

Contents

Abstract	vii
List of Figures	xiii
Nomenclature	xv
1 Introduction	1
1.0.1 Why do we need vectors?	1
1.0.2 Why vector energy-like function (Vector Lyapunov function)?	2
1.0.3 Why vector distance function (Vector Contraction analysis)?	3
1.1 Motivation	7
1.2 Literature Review	10
1.2.1 Stabilization in arbitrary time	10
1.2.2 Controller and Observer design	11
1.2.3 Interval Observer design	11
1.2.4 Multi-agent systems	12
1.3 Contributions	13
1.4 Organization of the Thesis	16
2 Preliminaries	19
2.1 Comparison functions	20
2.2 Stability notions	21
2.3 General conditions in interval observer design	23
2.4 Graph theory	23
3 Vector Control Lyapunov function based stabilization of nonlinear systems in arbitrary time	25

3.1	Introduction	25
3.2	Arbitrary time stability analyzed via vector Lyapunov function	26
3.3	Robust arbitrary time stabilization of large scale nonlinear systems	30
3.4	Simulation example	32
3.5	Conclusion	34
4	Controller and Observer design using generalized Contraction theory	37
4.1	Introduction	37
4.2	Convergence analysis via vector contraction analysis	38
4.2.1	A vector-valued norm	38
4.2.2	Main comparison results	42
4.3	Controller and Observer design using vector-based Contraction theory . . .	48
4.3.1	Controller design	48
4.3.2	Observer design	49
4.4	Simulation examples	51
4.5	Conclusion	58
5	Interval Observer design	59
5.1	Introduction	59
5.2	Main Results of Interval Observer design	60
5.3	Simulation Examples	64
5.4	Conclusion	72
6	Consensus problems in multi-agent systems	75
6.1	Introduction	75
6.2	Vector based contraction theory for consensus problems in multi-agent systems	76
6.2.1	Consensus problems in third-order dynamics multi-agent systems with acceleration and input constraints	77
6.2.2	Consensus of multi-agent systems with heterogeneous nodes under communication imperfections	80
6.2.3	Synchronization of multi-agent systems with connected and disconnected switching topologies	83

6.2.4	Synchronization in networked systems with time-varying couplings	85
6.2.5	Synchronization control design for multi-agent systems with under- actuated agent dynamics	85
6.3	Examples	86
6.4	Conclusion	92
7	Applications	95
7.1	Introduction	95
7.2	2 DOF Helicopter Model	95
7.2.1	Controller Design	99
7.2.2	Experimental Results	100
7.3	Two link manipulator	105
7.3.1	Simulation Results	107
7.4	Conclusion	108
8	Summary and Future Perspectives	109
8.1	Research Summary	109
8.2	Limitations and Proposed Future Investigation	111
A	List of Publications	113
A.1	Journal papers	113
A.2	Conference paper	114

List of Figures

2.1 (a) Class \mathcal{K} function ($\psi(r) = \tan^{-1}(r)$) (b) Class \mathcal{K}_∞ function ($\psi(r) = r^2$)	20
3.1 System state (3.18)-(3.19) at arbitrary time $t_a = 3$ sec	35
3.2 System state (3.18)-(3.19) at arbitrary time $t_a = 5$ sec	35
4.1 State evolutions of system (4.7) with time	45
4.2 Evolution of the states of the system (4.24)	53
4.3 Evolution of the states of system (4.31) (4.32) with time	55
4.4 Response of the estimated states along the actual states and the error of the system (4.37) with time	57
5.1 Interval observer for system (5.9) (1(a)(b)(c) with disturbance and 2(a)(b)(c) without disturbance)	66
5.2 Interval observer for system (5.9) with disturbance and measurement noise	67
5.3 Interval observer for system (5.14)	69
5.4 Interval observer for system (5.19)	70
5.5 Interval observer for system (5.23) using (a) method in [57] and (b) proposed method	73
6.1 Initial positions of the agents of system (6.4)	87
6.2 Solution trajectories of the states of the multi-agent system (6.4) controlled by (6.25)	88
6.3 Control (6.25) for the multi-agent system (6.4)	88
6.4 Graph with heterogeneous nodes for Example 6.7	89
6.5 Solution trajectories of the states of the multi-agent system (6.9)-(6.10) controlled by (6.26)	89

6.6	Piecewise continuous switching signal for Example 6.8	90
6.7	Three switching topologies for Example 6.8	90
6.8	State evolutions of switching system (6.27)	91
6.9	Network used for simulation of system (6.18)	92
6.10	Evolution of all the states of the network (6.18)	92
6.11	Undirected graph for the Example 6.10	93
6.12	Evolution of position states with time	93
6.13	Evolution of agent velocities with time	94
7.1	Helicopter Experimental Set up	96
7.2	Helicopter Model	97
7.3	Free-body diagram of a 2 DOF Helicopter Model	97
7.4	Yaw, Pitch and Control input responses of a 2 DOF Helicopter Model for the set arbitrary time $t_a = 15$ sec	101
7.5	Yaw, Pitch and Control input responses of a 2 DOF Helicopter Model for the set arbitrary time $t_a = 20$ sec	102
7.6	Yaw, Pitch and Control input responses of a 2 DOF Helicopter Model for the set arbitrary time $t_a = 30$ sec	103
7.7	Yaw, Pitch and Control input responses of a 2 DOF Helicopter Model for the set arbitrary time $t_a = 20$ sec	104
7.8	Two link manipulator	105
7.9	Evolution of system (7.4) states and control inputs with set arbitrary time $t_a = 5$ sec (a, b) and $t_a = 8$ sec (c, d)	108