

PREFACE

In this thesis, the numerical study of fractional diffusion equation has been done which has applications in porous media and tumor analyses. Chapter 1 contains the introduction part of the thesis. In this chapter, firstly the history of fractional calculus with evolution has been discussed. The definition of different types of fractional order derivatives like Caputo, Riemann-Liouville, and Atangana-Baleanu derivatives with constant and variable order is given which will be used through the article. In the last, the background, derivation of the fractional diffusion equation and its applications in different fields with a list of numerical methods dealing with it are incorporated in this chapter.

Chapter 2 contains the analysis of the one-dimensional reaction-diffusion equation. The Genocchi operational matrix of differentiation with collocation method has been used in solving the model. The validation of the method is shown by solving the particular cases of the reaction-diffusion model. For accuracy of the method, the error table has been incorporated and results are compared with the existing numerical results from previous literature. In the end, the application of this model in the groundwater contamination problem has been presented.

The two-dimensional version of the previous reaction-diffusion model along with Neumann boundary conditions has been studied in Chapter 3. The Genocchi operational matrix with collocation method has been used for solving this model. The validation and accuracy of the method are depicted through error tables and also from the plots between the exact and numerical solution.

In chapter 4, two models have been considered, out of which one is variable order non-linear reaction-diffusion model and another one variable order advection-diffusion

model. Here, the operational matrix of the ultraspherical wavelet is used in finding the numerical solutions of aforementioned models. The validation of the method is shown by solving the numerical examples for the particular cases of both models. The effect of reaction term on solution profile is depicted through figures.

In chapter 5, the applications of diffusion equation are shown in the tumor analysis with the absence and presence of chemotherapeutic treatment. A model of four coupled diffusion equations is considered with exponential kernel non-singular fractional order derivative for the analysis of the behavior of tumor cells, normal cells, and immune cells. The spectral method based on the Chebyshev polynomials is used for the investigation of the model. The results for tumor cells' behavior in presence of chemotherapeutic treatment and the dynamic behavior of all types of cells concerning different fractional order are shown with the help of figures. In the end, the response of the immune system against tumor cells has been depicted.