To almighty God, who has given me strength and healthiness.

To my beloved parents, who taught me honesty, simplicity, kindness and self-reliance.

To my teachers and friends, who constantly challenged me to go further.

CERTIFICATE

It is certified that the work contained in the thesis titled **Vector Framework based Stability and Stabilization of Nonlinear Dynamical Systems** by **Bhawana Singh** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree. It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION

I, **Bhawana Singh**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Shyam Kamal** and **Dr. Sandip Ghosh** from July-2017 to September-2021, at the Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the researchers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, thesis, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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Abstract

Classical stability analysis tools such as Lyapunov second method and contraction theory keep using scalar energy or distance-like function, which makes these tools somewhat restrictive specifically for large-scale systems. In this thesis, we develop further generalized results of these tools. In particular, we present stability analysis tools in the framework of vectors that relax the restrictions present in scalar cases and provide a simplified structure to perform stability or convergence analysis.

Firstly, we give special attention to the arbitrary time stability of nonlinear systems using vector Lyapunov functions. In fact, a vector comparison system, which is arbitrary time convergent, is constructed, and after that the stability of the original dynamical system is proved using differential inequalities and comparison principles. Moreover, the robust arbitrary time controllers for large-scale systems using vector control Lyapunov functions (VCLF) are designed. Also, aggregation of comparison systems is done to reduce their dimensionality in order to effectively apply the derived results on a class of underactuated systems. The theoretical results are implemented on an underactuated system.

Further, we introduce a new tool, vector contraction analysis that performs convergence analysis in a simplified way. The word vector came from the intuition that it employs vector-valued norm, which evidently induces a vector distance function between any pair of system trajectories to perform convergence analysis. It provides a very resilient framework as each component of vector-valued norm observes fewer strict conditions than that of a scalar distance function in classical contraction analysis and has a well-established structure of the vector-valued norm as compared to Lyapunov candidate function, which is very difficult to construct. In fact, every component need not to be negative definite to show convergence between any pair of trajectories. Rather the derivative of the vector-valued norm witnesses a component-wise inequality through some compar-

ison system with specified properties. In the wake of the exploitation of the comparison system, now the convergence analysis is performed by comparing the relative distances between any pair of the trajectories of the original nonlinear system and the comparison system. Some elementary mathematical tools such as quasi-monotonicity property and vector differential inequalities are collected to derive comparison results. To relax the quasi-monotone property, results are also derived in the framework of cone ordering. A level of mathematical sophistication is assumed in introducing and validating the properties of a new notion, vector-valued norm. To make the thesis easily perceivable to the readers, this theory is verified by academic examples followed by simulations.

Furthermore, we present tools for the design of controller and observer for a class of nonlinear systems using the proposed theory as this theory is quite appreciable for problems with convergence or synchronization aims, where there is no need of a specific attractor, rather convergence between trajectories is required. In addition, the proposed theory is able to solve the problem of interval observer design in a very efficient way in the essence that it does not require the formulation of error dynamics and need not require the Lyapunov candidate function to show convergence between any pair of system trajectories.

Finally, we exploit the proposed theory for the analysis of different challenging consensus problems in multi-agent agents such as consensus of third-order dynamics multi-agent systems with acceleration and input constraints, a consensus of multi-agent systems with heterogeneous nodes under communication imperfections, synchronization of multi-agent systems with connected and disconnected switching topologies, synchronization of agents in networked systems (like Hopfield networks) with time-varying couplings, and the synchronization of multi-agent systems with underactuated agent dynamics.

In the end, experimental and simulation validations of the proposed results are provided by considering a 2 DOF Helicopter model and a two-link manipulator.

Contents

List of Figures

Nomenclature

List of Greek and Roman Symbols

$\mathbb{R}$	Set of real numbers
$\mathbb{R}_{>q}$	Set of real numbers greater than q
$\mathbb{R}_{\geq q}$	Set of real numbers greater than or equal to q
$\mathbb{R}^n$	Set of $n \times 1$ column vector
$\mathbb{R} \to \mathbb{R}$	Element-wise mapping
K	Cone of the nonempty set
∂K	Boundary of the cone K
K^*	Adjoint cone of a cone K
K^0	Interior of a cone K
G	Graph
V	Vertex Set
D	Edge set
B	Adjacency matrix of G
L	Laplacian matrix of graph G
N_i	Set of indices of the neighbors of each node i

Abbreviations

SMC	Sliding mode control
VLF	Vector Lyapunov function
VCLF	Vector control Lyapunov function
TORA	Translational oscillator rotating actuator
MAS	Multi-agent systems