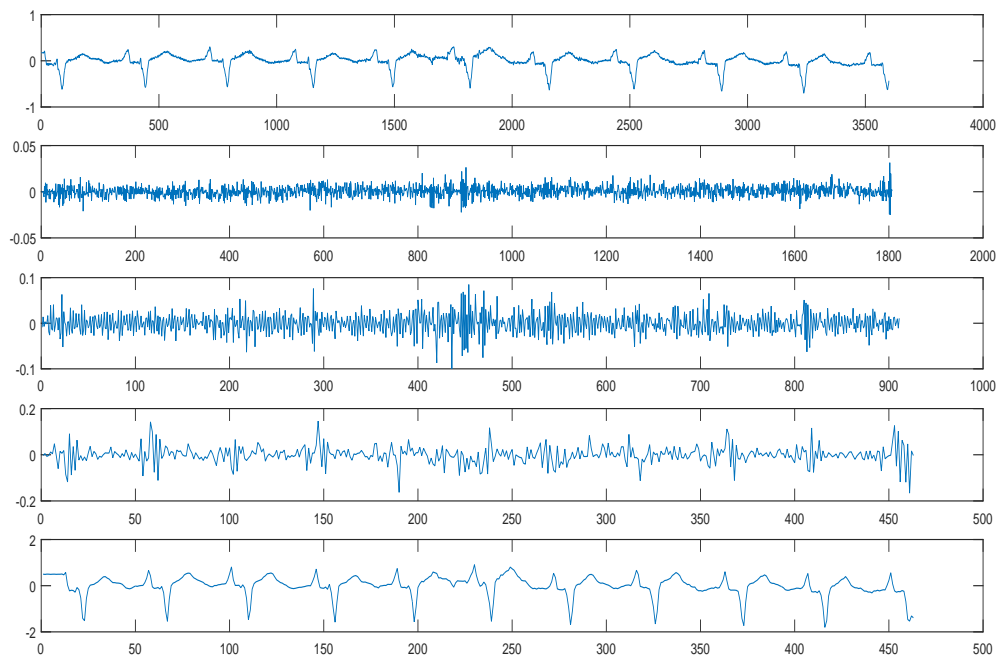


Figure 5.6: DWT decomposition of ECG record 108 using db8 wavelet

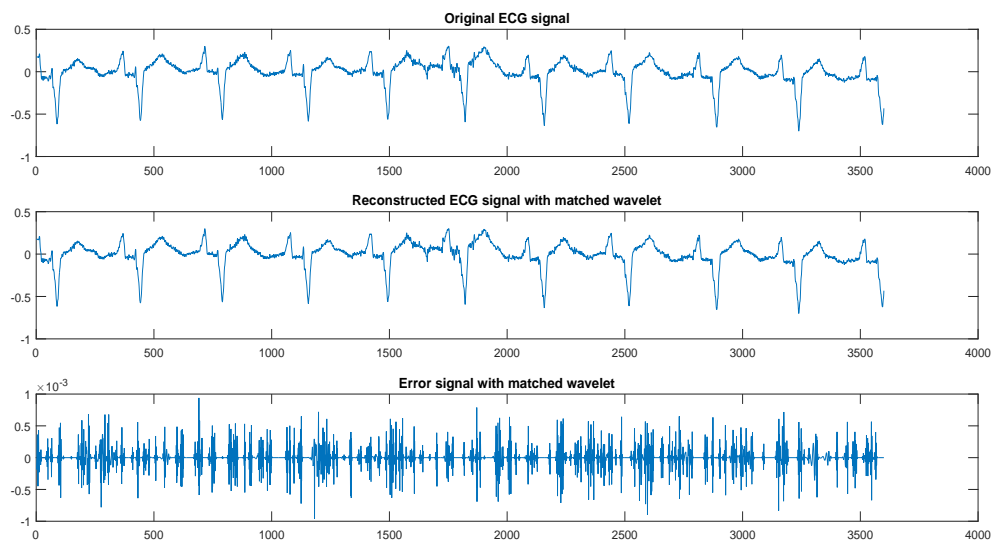


Figure 5.7: Original ECG signal, Reconstructed signal with matched wavelet and error signal

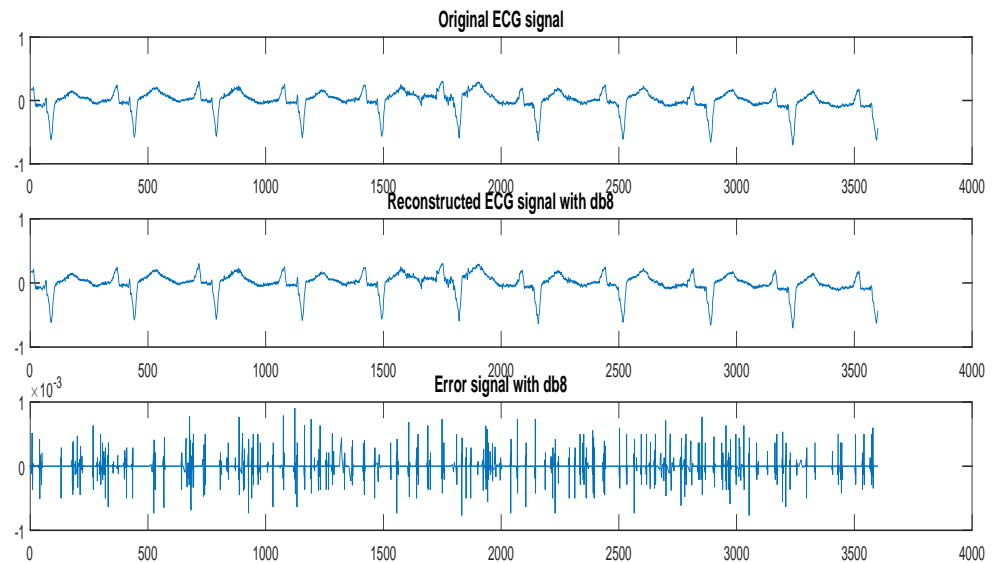


Figure 5.8: Original ECG signal, Reconstructed signal with db8 wavelet and error signal

ECG Record No.	CR(MW)	PRD(MW)	CR(db8)	PRD(db8)
108	14.05	8.81	7.69	0.10
100	12.17	9.15	13.88	0.20
101	13.34	9.03	15.39	0.21
102	16.77	8.91	13.14	0.18

Table 5.2: Fidelity assessment parameters with use of matched wavelet (MW) and db8 wavelet

## 5.5 Conclusions

A wavelet based methodology is presented for the ECG signal compression. In this methodology, matched wavelets was used for signal compression. Simulation results included in this chapter show the advantage of matched wavelet over other wavelets for ECG signal compression. It is found that the matched wavelet yields more compression with preserving all clinical information. All fidelity measuring parameters are improved. Therefore, it is concluded that it can be very effectively used in ECG signal compression.