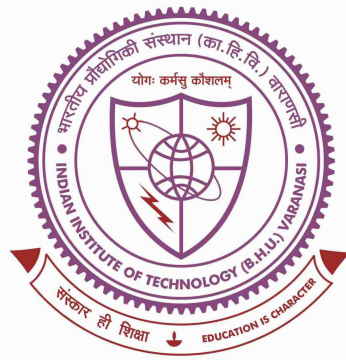# Lyapunov based Stability and Stabilization of Uncertain and Time-Delay Systems

Thesis submitted in partial fulfillment
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by

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Chapter 7

Conclusions and suggestions for future work

7.1   Research summary

The discussion presented in the entire work considers mainly the uncertainty and time-delay aspects of the problems arising in control theory. In this context, first a nonsmooth controller has been designed for the higher order uncertain chain of integrator type system and its asymptotic stability has been analyzed by constructing a new Lyapunov function. Second, for system with relative degree two, a control law has been formulated which is structurally similar to twisting algorithm and its closed loop stability has been analyzed by L-R approach. Finally, L-K method has been used to address some issues of stability analysis of linear systems with time varying delay. The following section emphasizes chapter wise main contributions of the present work.

7.2   Thesis Contributions

The main contribution of the work is summarized as follows:

A new controller has been introduced by modification in classical PI control in which the integral part of the PI controller is replaced by a discontinuous integrator. Due to this extra integrator, the overall control is absolutely continuous whereas the property of invariance concerning matched Lipschitz uncertainties is still preserved. The stability analysis of closed loop system has been carried by constructing a novel Lyapunov function

which is homogeneous and continuously differentiable.

An artificially delayed output feedback based controller is proposed which is structurally similar to the twisting algorithm. The main advantage of this controller is that it uses the information of output and artificially delayed version of it instead of the information of output and its derivative. The conditions on controller gains and the artificial delay are obtained by constructing a new Lyapunov-Razumikhin function.

For the stability analysis of system with time varying delay, a Lyapunov-Krasovskii functional has been formulated by introducing two new states. These new states have been used for the construction of delay product type functional and Lyapunov matrix based quadratic terms of the Lyapunov-Krasovskii functional. Based on this functional and in association with second order Bessel-Legendre inequality, two stability criteria are derived.

An augmented delay product based Lyapunov-Krasovskii functional has been constructed by including single and double integral states along with their interval normalized versions. On the basis of this LKF and negative determination lemma a stability criteria is derived leading to quadratic function based LMI conditions. Another new stability criterion is derived by avoiding quadratic function based LMI conditions by treating integral states and their normalized forms as individual states in the derivative of the LKF.

Two new states which are combination of interval normalized single and double integrals are utilized to form delay coefficient based quadratic terms. On the basis of these quadratic terms two Lyapunov-Krasovskii functionals are constructed for stability analysis of neural networks with time varying delay. A zero equality is formulated by including the single integral states and its normalized versions. Due to the augmentation of this inequality in the derivative of proposed LKFs provides less conservative results by exploiting the time relation among the integral states.

7.3 Suggestions for future work

As is common in all stability and stabilization problems, the proposed approaches also have some shortcomings, and there exists scope for further improvements. These issues and future work can be pointed out as:

- The Non-smooth PI control is designed for systems in the chain of integrators form

or systems that can be reduced to the chain of integrators form. So, for systems in more general form and for system that cannot be reduced to the chain of integrators form, we need to redesign the Non-smooth PI control.

- In this work, the disturbances are assumed to be matched. Although such an assumption on the disturbance is common in control literature, it remains an open area to formulate control for the case of mismatched disturbances.

- A delayed output feedback controller has been proposed in this work in which the controller gains are obtained using Lyapunov-Razumikhin method. Also, the maximum value of delay up to which this controller is stable has been calculated by the application Lyapunov-Razumikhin method. In future one can estimate the controller gains and upper bound of delay by using Lyapunov-Krasovskii approach.

- In this work, new Lyapunov-Krasovskii functionals are proposed for linear systems with time-varying delay. These LKFs can be utilized for the stability analysis of network control systems, load frequency control and wide area control of power systems.

- In this work, delay product type Lyapunov-Krasovskii functionals has formulated for stability analysis of delayed neural network. One can extend this idea for the extended passivity analysis for Markovian Jump Neural Networks, stability and stabilization of delayed fuzzy systems and in robust $H_{infinity}$ control for T-S fuzzy Systems with State and Input Time-varying Delays.

- Our present work is a continuous-time formulation of stability and stabilization. In future, we plan to propose the discrete-time formulation as well.