ON Q-TOPOLOGICAL SPACES, FUZZY CLOSURE SPACES AND THEIR SIERPINSKI OBJECTS

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by

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Conclusion and Future Scope

In 2008, Solovyov [36] introduced the notion of Q-topological spaces and studied the resulting category Q-**TOP** of Q-topological spaces. Solovyov also introduced the Q-Sierpinski space, T_0-Q-topological space, stratified Q-topological space, sober Q-topological space etc. and studied many properties related to these notions, including those of categorical nature.

The study of Q-topological spaces was continued further by Singh and Srivastava [30–33] and Noor et al. [27] etc.

In a large part of this thesis, we have continued the study of Q-topological spaces. In particular, we have studied exponential Q-topological spaces, injective objects and existence of injective hulls in the comma category Q-**TOP**$/(Y, \sigma)$, some coreflective hulls in the category **Str**-Q-**TOP** of stratified Q-topological spaces. In a somewhat different direction, we have also given in this thesis, a characterization of the category **FCS** of fuzzy closure spaces, as considered in Srivastava et al. [40], in terms of the Sierpinski fuzzy closure space.

Based on the nature of work done in this thesis, we are mentioning here some problems for future research.

Problem 1: We recall that a Q-bitopological space is a triple (X, τ_1, τ_2), where X is a set and τ_1, τ_2 are Q-topologies on X and a map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ between two Q-bitopological spaces is called Q-bicontinuous if $f : (X, \tau_i) \to (Y, \sigma_i)$ is Q-continuous for $i = 1, 2$ (cf. [27]). Noor et al. [27] determined two Sierpinski objects in the category Q-**BTOP** of Q-bitopological spaces. So it would be interesting to see whether we can characterize exponential objects in the category Q-**BTOP** also, in an analogous manner (as done in Theorem 2.4.10)?

Problem 2: Cagliari and Mantovani [7] gave a characterization of injective objects in the comma category $\mathbf{TOP}_0/B$, where $\mathbf{TOP}_0$ is the category of T_0-topological spaces. They characterized injective objects (with respect to the class of embeddings) in $\mathbf{TOP}_0/B$ as precisely being the retracts of the partial products of the two-point Sierpinski space $\mathbb{S}$. Solovyov [36], while introducing the category Q-**TOP**, also introduced T_0-Q-topological space and the Q-Sierpinski space. So it is natural to explore whether we can characterize injective objects in the comma category Q-$\mathbf{TOP}_0/(Y, \sigma)$ as precisely being the retracts of the partial products of the Q-Sierpinski space (where Q-$\mathbf{TOP}_0$ is the category of T_0-Q-topological spaces)?

Problem 3: Solovyov [36] introduced stratified Q-topological spaces. Singh and Srivastava [31] introduced the stratified Q-topological space $(Q, \langle \{id_Q\} \cup \{\underline{q} \mid q \in Q\}\rangle)$, which is a Sierpinski object in the category **Str**-Q-**TOP** of stratified Q-topological spaces. In Theorem 4.3.14, we have obtained the coreflective hull of $(Q, \langle \{id_Q\} \cup \{\underline{q} \mid q \in Q\}\rangle)$ in the category **Str**-Q-**TOP** of stratified Q-topological spaces. Solovyov also [36] introduced the Q-Sierpinski space and Singh and Srivastava [30] proved that it is a Sierpinski object in the category Q-**TOP** of Q-topological spaces. So it would be interesting to determine the coreflective hull of the Q-Sierpinski space in the category Q-**TOP** of Q-topological spaces.

Problem 4: Denniston et al. [12] studied the category **AfSys**(L) of L-affine systems (where L is a fixed member of a fixed variety of algebras $\mathbf{A}$) and introduced the Sierpinski (L-)affine system. They proved that the Sierpinski (L-)affine system is a Sierpinski object in the category **AfSys**(L). A natural question is: can we obtain a characterization of the category **AfSys**(L) among a suitable class of categories using the Sierpinski (L-)affine system (as done in Theorem 5.4.1)?

Problem 5: Another question is: can we obtain a characterization of exponential objects in the category **AfSys**(L) of of L-affine systems (where L is a fixed member of a fixed variety of algebras $\mathbf{A}$) introduced by Denniston et al. [12], similar to the one for exponential objects in the category of Q-topological spaces (as done in Theorem 2.4.10)?