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It is certified that the work contained in this thesis entitled **"STUDY OF DYNAMICS OF FLUID IN POROUS MEDIA"** by **"Prashant Pandey"** has been carried out under my supervision and that it has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements, Comprehensive examination, Candidacy and SOTA for the award of Ph.D. Degree.

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DECLARATION BY THE CANDIDATE

I, **Prashant Pandey**, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of **Prof. Subir Das** from a period **July 2017 to October 2, 2021** at **DEPARTMENT OF MATHEMATICAL SCIENCES**, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

Date: October 6, 2021

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Acknowledgment

First and foremost, I thank to Lord Shiva, the reigning deity of holy city Kashi for making my research journey fruitful, who always stands beside me in every up and down of the moments and gives me strength whenever I close my eyes and picture him within. My entire life is a thanksgiving phenomenon to Lord Shiva.

I would like to express my sincere gratitude and indebtedness to my mentor and my supervisor Prof. Subir Das, Professor, Department of Mathematical Sciences, IIT (BHU), Varanasi for his consistent support and guidance during the running of this Ph.D. degree. The meetings and conversations with my supervisor were vital in inspiring me to think outside the box, from multiple perspectives to form a comprehensive and objective critique.

I would like to pay my heartfelt regards to Prof. T. Som, Head, and Prof. Subir Das, Convener of DPGC of the Department of Mathematical Sciences, and the RPEC Members Prof. S. Mukhopadhyay and Prof. R. K. Mishra for their support throughout my research work. I acknowledge my deep sense of gratitude to all the Faculty Members of the Department of Mathematical Sciences for their valuable suggestions, appreciation, and encouragement. I am also grateful to all non-teaching staff members of the Department for their support.

I would also like to thank my seniors Dr. Vijay Kumar Yadav, Dr. Shubham Jaiswal, Dr. Pragya Singh, Dr. Vijay Kumar Shukla, Dr. Anup Singh, Dr. Rakesh Kumar and Dr. Anuwedita Singh for their sincere help. From the bottom of my heart I would like to say a special thank you to my roommate, friend, senior Dr. Om Namah Shivay and my colleague cum friend Mr. Kushal Dhar Dwivedi and Mr. Sachin Kumar who continuously making my life bearable and for being a savior of my life. I am also thankful to all my colleague researchers Mr. Rahul Kumar Chaturvedi, Mr. Prashant Kumar Pandey, Ms. Pragya Shukla, Mr. Umesh Kumar and Ms. Neha Trivedi of the Department of Mathematical Sciences for providing spontaneous support during writing the thesis and also for keeping me in good spirits.

I am grateful to the Council of Scientific and Industrial Research (CSIR), Govt. of India for providing the financial support.

Lastly, and most importantly a special thanks to my family, I feel a deep sense of gratitude to my grandmother Late Smt. Kapura Devi, Grandfather Late Mr. Prithvi Pal Pandey, Father Mr. Sheetala Deen Pandey, Mother Mrs. Neelam Pandey, younger brother Mr. Praveen Kumar Pandey, Mr. Prafulla Kumar Pandey and Mr. Pushpesh Pandey and my dear one who formed part of my vision and taught me the good things, continued care

and love that really matter in life. They are responsible for what I am today. Their continuous encouragements and endless patience have been the cornerstone of my life. Thus my gratitude for them is beyond expression through words.
Finally, I extend my thanks to very understanding better half Mrs. Priya Kumari for making so many sacrifices being my best friend and a true soul mate.
Last but not the least again; I bow down before the almighty Lord Shiva who made my dream come in true.
Above all, I bow down before the Almighty who made my dream a reality.

Date: December 5, 2021
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Preface

There are total six chapters in this thesis.The study of the thesis is based on one-dimensional, two-dimensional and coupled systems of solute transport model in porous media with space, time fractional derivative as well as variable order derivative.

In Chapter 1, literature review, some basic definitions and properties related to current thesis work have been discussed. A brief discussion about groundwater contamination and porous media is presented here. The historical background of fractional calculus theory and its various applications are added in this introductory chapter of the thesis.

In the Chapter 2, an operational matrix scheme with Laguerre polynomials is applied to solve a space-time fractional order non-linear Cahn-Hilliard equation, which is used to calculate chemical potential and free energy for a non-homogeneous mixture. Constructing operational matrix for fractional order differentiation, the collocation method is applied to convert Cahn-Hilliard equation into a system of algebraic equations, which have been solved using Newton method. The prominent features of the chapter is to provide the stability analysis of the proposed scheme and the pictorial presentations of numerical solution of the concerned equation for different particular cases and showcasing of the effect of advection and reaction terms on the nature of solute concentration of the considered mathematical model for different particular cases.

In the Chapter 3, an efficient operational matrix based on the famous Laguerre polynomials derived in Chapter 2 is applied for the numerical solution of two-dimensional non-linear time fractional order reaction-diffusion equation. Assuming the surface layers are thermodynamically variant under some specified conditions, many insights and properties are deduced e.g., nonlocal diffusion equations and mass conservation of the binary species which are relevant to many engineering and physical problems. The salient features of the chapter are finding the convergence analysis of the proposed scheme and also the validation and the exhibitions of effectiveness of the method using the order of convergence through the error analysis between the numerical solutions applying on the proposed method and the analytical results for two existing problems. The prominent feature of the present chapter is the graphical presentations of the effect of reaction term on the behavior of solute profile of the considered two-dimensional mathematical model for different particular cases.

The effective Laguerre collocation method developed in chapter 2 is used in Chapter 4 to obtain the approximate solution of a system of coupled fractional order non-linear reaction-advection-diffusion equations with prescribed initial and boundary conditions. In the proposed scheme, Laguerre polynomials are used together with operational matrix and collocation method to obtain approximate solutions of the coupled systems. The solution profiles of the coupled systems are presented graphically for different particular cases. The salient features of the present chapter are finding the stability analysis of the proposed method and also the demonstration of the lower variations of solute concentrations with respect to the column length in fractional order system as compared to integer order system. To show the higher efficiency, reliability and accuracy of the proposed scheme, a comparison between the numerical results of Burgers' coupled systems and its existing analytical result is reported. There are high compatibility and consistency between the approximate solution and its exact solution to a higher order of accuracy. The exhibition of error analysis for each case through tables and graphs confirms the super-linearly convergence rate of the proposed method.

In the Chapter 5, the Homotopy perturbation method is utilized with Laplace transform and He's polynomial to analyze and to obtain the approximate numerical solutions of a class of two-dimensional partial differential equations (PDEs) with Caputo fractional derivative. A time-fractional order mathematical model in two-dimension is analyzed with the given initial condition. The solution obtained is beneficial and significant to analyze the modeling of super-diffusive systems and sub-diffusive system, anomalous diffusion, transport process in porous media. This iterative technique presents the combination of homotopy perturbation technique and Laplace transform with He's polynomials, which can further be applied to numerous linear/nonlinear two-dimensional fractional order model to compute the approximate analytical solution. In the present method, the nonlinearity can be tackled by He's polynomials. The salient features of the chapter are the pictorial presentations of the approximate numerical solution of the two-dimensional fractional reaction-advection-diffusion equation for different particular cases and showcasing of the damping effects of reaction terms on the nature of probability density function of the considered two-dimensional nonlinear mathematical models for various situations. To validate the high efficiency and capability of the proposed numerical scheme, few test examples are reported with the computation of the errors between the analytical results and the results obtained by the numerical scheme for the concerned problem.