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Dedicated to the memory of

my supervisor

Late Prof. Om Prakash Singh

Acknowledgements

The presented thesis is an outcome of my research work during my Ph. D. program. I wish to express my gratitude to all those who contributed to the formation of this thesis.

First of all, I am very much grateful to my respected supervisor, **Late Prof. Om Prakash Singh**, for his excellent guidance, support and motivation. I am indebted to him for his affection, constant encouragement and inspiration. It was a great privilege to be his student and to work under the guidance of such a Great Teacher. I remember his enthusiasm and words of encouragement even during the days of his battle with cancer. Unfortunately, he is not with me today to see the completion of my thesis.

I acknowledge my deep sense of gratitude to **Prof. Devender Singh** (my co-supervisor) and **Prof. Rakesh Kumar Misra** (External Subject Expert for my Ph.D. program) of Department of Electrical Engineering for introducing me to the worlds of Signal Processing and Matched Wavelets. I am incredibly fortunate to have the opportunity to work with these two True Gentlemen. I wish to express my immense gratitude to them for their fruitful suggestions, constructive criticism and patience.

I am privileged to have association with **Prof. J. N. Pandey** of School of Mathematics and Statistics, Carleton University, Ottawa, Canada. I am indebted to him for the fruitful discussions which helped me to have a better understanding of the subject area of Wavelet Analysis. My sincere gratitude goes to him for his enthusiasm and help. I also acknowledge my heartfelt thanks and sincere gratitude to **Dr. Rajesh Kumar Pandey** and **Dr. Vineet Kumar Singh** for constant encouragement and support. My sincere thanks goes to both of them for their emotional support and motivation especially after the sad demise of my supervisor.

I would like to express my deep sense of gratitude to **Dr. L. P. Singh**, Professor and Head, Department of Mathematical Sciences for encouraging me and providing me all facilities to carry out my research work. I am indebted to him for his guidance after the death of my supervisor. I also acknowledge my deep regards to all faculty members of Department of Mathematical Sciences, especially **Prof. S. K. Pandey** in his role as Convener of DPGC, and the members of RPC, DPGC and DRC for their continued support and valuable suggestions during my research work. I wish to record special thanks to the non-teaching staffs of the institute, Mr. V. B. Gupta in particular, for their helping attitude during my tenure as a research scholar.

I am also thankful to CSIR, New Delhi, India for providing me financial support for research work during my Ph.D. program.

A special word of thanks goes to my co-scholars and friends **Chandra Shekher Singh, Ghanshyam Prasad, Sarita Yadav, Arpita Roy, Harendra Singh** and **Vijay Kumar Shukla** for their earnest help.

I am grateful to my close friends **Rakesh Raushan, Ashutosh Kumar Dubey, Shailendra Kumar Tiwari** and **Kamlesh Kumar** for being with me in my joys and sorrows.

Words are not enough to express my gratitude to my parents for their endless love and care for me. My deepest respect to them for the sacrifices they have made for me and accepting my decisions. I also want to thank the other members of my family, especially my elder brothers and sisters, for patiently hearing my problems, advising me well and giving me strength to continue even in most difficult situations.

Finally, I bow down before the **Almighty** for his infinite love and blessings.

Navin Kumar Jha

Varanasi, India

August 2016

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List of symbols

ϕ	The Scaling Function
ψ	The Wavelet Function or *Mother wavelet*
A	Constraint Matrix
W	Spectrum of the desired signal in passband
Y	Spectrum of the matched wavelet in passband
1	vector of length L whose entries are 1's only
$\mathbb{Z}$	Set of integers
$\mathbb{R}$	Set of real numbers
$L^2(\mathbb{R})$	Set of all square integrable functions $\mathbb{R}$
$L^2(\mathbb{R}^n)$	Set of all square integrable functions $\mathbb{R}^n$
$(W_\psi)f(a,b)$	Continuous wavelet Transform of function f with respect to wavelet ψ
Subscripts	
Λ_Ψ	Group delay of the matched wavelet
Λ_F	Group delay of the desired signal
C_ψ	Admissibility constant
Acronyms	
STFT	Short-time Fourier transform
CWT	Continuous Wavelet Transform
DWT	Discrete Wavelet Transform
MRA	Multi-Resolution Analysis
CR	Compression Ratio
PRD	Percent root mean square difference

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Preface

The thesis comprises of five chapters including an introductory chapter which primarily attempts to offer broad foreground to study our problem by providing the fundamentals of wavelet transform and the literature survey relevant to the proposed problems. The rest of the thesis is divided into two parts. In the first part (chapter 2), some results related to multidimensional wavelet transforms have been presented, while the second part (chapter 3, 4 and 5) is focused on construction of signal-dependent wavelets (matched wavelets) with their applications in some engineering problems.

The work presented in chapter 2 generalizes the conventional approach to the multidimensional wavelet transform with positive scales to the case of both positive and negative scales with respect to its inversion. We propose and prove a wavelet inversion formula for $f \in L^2(\mathbb{R}^n)$. It is shown that the dilation parameter a can be chosen to be in $\mathbb{R}^n$ with none of the components a_j vanishing in contrast to the approach chosen by Daubechies and Meyer who restricted a to be in $\mathbb{R}^+$. Further, we prove that if f and $\psi_{a,b}$ are continuous in $\mathbb{R}^n$, then the convergence, besides being in $L^2(\mathbb{R}^n)$, is also pointwise in $\mathbb{R}^n$. We have also shown that when f and ψ both belong to $L^2(\mathbb{R}^n)$ then the convergence of the wavelet inversion formula is pointwise at all points of continuity of f. This result significantly enhances the applicability of the wavelet inversion formula to the area of image processing.

In Chapter 3, need of signal-dependent wavelets (matched wavelets) is discussed. We also review the algorithm proposed by Chapa *et al.* (2000) for designing a wavelet matching to a specified signal and further modify it towards establishing application in some engineering problems.

Chapter 4 presents a method for inrush and fault detection for differential protection of power transformer. Matched wavelets have been constructed for both the inrush and the fault waveforms and further used as the mother wavelets in wavelet transform technique to analyse the output from a power transformer for distinguishing the inrush currents and fault currents. The simulated results show that the proposed technique facilitates good discrimination between magnetizing inrush and fault currents in transformer protection.

In chapter 5, a wavelet transform based method for compression of ECG signal has been presented. A wavelet matched to a cycle of an ECG signal in least squares sense has been constructed and further employed for compression using discrete wavelet transform. For performance measure, Percent Root Mean Square Difference (PRD) and Compression ratio (CR) results are obtained and presented. The results of ECG signal compression show better compression performance with matched wavelet compared to other mother wavelets.