Dedicated

to

my beloved parents Mr. and Mrs. B. Mahto,

amazing wife Julie Mahto

and

lovely daughter Harshita

CERTIFICATE

It is certified that the work contained in the thesis titled **Lyapunov based Stability and Stabilization of Uncertain and Time-Delay Systems** by **Sharat Chandra Mahto** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION

I, **Sharat Chandra Mahto**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. S. K. Nagar** and **Prof. R. K. Saket** from July-2015 to December-2020, at the Departement of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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Abstract

Time delay and external disturbance exist in many practical systems such as aircrafts, robotic manipulators, process control and manufacturing systems. It is the main source of instability or oscillation in many systems. The thesis work is based on Lyapunov analysis and control designs for systems with disturbance and time delay. The work can be divided into three main parts. The first part deals with the stabilization of systems with nonsmooth PI controller under uncertainty. The proposed control law has been designed by replacing the integral part of the proportional-integral controller by a discontinuous integrator such that it can handle all types of Lipschitz disturbances either non-vanishing or vanishing ones at the origin. Due to the addition of this extra integrator, overall control becomes absolutely continuous and the property of invariance concerning Lipschitz matched uncertainties is preserved. The main advantage of this controller is that it does not requires any additional information other than state variables. The stability of the proposed nonsmooth PI controller for an uncertain chain of an integrator is established for first order system via non-trivial strict Lyapunov function; then it is extended to an uncertain chain of an integrator.The performances of the controller are demonstrated using Matlab simulations of a magnetic suspension system.

In the second part an artificially delayed output feedback based controller which is structurally similar to the twisting algorithm has been designed for the uncertain system. The proposed controller uses the information of output and an artificially delayed version of it instead of using the information of output and its derivative. The condition on controller gains are obtained by the application of standard Lyapunov technique in the presence of time-varying disturbance. The maximum delay upto which the controller remains stable has been obtained by the application of Lyapunov-Razumikhin approach. Apart from time-varying disturbance rejection, the controller can also be used as an observer. Moreover, it can also mitigate the unmatched disturbance for an uncertain second

order system. The simulation results illustrates the convergence of states to equilibrium point when the controller gains and artificial delay are selected.

The third part mainly consists of the application of Lyapunov-Krasovskii theorem for the stability analysis of linear system with time-varying delay. Lyapunov-Krasovskii approach are used for obtaining delay-dependent stability criteria in terms of linear matrix inequalities. The key problem of this approach is that the stability criterion established involves conservativeness and requires consistent efforts to reduce such conservatism. In order to overcome the conservativeness, some novel delay product based Lyapunov-Krasovskii functionals are formed by introducing new states in the augmented vectors. These functionals have quadratic terms with time-varying delay coefficients so that their derivative leads to terms with the product of delay and its derivative so as to well reflect the information of delay. Further, some zero equalities are used by involving the integral states and its normalized versions with slack variable matrices of appropriate dimensions. These inequalities will be merged with the derivative of functionals to exploit the time relation among the integral states. The effectiveness of the proposed LKFs and zero equalities are demonstrated by considering several numerical examples.

Contents

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Nomenclature

List of Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_+$	Set of positive real numbers
$\mathbb{R}^n$	The set of real n vectors
$R^{m\times n}$	The set of real $m \times n$ matrices
$\mathbb{N}$	Set of positive numbers
$\mathbb{R} \to \mathbb{R}$	Element-wise mapping
$\mathbb{R} \rightrightarrows \mathbb{R}$	Set-wise mapping
$\mathbb{C}_{[-\tau,0]}$	Banach space of continuous functions
P	Positive definite matrix
I	An identity matrix with appropriate dimension
0	A null matrix with appropriate dimension
X^T	Transpose of matrix X
$\mathrm{diag}(*)$	A diagonal matrix
$\mathrm{Col}(*)$	A column matrix
$*$	The symmetric terms in a matrix is denoted by $*$
$\|\cdot\|$	Standard norm

Abbreviations

PI	Proportional integral
PID	Proportional integral derivative
SMC	Sliding mode control
LF	Lyapunov functions
CLRF	Control Lyapunov-Razumikhin functions
STA	Super twisting algorithm
LK	Lyapunov-Krasovskii
LR	Lyapunov-Razumikhin
LKF	Lyapunov-Krasovskii functional
DPF	Delay product type functional
LMI	Linear matrix inequality
NLV	Number of LMI variables
BLI	Bessel-Legendre inequality
NDL	Negative determination lemma
DNN	Delayed neural network
LAUB	Largest admissible upper bound