CHAPTER 6

SUMMARY AND SUGGESTIONS FOR FUTURE WORK

6.1 Summary

Classical coupled thermoelasticity theory has been developed to demonstrate the interrelation between thermal and elastic fields to understand the impact of one on the other and vice versa. However, this theory predicts an infinite speed of thermal waves. This behaviour gives satisfactory explanations for the thermoelastic processes involving small heat flux or large time intervals. However, it still offers a poor or inadequate interpretation of thermoelastic responses to the case of fast transient loadings or high heat flux. Fast transient processes have applications in structural design, aircraft engines, nuclear, chemical, and acoustic engineering, among many others. It prompted an urgent interest in researchers to modify the classical thermoelasticity theory. Several generalized thermoelasticity theories therefore came into existence. The theory proposed as generalized thermoelasticity with one relaxation time (LS theory) by following the Maxwell and Cattaneo heat conduction, the theory with temperature-rate dependent constitutive relations, the theory developed by focusing on low temperatures, theory by considering the absence of energy dissipation (GN theory), a dual-phase-lag (DPL) theory, a three-phase-lag (TPL) theory or the theory based on anomalous heat conduction characterized by fractional calculus are some of the well studied theories that are reported in the literature during last few decades. The current thesis is primarily concerned with the advancement and investigation of various aspects of temperature-rate dependent (TRD) theory in the context of various thermoelastic problems. The advancement of TRD theory in the context of a two-temperature model and for a porothermoelastic system of two-phase medium is provided, together with the theoretical and numerical results of various coupled thermomechanical problems. The analysis of the material properties on the thermoelastic behaviour by using an alternative numerical approach is also dealt with in this thesis. The key aspects of the present work are summarized as follows:

Chapter 1 describes a brief history of the development of various modifications to the classical thermoelasticity and a detailed literature review on the work relevant to the present work. In **Chapter 2**, a coupled thermoelastic problem of the homogeneous and isotropic hollow disk is investigated under the TRD theory. This chapter is mainly attempted to achieve two major goals. Firstly, it aims to analyze the thermoelastic responses of the hollow disk, which is subjected to thermal shock at its inner boundary. A comparative study of the prediction of TRD theory with LS theory due to the action of the present thermomechanical loading is carried out to understand the behaviour of the field variables under TRD theory more accurately. The differences between the predictions by these two theories are highlighted, and it is revealed that the effect of thermal shock is more prominent near the inner boundary under both theories. However, the effective domain of influence of TRD theory is larger as compared to LS theory. The second goal of the present chapter is to apply a numerical technique based on a complete finite element approach to obtain the solution of the problem. This method involves the finite element approaches for both space as well as for time domain. The present result is validated by comparing the results with the corresponding results obtained under two other numerical schemes, namely, trans-FEM method and FEM with Newmark time integration technique. Further, the efficiency of the present method is shown in terms of the CPU time taken under these three methods (complete FEM, FEM with Newmark and trans-FEM) to calculate the numerical solution of the present problem. It is shown that the present method is more efficient than the trans-FEM method. The computation times of complete FEM and FEM with Newmark method that is based on the predictor-corrector scheme are shown to be almost comparable. It is believed that the present method can be used as an alternative numerical approach for solving a coupled dynamical thermoelastic problem.

Chapter 3 is concerned with the investigation of generalization of TRD theory (strain and temperature-rate dependent or STRD theory) involving effects of stain-rate along with the temperature-rate in the thermoelasticity theory. In this chapter, a problem of functionally graded (FG) hollow disk subjected to thermal shock at both of its inner and outer boundaries is considered in such a way that the disk is composed of two different materials: metal and ceramic, and material properties of the disk vary along the radial direction. Based on the considered FG model, different FG configurations can be achieved by changing the value of non-homogeneity index term n, such that, at $n = 0$, the disk is completely metallic, while with the increasing values of n, the metallic property of the disk decreases and for $n \to \infty$, the disk becomes completely ceramic. The complete finite element method, as described above, is employed to obtain the solution of the problem in the space and time domain. In order to investigate the effects of the strain-rate term, the results under this theory are compared with the corresponding results under the TRD theory. Detailed analysis highlighting these effects on space-time distributions of displacement, temperature and stress components and also the effects of the non-homogeneity index term are presented under both STRD and TRD theories. Significant differences in the distributions of field variables are observed, and it is noted that the domain of influence under the STRD theory is larger in comparison with that of the TRD theory. By studying the behaviour of field variables under the different FG configurations, variation in the thermomechanical responses due to variable material properties can be observed. Here, it is noted that the effect of thermal shock is more significant for materials with larger metallic properties compared to materials with larger ceramic properties. There is a significant difference in the behaviour of physical field variables for different FG configurations, and the effective domain of influence is shown to increase prominently with the increase of the non-homogeneity index term.

Chapter 4 is devoted to the development and analysis of temperature-rate dependent thermoelasticity in the context two-temperature theory. The governing equations of the temperature-rate dependent two-temperature (TRDTT) thermoelasticity theory for the homogeneous and isotropic media has been introduced by Quintanilla [2008, 2009] while establishing the uniqueness and exponential stability results on the two-temperature thermoelasticity theory. However, the foundation of the TDRTT theory from the fundamental laws of thermodynamics was not available in the literature. **Sub-chapter 4.1** attempts to derive the governing equations of TRDTT theory by considering the more general form of the entropy inequality and the concept of generalized temperature. First, the set of nonlinear governing equations of the TRDTT thermoelasticity theory are derived from the modified first and second laws of thermodynamics in terms of generalized Helmholtz free energy function by taking the necessary constitutive assumptions. Then, following the linearity assumptions of the theory for small deformation, the set of linear governing equations of TRDTT thermoelasticity theory is obtained in terms of physical field variables and material parameters for the homogeneous and anisotropic medium. While formulating the theory, unlike the existing two-temperature relation, a modified two-temperature relation involving the temperature as well as temperature-rate term is obtained. This result is also convincing from the logic that the effect of temperature-rate terms should also be incorporated in the two-temperature relation under TRD theory. It is observed that the new two-temperature relation is reduced to the existing two-temperature relation for a particular case when the effect of temperature-rate term is removed from the theory. The uniqueness theorem of the new TRDTT theory is also established for a general mixed initial and boundary value problem of homogeneous and anisotropic thermoelastic solid. Lastly, the effect of this new two-temperature relation is analyzed from the study of thermoelastic interactions in a one dimensional half-space problem of the homogeneous and isotropic medium due to thermal shock under the present theory. The effect of two-temperature parameter, and relaxation parameters are highlighted by the results of field variables under this new theory. Comparison of the results with the corresponding results under other existing theories is investigated, and it is observed that differences due to the present and old two-temperature relations are not much significant for the materials with a much smaller value of relaxation parameter, t_1 and the temperature rate term in the two-temperature relation for such materials can be neglected. However, the effect of temperature-rate term in two-temperature relation is noted to be much more prominent for the materials with larger values of t_1. The effect of the two-temperature parameter, a_1 is also observed to be significant. For higher values of a_1, the effective domain of influence is found to increase.

Subchapter 4.2 continues to investigate the TRDTT theory for a different thermoelastic problem. In this subchapter, a two-dimensional infinite homogeneous and isotropic thermoelastic medium is considered to study the variations in the distributions of the physical field variables due to the presence of an opening mode crack in the medium. The unified set of governing equations under the TRDTT and TRD theories are derived to demonstrate the effects of two-temperature in the TRD theory. Firstly, the Laplace and Fourier transformations are used to simplify the present 2D problem, and then the solution of the field variables are derived in the Laplace and Fourier transform domains involving unknown parameters. A dual integral equation is formulated by applying the boundary conditions on the obtained solution in terms of an unknown parameter. An implementation of the regularization method along with the numerical method for solving the integral equations is demonstrated for solving a Fredholm integral equation of the first kind. Substituting the values of the unknown parameter in the derived solution and by taking inverse Fourier transformation, the numerical solution in the Laplace transform domain is obtained. Lastly, a technique of numerical inversion of the Laplace transform is applied to formulate the solution in the real space and time domain. The impact of thermal and mechanical loadings on the field variables in the weakened medium is examined with graphical results under both TRDTT and TRD theories. It is noted that except for the horizontal and vertical stress components, the effect of boundary loadings are not prominent for the field variables outside the crack length. However, the stress components exhibit the effect of boundary loadings even outside the crack length and show oscillatory nature in the vicinity of the crack. Comparative analysis on the behaviour of the field variables under TRDTT and TRD thermoelasticity theories indicates that the effect of the two-temperature theory is prominent.

Chapter 5 aims at the development of the TRD thermoelasticity theory for the poroelastic medium. Porothermoelasticity theory has application to the fields of geophysics, mining and other branches of science involving the problems of thermal and elastic interactions in the two phase medium. **Subchapter 5.1** attempts to establish a mathematical foundation for the temperature-rate dependent porothermolelasticity (TRDPTE) theory and present the basic governing equations of the theory. The general entropy inequality is considered by following Muller's entropy inequality (Muller (1967)) involving generalized temperature and Biot's entropy inequality of two-phase medium to derive the general nonlinear governing equations of the TRDPTE theory. Further, the linearity restrictions are imposed to formulate the linear governing equations for the TRDPTE theory of homogeneous and anisotropic medium. **Subchapter 5.2** deals with the development of the TRDPTE theory and establishes some theoretical results based on this theory. First, a uniqueness theorem for the general mixed initial and boundary value problem of homogeneous and anisotropic medium is presented to show the uniqueness of the solution under TRDPTE theory for any porothermoelastic problem. Next, a variational principle is formulated for the homogeneous and anisotropic porothermoelastic medium that represents an alternative form of the considered porothermoelastic system. This variational principle could be useful in the development of numerical approaches like FEM for solving the porothermoelastic systems. Lastly, a convolutional type reciprocity theorem is established to provide a reciprocity relation between two different systems of thermoelastic loading and configurations. Such relations are helpful in the development of the boundary element method for solving dynamical problems of porothermoelastic medium.

In **Subchapter 5.3**, the application of the present TRDPTE theory is illustrated by solving a half space problem subjected to thermal shock on its boundary. The thermoelastic interactions inside a liquid saturated porous medium are investigated. Numerical results for a kerosene saturated sandstone medium is presented, and the behaviour of the field variables for the two phase medium is analyzed. The present results are compared with the corresponding results of the field variables in the absence of porosity, and it is noticed that the temperature-rate term exhibits a larger influence in the presence of porosity. The absolute values of the field variables are observed to be larger under the influence of porosity. Moreover, the field variables attain their peak values at a larger distance from the boundary of the half space, which is subjected to thermal shock. While comparing the results under TRDPTE theory with the results under classical porothermoelasticity theory, it is observed that the effect of thermal shock is significant, and the domain of influence under classical porothermoelasticity is larger as compared to the TRDPTE theory, which is due to the infinite speed behaviour of classical theory as compared to the theory of TRDPTE that admits finite speed of heat signals.

6.2 Suggestions for Future Work

Thermoelasticity theory has a variety of applications in the fields of engineering and science. The gradual development and application of the thermoelasticity theories to the various thermoelastic problems that are available in the literature indicate the importance of the subject. The study of mathematical consistency, i.e., the well-posedness and physical relevance of a theory, plays an important role in determining the suitability of the proposed models regarding the application to real world problems. It also motivates to generalize further or modify the existing models. Therefore, it is worth developing a unified approach to study the well-posedness and physical relevance of various thermoelastic models. Among all the generalized thermoelasticity theories, only a few theories are reported that permit the finite wave speed of thermal signals. The present thesis demonstrates some mathematical aspects of TRD thermoelasticity theory and various generalizations of this theory like, STRD theory, TRDTT theory and TRDPTE theory. Due to the complexity involved in the formulation of the governing equations under TRD theory, the mathematical foundation of this theory in the context of other fields like electro-thermoelasticity, visco-thermoelasticity, magneto-thermoelasticity, thermo-piezo-electricity are not available. Therefore, it is worth studying the development of the TRD theory under these couple-thermomechanical fields. The formulation of numerical approaches for solving the dynamical problems of thermomechanics providing accurate and efficient results, help in understanding the behaviour of the mathematical models more accurately. In the present thesis, different numerical techniques like trans-FEM, complete FEM, FEM with Newmark method, state-space approach, direct methods involving integral transforms are used for solving the different thermoelastic problems. In general, trans-FEM is the most considerable method that is used for solving structural and engineering problems. Therefore, the application of advanced FEMs like meshfree or meshless FEM, wave enriched FEM, etc., for solving the thermoelastic problems will be useful for accurate prediction of thermoelastic responses under various practical problems. However, limited work is reported in the literature in this direction. Moreover, the computation cost of the FEMs are in general much larger than other numerical methods (like different finite difference methods), and the computation cost of advanced FEM techniques are even higher than that of the FEMs. Therefore, it is worthy pursuing further research work towards the challenging task of developing more efficient and accurate numerical techniques and improving the existing numerical tools for investigating thermomechanical problems. The thesis is therefore concluded with the suggestions for future studies in connection to the developed theories for advancing our understanding in these areas.