Chapter 7

Overall Conclusion and scope for the future work

It has shown experimentally that we can not model every diffusion phenomenon with the classical diffusion equation. When the fractional calculus came into light, the researchers find a more effective way to model the physical and biological diffusion phenomena accurately, but the complexity of the model came simultaneously. The classical methods became almost useless to deal with these fractional order models. This became a challenge among the researchers to develop new methods that can accurately deal with these complex models. Finding the exact solutions to those complex models are not always possible due to complexity, so researchers seek their attention towards numerical solutions. During the literature survey in chapter 1, it is shown that the researchers have developed many efficient numerical techniques to deal with the complex fractional diffusion models. Nowadays, there are enough numerical techniques to deal with complex diffusion models, so now the challenge is not only to develop a numerical method but also that new method should perform better than previously existing methods. This thesis is an effort to develop efficient methods that perform better as compared to old methods and observe the behavior of that taken particular type of fractional diffusion model due to change in various parameters present in the model.

In chapters 2 and 3, the numerical techniques with operational matrices have been discussed with the help of Fibonacci polynomial to deal with one and two-dimensional fractional-order space-time reaction advection-diffusion equations, respectively. Further showed that discussed numerical technique is performing better than previously solved methods and observed how diffusion profile is increasing and decreasing with the change in the order of space and time derivatives. For the future scope, one can try to develop the other polynomials or can generalize these methods to solve the model with different types of fractional order derivatives arising in non-homogeneous media.

It is always challenging to deal with a non-linear diffusion equation. Things become more complicated when fractional derivatives present in the model. In chapter 4 a time-fractional diffusion model in a homogeneous medium with non-linearity in diffusion and reaction terms has been solved with the help of cubic B-spline and L_1 formula. Furthermore, it has been shown that the developed scheme is convergent and unconditionally stable. After showing that the discussed method is performing better than the previously existing method, the said method is used to solve a considered highly non-linear model to observe the behavior of diffusion profile due to variation in the degree of non-linearity in the diffusion coefficient in the presence and absence of advection and reaction terms. For the future scope, one can try to solve the model with different splines to observe whether its performance is better than this proposed method.

In chapter 5, a very efficient nonstandard/standard method has been developed with the help of Fibonacci polynomial to deal with space fractional order diffusion equation for homogeneous medium with non-linearity in reaction terms. The accuracy of the developed scheme has been shown both analytically and numerically. Through comparison, it has been shown that the proposed numerical method is performing more accurately than previously used methods. Also, the behavior of the diffusion model for different spatial fractional-order for different particular cases has been observed. For the future scope, one can develop the scheme with other polynomials and compare the scheme with other existing methods.

So all the above mentioned chapters, every model is taken in homogeneous medium with fractional order derivatives in Caputo sense, but in the chapter 6 a multi-term variable fractional order diffusion equation in heterogeneous medium has been taken. A finite-difference with collocation scheme is developed with the help of the Fibonacci polynomial. After validation of the scheme, the behavior of the considered diffusion model has been observed with different variable order derivatives. For the future scope, one can develop a scheme with different polynomials or use this developed method to the different types of variable order models, especially for nonlinear types of models.