CERTIFICATE

It is certified that the work contained in this thesis titled " ***Study of Nonlinear Wave Propagation Problems in Gaseous Media***" by ***Pooja Gupta*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

20/12/2021

Dr. L.P. Singh
(Supervisor)
Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

पर्यवेक्षक/Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

Forwarded
20/12/21

विभागाध्यक्ष/HEAD
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

DECLARATION BY THE CANDIDATE

I, ***Pooja Gupta***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. L.P. Singh*** from ***December, 2016*** to ***December, 2021*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Pooja Gupta

Date: 20.12.2021

(Pooja Gupta)

Place: Varanasi

(Roll no. 16121501)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.

L.P. Singh 20/12/2021

(Dr. L.P. Singh)
Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

पर्यवेक्षक/Supervisor
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

T. Som 20/12/21

(Prof. T. Som)
Professor and Head
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

विभागाध्यक्ष/HEAD
गणितीय विज्ञान विभाग
Department of Mathematical Sciences
भारतीय प्रौद्योगिकी संस्थान
Indian Institute of Technology
(काशी हिन्दू विश्वविद्यालय)
(Banaras Hindu University)
वाराणसी/Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: ***Study of Nonlinear Wave Propagation Problems in Gaseous Media.***

Name of the Student: ***Pooja Gupta***

Copyright Transfer

Pooja Gupta

Date: 20. 12. 2021

(Pooja Gupta)

Place: Varanasi

(Roll no. 16121501)

DEDICATED

TO

MY BELOVED PARENTS

AND

TEACHERS

ACKNOWLEDGEMENTS

With the blessings of Ishtadev Baba Vishwanath, Shri Hanuman ji and Maa Durga ji, I am going to write this thesis. Working for this thesis has certainly been a long and arduous journey, but it would not have 'been' at all possible were it not for the help and support of a rather special group of people.

It is a privilege to extend my ardent regard and gratitude towards my honorable supervisor ***Prof. L.P. Singh, Department of Mathematical Sciences, Indian Institute of technology, (Banaras Hindu University) Varanasi*** for his incessant encouragement, priceless assistance, guidance and constructive criticism throughout my stay at IIT(BHU). His role in shaping my understanding and helping me to look at everything with a positive approach has helped me a great deal, to which I'll be lifelong indebted to him. If not for exemplary exquisite guidance and insightful supervision, I would not have been able to successfully complete the work this well. His foreknowledge and thoughtfulness have made me learn a lot not only in the Ph.D. programme but also in difficult situations of my life. I am extremely thankful and obliged to his directions. I look forward to learn and grow more under his gracious presence in my life.

I owe my sincere gratitude to ***Prof. T. Som, Head, Department of Mathematical Sciences*** for providing all the necessary facilities and constant support to carry out my doctoral research work. I would like to express my deep sence of gratitude to ***Dr. Vineet Kumar Singh, Convener DPGC, Department of Mathematical Sciences*** for encouraging me during my research work. I also acknowledge my sincere thanks to DPGC members and all faculty members of the Department for their academic support, encouragement and moral support.

I am also thankful to my RPEC members ***Prof. P.K. Singh, Department of Civil engineering Indian Institute of Technology, (Banaras Hindu University) Varanasi*** and ***Dr. R. K. Pandey***, for their valuable suggestions throughout the period of research work.

I would like to acknowledge my sincere thanks to my research group *Dr. Raghwendra Singh, Dr. Raj Kumar Gupta, Dr. Triloki Nath, Dr. J.P. Chaudhary, Mr. Rahul Kumar Chaturvedi, Mr. Shobhit Kumar Srivastava, Mr. Dheeraj Shukla, Mr. Pradeep Maddhesiya, Ms. Shweta and Mr. Gaurav Upadhyay* for providing me a helping hand whenever I asked for and all the research scholars of the department for their moral supports.

Further, words would not be sufficient to express my thanks to my loving friends for their persistence help, unfolding moral support and for being with me in all situations.

I am also grateful to my Institute, IIT(BHU), for providing necessary resources throughout my research. I express my thanks to all non teaching staff members of the department for their support.

I gratefully acknowledge financial support by Department of Science & Technology (DST), India for awarding me INSPIRE fellowship during my research work.

Above everything, this Ph.D would not have been possible without the constant support, encouragement, blessings and good wishes of my family. I would never be able to repay their unconditional support and personal sacrifices, which made it possible for me to achieve this goal. I express my sincere and cordial gratitute to my father ***Shri Jagadish Prasad Modanwal*** and my mother ***Smt. Usha Gupta***. I have no words to express my affectionate thanks to ***my parents, my sisters,*** and ***brother*** for their innocent and enormous love.

This acknowledgement would be incomplete if the name of great visionary ***Pt. Madan Mohan Malaviya*** is not mentioned, who made this divine centre of knowledge. Deepest regards to him. I am also indebted to the Department of Mathematics, Banaras Hindu University, Varanasi.

Above all, I bow down before the Almighty who made my dream a reality.

Pooja Gupta

Contents

List of Figures

PREFACE

A wave of sudden rarefaction, though mathematically possible, is an unstable condition of motion, any deviation from absolute suddenness tending to make the disturbance become more and more gradual. Hence the only wave of sudden disturbance whose permanency of type is physically possible, is one of sudden compression. A wave is an oscillation that travels through a medium by transferring energy from one particle or point to another without causing any permanent displacement of the medium. It may be any features of the disturbance, such as a maximum or an abrupt change in some quantity, provided that it can be clearly recognized.

The present thesis, embodies the results of researches carried out by me at the Department of Mathematical Sciences, Indian Institute of Technology(BHU), Varanasi, during the period December 2016 to December 2021 under the supervision of Prof. L. P. Singh. In the present work certain aspects of nonlinear wave propagation problems have been studied in various gasdynamic regimes. The thesis is categorically divided into seven chapters.

Chapter-1 is introductory which describes the general understanding of the nonlinear wave propagation problems and the historical background. This chapter gives an idea of when and how a discontinuity appears and propagates. The physical properties of non-ideal gases, electrically conducting gases and radiating gases are discussed and briefly reviewed.

Chapter-2 concerns with the study of the Riemann problem for magnetogasdynamic equations governing an inviscid unsteady one-dimensional flow of non-ideal polytropic gas subjected to the transverse magnetic field with infinite electrical conductivity. The mathematical form of the Riemann problem is formulated and the generalized Riemann invariants are determined. By using the Lax entropy condition and R-H conditions, we derive the elementary wave solutions i.e. shock wave, simple wave and contact discontinuities without any restriction on the magnitude of initial data states and discussed about their properties. The elementary wave solutions are obtained in the form of explicit expressions. Further, the density and velocity distribution in the flow field for the cases of compressive wave and rarefaction wave is discussed. Here we also compare/contrast the nature of solution in non-ideal magnetogasdynamic flow and ideal gas flow.

In **Chapter-3**, the evolutionary behavior of weak shock waves propagating in an unsteady one-dimensional flow in non-ideal radiating gas is analyzed. The effect of thermal radiation under optically thin limit is included in the energy equation of the governing system. The method of asymptotic analysis is used to derive the transport equation describing the propagation of waves under the high frequency conditions which is also used to determine the time of first wave breaking conditions. The equation governing the propagation of acceleration waves is also obtained. Further, the propagation of disturbance in the shape of saw-tooth profile is discussed. The effect of parameter of non-idealness under the influence of radiative heat transfer, on the decay of sawtooth profile is analyzed.

In **Chapter-4**, the theory of weakly non-linear geometrical acoustics is used to derive the high frequency small amplitude asymptotic solution of the one-dimensional quasilinear hyperbolic system of partial differential equations characterizing compressible, unsteady flow with generalized geometry in ideal gas flow with dust particles. The method of multiple time scales is applied to derive the transport equations for the amplitude of resonantly interacting high-frequency waves in a dusty gas. These transport equations are used for the qualitative analysis of non-linear wave interaction process and self-interaction of non-linear waves which exist in the system under study. Further, the evolutionary behavior of weak shock waves propagating in ideal gas flow with dust particles is examined here. The progressive wave nature of non-resonant waves terminating into the shock wave and its location is also studied. Further, we analyze the effect of the small solid particles on the propagation of shock wave.

In **Chapter-5**, the analytical solution of Riemann problem for a quasilinear hyperbolic system of partial differential equations governing the one dimensional and unsteady flow of van der Waals gas is discussed. By utilizing Rankine-Hugoniot conditions and Lax entropy condition, we derive classical wave solution of Riemann problem and analyze their properties. Also, it is observed here that van der Waals gasdynamics system is more complex in comparison to ideal gasdynamics case. Further, the effect of presence of intermolecular forces of attraction between the particles and variation of covolume of the gas on the density and velocity distribution across the simple wave, shock wave and contact discontinuities is discussed. Also, we have shown that our results are in good agreement with already established results for an ideal gas.

Chapter-6 concerns with the evolutionary behaviour of weakly nonlinear waves propagating in one-dimensional unsteady flow of perfectly conducting compressible fluid subjected to a transverse magnetic field with dust particles. The method of progressive wave approach is utilized to determine the evolution equation characterizing the propagation of wave which also leads to determine the condition for first wave breaking at finite time. Further, we analyze the behaviour of acceleration wave propagating in the medium considered. Also, the propagation of disturbances in the form of sawtooth wave (half N-wave) is studied. It is observed that the presence of mass fraction of solid particles and ratio of specific heat of the solid particles to specific heat of the gas at constant pressure both causes to slow down the decay process. Also, the effect of axial magnetic field is to increase the growth rate of sawtooth wave (Half N-wave) as compared to in the absence of axial magnetic field.

Further, the effect of magnetic field is to slow down the decay process in the presence of mass fraction of solid particles.

Chapter-7 concerns with the summary and future scope of the work done in the thesis. This thesis deals with detailed analytical and numerical solution of the specific problems, formulated mathematically as IVPs/BVPs, associated with quasilinear hyperbolic system of partial differential equations. The analytical solution of the Riemann problem for non-ideal polytropic gas with an added effect of transverse magnetic field and van der Waals Gasdynamics is obtained. The main features of weakly non-linear waves propagating in a compressible, inviscid non-ideal radiating gas and dusty gas flow is studied by using the method of progressive wave analysis. The small amplitude high frequency asymptotic solution for the system of nonlinear partial differential equations characterizing one-dimensional compressible unsteady, planar and non-planar flows in a dusty gas is derived by using the multiple scales method. Our study is restricted to a one-dimensional system of non-linear partial differential equations in gasdynamics. However, this analysis can be extended for two or higher dimensional non-linear partial differential equations in gasdynamics.