PREFACE

The thermoelasticity theory, which involves the simultaneous study of the thermal and deformation changes in the solid, is developed to support the general observation that deformation of an elastic solid causes some temperature changes inside the body and thermal changes in the solid may also cause some deformation in the elastic solid. This theory has developed considerable interest among the engineers and researchers to study the vast range of its applications in the various disciplines of science and technology. Thermal stress analysis is very important in a wide range of structural challenges, including high-speed plane manufacture, space vehicle, rocket, jet engine design, nuclear reactor design, and so on. Moreover, the thermoelasticity theory is increasingly being used for a range of engineering problems, including the development of material parts that can endure sudden thermal and mechanical stresses and function at high temperatures. Therefore, understanding this aspect and carrying out further research are necessary for the design and development of structures in a variety of engineering sectors, such as nuclear, chemical, and metallurgy. Various other branches of mechanics and physics like porothermoelasticity, viscothermoelasticity, pizothermoelasticity, magnetothermoelasticity, and many other sub-branches, which have drawn attention of researchers, are also founded as extensions of the thermoelasticity theory.

The classical thermoelasticity theory developed by Biot (1956) has been widely considered to investigate the various thermoelastic problems. However, the classical thermoelasticity theory is based on Fourier's law of heat conduction and suffers from the paradox of infinite speed of thermal wave. This theory has been found to be relevant to the problems with large spatial dimensions or short time responses; however, for thermoelastic processes such as laser-material interactions, where severe temperature gradients or extremely short-time behaviour are involved, the theory predicts unsatisfactory outcomes. Hence, starting from Maxwell in the late 19th century, several models predicting finite thermal wave speed as Lord-Sulman (LS) theory (1967), Green-Lindasy (GL) theory (1972), Green-Naghdi (GN) theory (1991-1993), dual-phase-lag (DPL) theory (1998), three-phase-lag (TPL) theory (2007), have been proposed over the time. The GL theory, which is also referred to as temperature-rate dependent (TRD) theory, is one of the most considered thermoelasticity theories for investigating the thermomechanical interactions. The theory is also well established from the fundamental laws of irreversible thermodynamics, and therefore, it is one of the most suitable theories for studying the dynamical problems of thermoelasticity.

The present thesis is concerned with the study of the TRD theory and its applications to the various problems of thermomechanics. Formulation of the TRD theory involves the dependence on temperature-rate term for the constitutive response functions and involvement of this new term increases the complexity for derivation of the governing equations. Therefore, governing equations for the extensions of this theory in different thermomechanical contexts like two-temperature theory, porothermoelasticity theory with non-local effects, micropolar theory and many other are either directly stated by the researchers without any theoretical justification or not available in the literature. This thesis therefore attempts to provide a strong mathematical foundation to some thermomechanical extensions of the TRD theory and also to establish some theoretical as well as numerical results on these theories.

The study of the coupled dynamical problems consists of a complicated system of partial differential equations (pde) and the solution to this system of pde cannot be derived analytically except for a few simplified thermoelastic problems. Therefore, the development of new effective and efficient numerical approaches is essential for the advancement of technologies that require the fast and accurate prediction of the solution of real life problems. The study and implementation of various numerical approaches for solving different thermoelastic problems are also addressed in the present thesis.

The work carried out in the thesis is divided into six chapters and outlines of various chapters are as follows:

Chapter 1 starts with an introduction to the subject, including the brief history on the development of various modifications to the classical thermoelasticity theory, and then moves on to a comprehensive literature review of works relevant to the current thesis. The chapter concludes with a summary of the objective of the thesis.

Chapter 2 discusses the application of TRD theory for a problem of a homogeneous and isotropic hollow disc subjected to thermal shock at its inner boundary and caries out the comparative study of TRD theory with the LS thermoelasticity theory in the context of the present problem. An application of a complete finite element approach for solving the coupled dynamical thermoelastic problem is demonstrated by an implementation of this method for the present problem. Furthermore, the validation and efficiency of the results under the complete FEM method are discussed by comparison of the corresponding results under trans-FEM method and FEM with Newmark scheme.

Chapter 3 aims at the investigation of the strain and temperature-rate dependent thermoelasticity theory. A problem of non-homogeneous and isotropic functionally graded hollow disk under the thermal shocks applied at both of its inner and outer boundaries is considered and a detailed comparative study of STRD theory and TRD theory is carried out to analyze the effects of inclusion of strain rate terms in TRD thermoelasticity theory. Functionally graded materials (FGM) are composite materials designed to withstand the sudden applied thermal and mechanical loads. In this chapter, the FGM material composed of metal and ceramic material is taken to discuss the change in the behaviour of the field variables due to variable material properties. The complete finite element approach is applied to derive the numerical solution of the present problem. Some important findings are highlighted.

Chapter 4 is focused on the development and application of the temperature rate-dependent two-temperature (TRDTT) thermoelasticity theory for different thermoelastic problems. This chapter is divided into two subchapters. The **Subchapter 4.1** attempts to establish the foundation of the TRDTT theory from the fundamental laws of irreversible thermodynamics. The modified entropy production inequality involving two generalized temperatures corresponding to thermodynamic and conductive temperatures are presented to derive the governing equations of the TRDTT theory. A general uniqueness theorem is proved for the mixed initial and boundary value problem of thermoelasticity of the homogeneous and anisotropic medium in the present context. Furthermore, an application of the TRDTT theory is illustrated by solving a half-space problem of homogeneous and isotropic thermoelastic medium. **Subchapter 4.2** discusses the application of the TRDTT theory for a coupled dynamical thermomechanical problem in the presence of a mode-I crack inside a two dimensional medium to examine the effects of the two-temperature model on the basis of TRD theory. A thermoelastic problem of 2D infinite homogeneous and isotropic medium in the presence of an opening mode crack of finite length is considered. A suitable numerical method involving the Laplace and Fourier transformation techniques followed by a regularization method together with the numerical method for solving the dual integral equation is applied to obtain the solution of the present problem. The behaviour of the physical field variables due to the presence of the crack in the two-temperature model are discussed in detail.

Chapter 5 comprises of three subchapters that are focused on the investigation and development of the porothermoelasticity theory in the context of temperature-rate dependent theory. **Subchapter 5.1** formulates the temperature-rate dependent porothermoelasticity (TRDPTE) theory and derives the basic governing equations and constitutive relations of the theory from the fundamental laws of thermodynamics. A generalized entropy inequality for TRD porothermoelasticity theory is presented by following Biot's theory of porothermoelasticity (1973) and Muller's entropy (1968) inequality, and the system of non-linear governing equations of the TRDPTE theory is derived. Lastly, the linearity conditions are applied to derive the linear governing equations for the TRDPTE theory. **Chapter 5.2** attempts to establish some theoretical results on the TRDPTE theory, which are helpful in understanding the theory, and provide the basis for the development of some numerical methods as well. Firstly, a uniqueness theorem for the general porothermoelastic problem of homogeneous and anisotropic medium is proved. Then, a variational principle is presented by deriving an expression for the total energy function of the porothermoelastic system. Lastly, a reciprocity theorem is established that provides the relationship between two different sets of porothermoelastic loadings and corresponding porothermoelastic configurations. **Subchapter 5.3** investigates the present TRDPTE theory by applying it to a one dimensional half space problem subjected to thermal shock on the boundary. Laplace transformation technique for the time domain along with a direct approach is employed to solve the half space problem. The effects of porosity is examined for classical theory and TRD theory by the comparative study of the behaviour of the field variables under Biot's theory and TRD theory with and without porosity.

Chapter 6 provides a summary of the work presented in the thesis and suggestions for further research in these related topics.