STUDY OF NONLINEAR WAVE PROPAGATION PROBLEMS IN GASEOUS MEDIA

Thesis submitted in partial fulfillment

for the Award of Degree

Doctor of Philosophy

by

POOJA GUPTA

DEPARTMENT OF MATHEMATICAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY

(BANARAS HINDU UNIVERSITY)

VARANASI-221005

Roll Number: 16121501

December 2021

Chapter 7

Summary and Future scope

> "God used beautiful mathematics
> in creating the world."
>
> –Paul Dirac

7.1 Overall Summary

In this thesis the phenomenon of shock wave propagation is explored in different gaseous media. This thesis deals with detailed analytical and numerical solution of the specific problems, formulated mathematically as IVPs/BVPs, associated with quasilinear hyperbolic system of partial differential equations. The analytical solution of the Riemann problem for non-ideal polytropic gas with an added effect of transverse magnetic field and van der Waals Gasdynamics is obtained. The explicit formulas for the shock waves and rarefaction waves are determined. By using the Lax entropy condition and R-H conditions, the elementary wave solutions i.e. shock wave, simple wave and contact discontinuities without any restriction on the magnitude of initial data states are derived. The density and velocity profiles for compressive waves and rarefaction waves is presented. In the flow-field, the velocity and density distributions for the compressive and rarefaction waves are discussed. The effects of intermolecular forces of attraction between the particles, non-ideal gas parameter and magnetic field strength parameter on the velocity and density profile across the elementary waves are studied.

The main features of weakly non-linear waves propagating in a compressible, inviscid non-ideal radiating gas and dusty gas flow is studied by using the method of progressive wave analysis. An evolution equation is derived which describes the propagation of disturbance in high frequency domain and determined the condition for the formation of shock wave at a finite time. A sufficiently weak shock is taken at the front and analyzed the motion of the weak shock wave in the form of sawtooth wave (half-N wave). The length and velocity of sawtooth wave for planar and cylindrically symmetric flows in non-ideal radiating and dusty gas are discussed and shown graphically. The effect of non-idealness parameter, radiation, mass fraction of solid particles (k_p), ratio of specific heat of solid particles to specific heat of gas at constant pressure (β) and axial magnetic field (μ) on the length and velocity of sawtooth wave for planar and cylindrically symmetric flow are studied in detail.

The small amplitude high frequency asymptotic solution for the system of nonlinear partial differential equations characterizing one-dimensional compressible unsteady, planar and non-planar flows in a dusty gas is derived by using the multiple time scales method. The theory of weakly non-linear geometrical acoustics is utilized to examine the resonant interaction of waves and to analyze the evolution of shock wave in a dusty gas flow. The transport equations for the wave amplitude along the rays for the dusty gas flow, comprising of a system of inviscid Berger's equations with known kernel, has been derived. Finally, the existence of shock wave in a dusty gas is also discussed .

7.2 Future Scope

This section provides future work to consolidate the study presented in this thesis. Our study is restricted to a one-dimensional system of non-linear partial differential equations in gasdynamics. However, this analysis can be extended for two or higher dimensional non-linear partial differential equations in gas dynamics. The key areas that can be focused for future research are identified here. We can highlight some of the proposed extensions of the work made in the thesis as follows:

- Analytical study of the Riemann problem in a higher dimensional system of non-linear partial differential equations in various gas dynamics regimes.
- The closed form solution of the Generalized Riemann Problem for the higher dimensional Euler's equation for various gas dynamic regimes by using the Differential Constraint Method. On the behalf of applications of Differential Constraint Method, one can use this method to determine the exact solution to the Generalized Riemann Problem for two-phase flow in gasdynamics.
- To discuss the interaction of elementary waves for Riemann problem in a higher dimensional system of non-linear partial differential equations in various gas dynamics regimes.
- Numerical study of the Riemann problem in higher dimensional Euler's equation for various gas dynamic regimes using different types of Riemann solvers.
- To analyze the evolutionary behavior of weak shock waves propagating in higher-dimensional flow in different gaseous media by using the method of asymptotic analysis.

- To derive the high frequency small amplitude asymptotic solution of the higher-dimensional quasilinear hyperbolic system of partial differential equations characterizing compressible, unsteady flow with generalized geometry in different gaseous media using the theory of weakly non-linear geometrical acoustics .

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