CERTIFICATE

It is certified that the work contained in this thesis titled "***Numerical study of fractional diffusion equations and its applications***" by ***Sachin Kumar*** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

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DECLARATION BY THE CANDIDATE

I, ***Sachin Kumar***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. Subir Das*** from ***July, 2017*** to ***July, 2021*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi.*** The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: October 6, 2021

Place: Varanasi

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It is certified that the above statement made by the student is correct to the best of my/our knowledge.

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Sachin Kumar

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PREFACE

In this thesis, the numerical study of fractional diffusion equation has been done which has applications in porous media and tumor analyses. Chapter 1 contains the introduction part of the thesis. In this chapter, firstly the history of fractional calculus with evolution has been discussed. The definition of different types of fractional order derivatives like Caputo, Riemann-Liouville, and Atangana-Baleanu derivatives with constant and variable order is given which will be used through the article. In the last, the background, derivation of the fractional diffusion equation and its applications in different fields with a list of numerical methods dealing with it are incorporated in this chapter.

Chapter 2 contains the analysis of the one-dimensional reaction-diffusion equation. The Genocchi operational matrix of differentiation with collocation method has been used in solving the model. The validation of the method is shown by solving the particular cases of the reaction-diffusion model. For accuracy of the method, the error table has been incorporated and results are compared with the existing numerical results from previous literature. In the end, the application of this model in the groundwater contamination problem has been presented.

The two-dimensional version of the previous reaction-diffusion model along with Neumann boundary conditions has been studied in Chapter 3. The Genocchi operational matrix with collocation method has been used for solving this model. The validation and accuracy of the method are depicted through error tables and also from the plots between the exact and numerical solution.

In chapter 4, two models have been considered, out of which one is variable order non-linear reaction-diffusion model and another one variable order advection-diffusion model. Here, the operational matrix of the ultraspherical wavelet is used in finding the numerical solutions of aforementioned models. The validation of the method is shown by solving the numerical examples for the particular cases of both models. The effect of reaction term on solution profile is depicted through figures.

In chapter 5, the applications of diffusion equation are shown in the tumor analysis with the absence and presence of chemotherapeutic treatment. A model of four coupled diffusion equations is considered with exponential kernel non-singular fractional order derivative for the analysis of the behavior of tumor cells, normal cells, and immune cells. The spectral method based on the Chebyshev polynomials is used for the investigation of the model. The results for tumor cells'behavior in presence of chemotherapeutic treatment and the dynamic behavior of all types of cells concerning different fractional order are shown with the help of figures. In the end, the response of the immune system against tumor cells has been depicted.