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It is certified that the work contained in the thesis titled "**On Q-Topological Spaces, Fuzzy Closure Spaces and Their Sierpinski Objects**" by **Ms. Harshita Tiwari** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. Degree.

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I, **Harshita Tiwari**, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of **Prof.(Mrs.) Rekha Srivastava** from **July 2017** to **April 2021** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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Contents

Preface

The present thesis is concerned with a study of Q-topological spaces, fuzzy closure spaces and their Sierpinski objects.

This thesis is organized into five chapters.

The first chapter is introductory. It contains a brief introduction of the subject related to the thesis, necessary definitions and results which are used in the thesis.

Next three chapters are devoted to a study of Q-topological spaces.

Chapter two is on exponential Q-topological spaces. In this chapter, we have given a characterization of exponential objects in the category Q-**TOP** of Q-topological spaces with the help of the Q-Sierpinski space.

Chapter three is on injective objects and existence of injective hulls in the comma category Q-**TOP**$/(Y, \sigma)$. In this chapter, we have given a characterization of injective objects (with respect to the class of embeddings in the category Q-**TOP** of Q-topological spaces) in the comma category Q-**TOP**$/(Y, \sigma)$, when (Y, σ) is a stratified Q-topological space, with the help of their T_0-reflection. We have also proved, in this chapter, that for any Q-topological space (Y, σ), the existence of an injective hull of $((X, \tau), f)$ in the comma category Q-**TOP**$/(Y, \sigma)$ is equivalent to the existence of an injective hull of its T_0-reflection $((\tilde{X}, \tilde{\tau}), \tilde{f})$ in the comma category Q-**TOP**$/(\tilde{Y}, \tilde{\sigma})$ (and in the comma category Q-**TOP**$_0/(\tilde{Y}, \tilde{\sigma})$, where Q-**TOP**$_0$ denotes the category of T_0-Q-topological spaces).

Chapter four is on some coreflective hulls in the category **Str**-Q-**TOP** of stratified Q-topological spaces. In this chapter, we have obtained the coreflective hull of $(Q, \langle\{id_Q\} \cup \{\underline{q} \mid q \in Q\}\rangle)$ in the category **Str**-Q-**TOP** of stratified Q-topological spaces. We have also obtained, in this chapter, the coreflective hulls of the categories **Str**-**Dis**-Q-**TOP** of discrete Q-topological spaces and **Str**-**Ind**-Q-**TOP** of stratified indiscrete Q-topological spaces in the category **Str**-Q-**TOP**.

Chapter five is on a characterization of the category **FCS** of fuzzy closure spaces. In this chapter, we have introduced the Sierpinski fuzzy closure space and given a characterization of the category **FCS** using Sierpinski fuzzy closure space.

In the last, we have given conclusion and future scope of the work presented in the thesis.