Vector Framework based Stability and Stabilization of Nonlinear Dynamical Systems

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by

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Chapter 8

Summary and Future Perspectives

In the brief, we summarize the work done in the thesis by pointing out the main results, limitations and proposed future investigations.

8.1 Research Summary

To weaken the hypothesis of Lyapunov stability and to enlarge the class of Lyapunov theory, we presented the generalized control design approach to stabilize nonlinear systems in arbitrary time using the framework of vector Lyapunov functions and comparison systems. We have considered large-scale dynamical systems with external bounded matched disturbances and we found that it is quite tedious to construct a single scalar Lyapunov function for these large-scale systems to analyze stability. Thus, it is relevant to break these systems into smaller subsystems and construct a scalar Lyapunov function corresponding to each subsystem. Hence, the proposed results offer a flexible framework since each component of vector Lyapunov functions can satisfy less rigid requirements as compared to a single scalar function. We have shown that it is also robust to bounded disturbances by the inclusion of sliding mode control. In addition, we aggregated the comparison system to reduce its dimension in order to make the proposed approach efficient and straightforward, specifically for underactuated systems. Finally, we assessed through an example accompanied by simulations the efficacy of the mathematical results.

However, it was felt that trouble comes in some situations that how to find a suitable Lyapunov function candidate. Moreover, one other restriction is also observed in Lyapunov methods, they provide stability relative to some attractor or equilibrium point.

On the contrary, an alternative theory, contraction analysis is also somewhat restrictive, particularly for a large-scale nonlinear system, since it is quite complicated to check negative definiteness property or its mild variations of the Jacobian matrix for a large-scale nonlinear system. Hence, to overcome the problems faced in Lyapunov stability and standard contraction analysis, we introduced a new tool, vector contraction analysis utilizing a notion of the vector-valued norm, which evidently induces a vector distance between any pair of trajectories of the system, performs convergence analysis in a simplified way through a comparison system. In the wake of the exploitation of the comparison system, now the convergence analysis is performed by comparing the relative distances between any pair of the trajectories of the original nonlinear system and the comparison system. Some elementary mathematical tools such as quasi-monotonicity property and vector differential inequalities are collected to derive comparison results. Moreover, this theory has been verified and approved by academic examples followed by simulations.

Furthermore, we exploited the proposed analysis for the design of controller and observer for a class of nonlinear systems. General results are derived to estimate the control input and observer gains through a suitable selection of the comparison system. The theoretical results are illustrated through the example of controller design to synchronize two duffing systems followed by the observer design problem of Chua's circuit. Furthermore, the proposed theory is able to solve the problem of interval observer design in a very efficient way in the essence that it does not require the formulation of error dynamics and need not require the Lyapunov candidate function to show convergence between any pair of system trajectories. Dynamic output feedback control has been designed using state bounds from the constructed interval observer to prove it to be globally asymptotic stable. In the end, examples are illustrated to show the efficacy of the theoretical results.

Finally, we exploited the proposed theory for the analysis of different consensus problems in multi-agent agents such as consensus of third-order dynamics multi-agent systems with acceleration and input constraints, a consensus of multi-agent systems with heterogeneous nodes under communication imperfections, synchronization of multi-agent systems with connected and disconnected switching topologies, synchronization of agents in networked systems (like Hopfield networks) with time-varying couplings, and the synchronization of multi-agent systems with underactuated agent dynamics. The proposed procedure offers an advantage that proving the contraction for the comparison system is much easier than for the original complex multi-agent system in the sense of the Jacobian matrix which is quite complex for complex (large scale and nonlinear) multi-agent systems.

In the end, we demonstrated our proposed results on an experimental laboratory set up of a Helicopter model and validated experimental results with simulations. After that, we implemented on a Robotic system (two-link manipulator) and validated the developed theoretical results through simulations.

8.2 Limitations and Proposed Future Investigation

Since, it is common in all designs, the proposed results also have shortcomings and there exists a possible scope for further enhancement. These limitations and proposed further possibilities are entitled as

- An unpleasant fact in the proposed vector framework based stability analysis tool is the requirement of a quasi-monotone property of the comparison system, when dealing with component-wise inequalities between vectors. The proposed designs can be extended in the frame of cone ordering, or some other framework can be investigated in order to get rid off from this restriction.

- Our proposed results are based on continuous-time frameworks. These results can be extended to discrete-time frameworks as well.

- In this work, only matched disturbances have been taken into consideration when dealing with vector Lyapunov functions. However, when dealing with vector contraction analysis, disturbances have not been taken care off. Thus, one can extend the proposed results by considering different types of disturbances.

- The proposed work can be extended to the problems of synchronization, flocking, formation, and other relevant kinds of consensus aims. Implementation of the derived results on more realistic consensus problems such as agents with disturbances, delays can be done. Interval observers can also be designed for systems having delays. Moreover, this work can be compared with other existing approaches.

- The proposed vector framework based contraction theory can be reframed to provide rated convergence such as finite-time, fixed-time and prescribed-time stability.
- The proposed results can be implemented on more practical and experimental systems.