PREFACE

A wave of sudden rarefaction, though mathematically possible, is an unstable condition of motion, any deviation from absolute suddenness tending to make the disturbance become more and more gradual. Hence the only wave of sudden disturbance whose permanency of type is physically possible, is one of sudden compression. A wave is an oscillation that travels through a medium by transferring energy from one particle or point to another without causing any permanent displacement of the medium. It may be any features of the disturbance, such as a maximum or an abrupt change in some quantity, provided that it can be clearly recognized.

The present thesis, embodies the results of researches carried out by me at the Department of Mathematical Sciences, Indian Institute of Technology(BHU), Varanasi, during the period December 2016 to December 2021 under the supervision of Prof. L. P. Singh. In the present work certain aspects of nonlinear wave propagation problems have been studied in various gasdynamic regimes. The thesis is categorically divided into seven chapters.

Chapter-1 is introductory which describes the general understanding of the nonlinear wave propagation problems and the historical background. This chapter gives an idea of when and how a discontinuity appears and propagates. The physical properties of non-ideal gases, electrically conducting gases and radiating gases are discussed and briefly reviewed.

Chapter-2 concerns with the study of the Riemann problem for magnetogasdynamic equations governing an inviscid unsteady one-dimensional flow of non-ideal polytropic gas subjected to the transverse magnetic field with infinite electrical conductivity. The mathematical form of the Riemann problem is formulated and the

generalized Riemann invariants are determined. By using the Lax entropy condition and R-H conditions, we derive the elementary wave solutions i.e. shock wave, simple wave and contact discontinuities without any restriction on the magnitude of initial data states and discussed about their properties. The elementary wave solutions are obtained in the form of explicit expressions. Further, the density and velocity distribution in the flow field for the cases of compressive wave and rarefaction wave is discussed. Here we also compare/contrast the nature of solution in non-ideal magnetogasdynamic flow and ideal gas flow.

In **Chapter-3**, the evolutionary behavior of weak shock waves propagating in an unsteady one-dimensional flow in non-ideal radiating gas is analyzed. The effect of thermal radiation under optically thin limit is included in the energy equation of the governing system. The method of asymptotic analysis is used to derive the transport equation describing the propagation of waves under the high frequency conditions which is also used to determine the time of first wave breaking conditions. The equation governing the propagation of acceleration waves is also obtained. Further, the propagation of disturbance in the shape of saw-tooth profile is discussed. The effect of parameter of non-idealness under the influence of radiative heat transfer, on the decay of sawtooth profile is analyzed.

In **Chapter-4**, the theory of weakly non-linear geometrical acoustics is used to derive the high frequency small amplitude asymptotic solution of the one-dimensional quasilinear hyperbolic system of partial differential equations characterizing compressible, unsteady flow with generalized geometry in ideal gas flow with dust particles. The method of multiple time scales is applied to derive the transport equations for the amplitude of resonantly interacting high-frequency waves in a dusty gas. These transport equations are used for the qualitative analysis of non-linear wave interaction process and self-interaction of non-linear waves which exist in the system under study. Further, the evolutionary behavior of weak shock waves propagating in

ideal gas flow with dust particles is examined here. The progressive wave nature of non-resonant waves terminating into the shock wave and its location is also studied. Further, we analyze the effect of the small solid particles on the propagation of shock wave.

In **Chapter-5**, the analytical solution of Riemann problem for a quasilinear hyperbolic system of partial differential equations governing the one dimensional and unsteady flow of van der Waals gas is discussed. By utilizing Rankine-Hugoniot conditions and Lax entropy condition, we derive classical wave solution of Riemann problem and analyze their properties. Also, it is observed here that van der Waals gasdynamics system is more complex in comparison to ideal gasdynamics case. Further, the effect of presence of intermolecular forces of attraction between the particles and variation of covolume of the gas on the density and velocity distribution across the simple wave, shock wave and contact discontinuities is discussed. Also, we have shown that our results are in good agreement with already established results for an ideal gas.

Chapter-6 concerns with the evolutionary behaviour of weakly nonlinear waves propagating in one-dimensional unsteady flow of perfectly conducting compressible fluid subjected to a transverse magnetic field with dust particles. The method of progressive wave approach is utilized to determine the evolution equation characterizing the propagation of wave which also leads to determine the condition for first wave breaking at finite time. Further, we analyze the behaviour of acceleration wave propagating in the medium considered. Also, the propagation of disturbances in the form of sawtooth wave (half N-wave) is studied. It is observed that the presence of mass fraction of solid particles and ratio of specific heat of the solid particles to specific heat of the gas at constant pressure both causes to slow down the decay process. Also, the effect of axial magnetic field is to increase the growth rate of sawtooth wave (Half N-wave) as compared to in the absence of axial magnetic field.

Further, the effect of magnetic field is to slow down the decay process in the presence of mass fraction of solid particles.

Chapter-7 concerns with the summary and future scope of the work done in the thesis. This thesis deals with detailed analytical and numerical solution of the specific problems, formulated mathematically as IVPs/BVPs, associated with quasilinear hyperbolic system of partial differential equations. The analytical solution of the Riemann problem for non-ideal polytropic gas with an added effect of transverse magnetic field and van der Waals Gasdynamics is obtained. The main features of weakly non-linear waves propagating in a compressible, inviscid non-ideal radiating gas and dusty gas flow is studied by using the method of progressive wave analysis. The small amplitude high frequency asymptotic solution for the system of nonlinear partial differential equations characterizing one-dimensional compressible unsteady, planar and non-planar flows in a dusty gas is derived by using the multiple scales method. Our study is restricted to a one-dimensional system of non-linear partial differential equations in gasdynamics. However, this analysis can be extended for two or higher dimensional non-linear partial differential equations in gasdynamics.