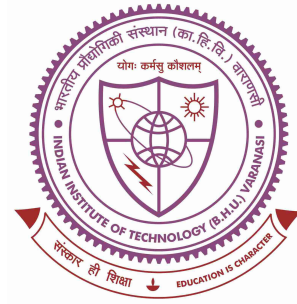


**ON Q -TOPOLOGICAL SPACES, FUZZY
CLOSURE SPACES AND THEIR SIERPINSKI
OBJECTS**



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by
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Conclusion and Future Scope

In 2008, Solovyov [36] introduced the notion of Q -topological spaces and studied the resulting category $Q\text{-TOP}$ of Q -topological spaces. Solovyov also introduced the Q -Sierpinski space, T_0 - Q -topological space, stratified Q -topological space, sober Q -topological space etc. and studied many properties related to these notions, including those of categorical nature.

The study of Q -topological spaces was continued further by Singh and Srivastava [30–33] and Noor et al. [27] etc.

In a large part of this thesis, we have continued the study of Q -topological spaces. In particular, we have studied exponential Q -topological spaces, injective objects and existence of injective hulls in the comma category $Q\text{-TOP}/(Y, \sigma)$, some coreflective hulls in the category $\mathbf{Str}\text{-}Q\text{-TOP}$ of stratified Q -topological spaces. In a somewhat different direction, we have also given in this thesis, a characterization of the category \mathbf{FCS} of fuzzy closure spaces, as considered in Srivastava et al. [40], in terms of the Sierpinski fuzzy closure space.

Based on the nature of work done in this thesis, we are mentioning here some problems for future research.

Problem 1: We recall that a Q -bitopological space is a triple (X, τ_1, τ_2) , where X is a set and τ_1, τ_2 are Q -topologies on X and a map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ between two Q -bitopological spaces is called Q -bicontinuous if $f : (X, \tau_i) \rightarrow (Y, \sigma_i)$ is Q -continuous for $i = 1, 2$ (cf. [27]). Noor et al. [27] determined two Sierpinski objects in the category $Q\text{-BTOP}$ of Q -bitopological spaces. So it would be interesting to see whether we can characterize exponential objects in the category $Q\text{-BTOP}$ also, in an analogous manner (as done in Theorem 2.4.10)?

Problem 2: Cagliari and Mantovani [7] gave a characterization of injective objects in the comma category \mathbf{TOP}_0/B , where \mathbf{TOP}_0 is the category of T_0 -topological

spaces. They characterized injective objects (with respect to the class of embeddings) in \mathbf{TOP}_0/B as precisely being the retracts of the partial products of the two-point Sierpinski space \mathbb{S} . Solovyov [36], while introducing the category $Q\text{-TOP}$, also introduced T_0 - Q -topological space and the Q -Sierpinski space. So it is natural to explore whether we can characterize injective objects in the comma category $Q\text{-TOP}_0/(Y, \sigma)$ as precisely being the retracts of the partial products of the Q -Sierpinski space (where $Q\text{-TOP}_0$ is the category of T_0 - Q -topological spaces)?

Problem 3: Solovyov [36] introduced stratified Q -topological spaces. Singh and Srivastava [31] introduced the stratified Q -topological space $(Q, \langle \{id_Q\} \cup \{\underline{q} \mid q \in Q\} \rangle)$, which is a Sierpinski object in the category $\mathbf{Str}\text{-}Q\text{-TOP}$ of stratified Q -topological spaces. In Theorem 4.3.14, we have obtained the coreflective hull of $(Q, \langle \{id_Q\} \cup \{\underline{q} \mid q \in Q\} \rangle)$ in the category $\mathbf{Str}\text{-}Q\text{-TOP}$ of stratified Q -topological spaces. Solovyov also [36] introduced the Q -Sierpinski space and Singh and Srivastava [30] proved that it is a Sierpinski object in the category $Q\text{-TOP}$ of Q -topological spaces. So it would be interesting to determine the coreflective hull of the Q -Sierpinski space in the category $Q\text{-TOP}$ of Q -topological spaces.

Problem 4: Denniston et al. [12] studied the category $\mathbf{AfSys}(L)$ of L -affine systems (where L is a fixed member of a fixed variety of algebras \mathbf{A}) and introduced the Sierpinski (L -)affine system. They proved that the Sierpinski (L -)affine system is a Sierpinski object in the category $\mathbf{AfSys}(L)$. A natural question is: can we obtain a characterization of the category $\mathbf{AfSys}(L)$ among a suitable class of categories using the Sierpinski (L -)affine system (as done in Theorem 5.4.1)?

Problem 5: Another question is: can we obtain a characterization of exponential objects in the category $\mathbf{AfSys}(L)$ of L -affine systems (where L is a fixed member of a fixed variety of algebras \mathbf{A}) introduced by Denniston et al. [12], similar to the one for exponential objects in the category of Q -topological spaces (as done in Theorem 2.4.10)?