Contents

Contents	v
List of Figures	viii
List of Tables	x
Preface	xi

1	Intr	roduction	1
	1.1	Porous Media	1
	1.2	Fractional Calculus	5
	1.3	Definitions of fractional order derivative and integral	$\overline{7}$
		1.3.1 Riemann-Liouville Arbitrary Order Integral	7
		1.3.2 Riemann-Liouville Arbitrary Order derivative	8
		1.3.3 Caputo Arbitrary Order Derivative	8
		1.3.4 Some properties of Fractional operators	8
	1.4	Fibonacci Polynomial	9
	1.5	Advection Reaction-Diffusion Equation in Porous Media	10
2	Nur	merical solution of nonlinear space-time fractional order advection-	
	read	ction-diffusion equation	14
	2.1	Introduction	14
	2.2	Preliminaries	18
	2.3	Generalization of Fibonacci Operational Matrix to fractional order derivative	18
	2.4	Numerical method to solve the fractional order partial differentiation equation	20
	2.5	Error Analysis	22
	2.6	Solution of the space-time fractional order ARDE	25
	2.7	Results and Discussion	26
	2.8	Conclusion	31
3	Fib	onacci collocation method to solve two-dimensional nonlinear frac-	
	4.1	al order advection-reaction diffusion equation	วก
	tion	a order advection-reaction diffusion equation	34
	3.1	Introduction	32 32

		3.2.1 Three-dimensional Fibonacci polynomial	35
		3.2.2 Kronecker product	35
		3.2.3 Function approximation	36
	3.3	Derivative in terms of Fibonacci operational matrices	37
	3.4	Method of solution of 2D fractional order diffusion equation	39
	3.5	Error analysis and accuracy of the method	41
		3.5.1 Numerical Examples	42
	3.6	Solution of the two dimensional space-time ARDE	47
		3.6.1 Results and Discussion	50
	3.7	Conclusion	51
1	N	morical solution of highly non-linear fractional order reaction advec	
4	tior	diffusion equation using cubic B-spline collocation method	52
	4.1	Introduction	52
	4.2	Description of the Cubic B-spline function	55
	4.3	Description of the method	56
	4.4	Initial State	58
	4.5	Stability Analysis	59
		4.5.1 Proposition	60
	4.6	Convergence Analysis	61
		4.6.1 Lemma	61
	4.7	Numerical Examples	63
	4.8	Solution of the proposed FRADE model	67
	4.9	Conclusion	70
5	Nu	merical Solution of Nonlinear Diffusion Equation by Using Non-Standa	ard
	/ S	tandard Finite Difference and Fibonacci Collocation Methods	72
	5.1	Introduction	72
	5.2	Preliminaries	75
		5.2.1 NSFD method	75
	5.3	NSFD Scheme Construction	76
	5.4	Stability Analysis	79
	5.5	Illustrative Examples	80
	5.6	Numerical Solution of Proposed ARDE and discussion	85
	5.7	Conclusion	88
6	Fin	ite difference with collocation method to solve multi term variable-	
	ord	er fractional reaction-advection-diffusion equation in heterogeneous	
	mee	dium	89
	6.1	Introduction	89
	6.2	Preliminaries	91
		6.2.1 Caputo variable order fractional derivative	91
		6.2.2 Riemann-Liouville left and right variable fractional derivatives \ldots	91
	6.3	Construction of finite difference scheme	92
	6.4	Error Analysis	97

	$\begin{array}{c} 6.5 \\ 6.6 \end{array}$	Numerical results and discussions	99 102
7	Ove	all Conclusion and scope for the future work	103

Bibliography	105
List of Publications	127

List of Figures

1.1	Top: Examples of natural and organic porous materials: (a) beach sand, (b) sandstone, (c) limestone, (d) rye bread, (e) wood, and (f) human lung. Bottom: Granular porous materials used in the construction industry, 0.5- cm-diameter Liapor® spheres (left), and 1-cm-size crushed limestone(right) [1]	2
2.1	Variations of $u(x,t)$ vs. x for conservative system at $t = 0.6$ when $v=0.2$.	26
2.2	Variations of $u(x,t)$ vs. x for conservative system at $t = 0.6$ when $v=0$.	27
2.3 2.4	Variations of $u(x,t)$ vs. x for non-conservative system at $t = 0.6$ when $v=0$. Variations of $u(x,t)$ vs. x for non-conservative system at $t = 0.6$ when	27
	v=0.2	27
2.5	Variations of $u(x,t)$ vs. x for conservative system at $t = 0.6$ when $v=0.2$.	28
2.6	Variations of $u(x,t)$ vs. x for conservative system at $t = 0.6$ when $v=0$.	28
2.7	Variations of $u(x,t)$ vs. x for non-conservative system at $t = 0.6$ when $v=0$.	28
2.8	Variations of $u(x,t)$ vs. x for non-conservative system at $t = 0.6$ when	
	v=0.2.	29
2.9	Variations of $u(x,t)$ vs. x for non-conservative system for different value of	
	t when v = 0.2.	29
2.10	Variations of $u(x,t)$ vs. x for linear diffusive term at $\alpha = 1, \beta = 1.8$,	90
9.11	k = -1, v = 0.2 at $t = 0.5$.	30
2.11	variations of $u(x,t)$ vs. x for non-intear diffusive term at $\alpha = 1, \beta = 1.8, k = -1, v = 0.2$ at $t = 0.5$	30
2.12	Numerical solution of $u(x, t)$ vs x for linear diffusive term at $\alpha = 0.8$	00
2.12	$\beta = 2, k = -1, v = 0.2$ at $t = 0.5, \dots, \dots, \dots, \dots$	30
2.13	Numerical solution of $u(x,t)$ vs. x for non-linear diffusive term at $\alpha = 0.8$,	
	$\beta = 2, k = -1, v = 0.2$ at $t = 0.5$.	31
3.1	The absolute error of example.1 at time $t = 0.5$ and $n = 10$	43
3.2	The absolute error of example.2 at time $t = 0.5$ and $n = 10$	45
3.3	The absolute error of Example 3 at time $t = 0.5$ and $n = 10$	47
3.4	Solute concentration for non-conservative system with sink term when $\alpha =$	
	1, $v = 0$ at $t = 0.5$.	48
3.5	Solute concentration for non-conservative system with sink term for $\alpha = 1$,	
	v = 1 at $t = 0.5$	49
3.6	Pollute concentration for non-conservative system with sink term for $\beta = 2$	16
	v = 1 at $t = 0.5$.	49

3.7	Pollute concentration for conservative and non-conservative systems for $\alpha =$	
	$1 \ \beta = 2 \text{ at } t = 0.5. $	50
4.1	Plot of absolute error vs. x for different values of K at $t = 1, \ldots, \ldots$	65
4.2	Plot of absolute error vs. x for different values of α at $t = 1, \ldots, \ldots$	66
4.3	Plots of solute concentration vs. x for $K = 0.2$, $v = 0.2$, $l = 0$ at $t = 0.5$.	67
4.4	Plots of solute concentration vs. x for $K = 0.2$, $v = 0.2$, $l = 1$ at $t = 0.5$.	68
4.5	Plots of solute concentration vs. x for $K = 0.2$, $v = 0.2$, $l = 2$ at $t = 0.5$.	68
4.6	Plots of solute concentration vs. x for $\alpha = 0.25$, $K = 0.2$, $v = 0.2$, at $t = 0.5$.	68
4.7	Plots of solute vs. x concentration for $\alpha = 0.5$, $K = 0.2$, $v = 0.2$, at $t = 0.5$.	69
4.8	Plots of solute concentration vs. x for $\alpha = 0.75$, $K = 0.2$, $v = 0.2$, at $t = 0.5$.	69
4.9	Plots of solute concentration vs. x for $\alpha = 1.0$, $K = 0.2$, $v = 0.2$, at $t = 0.5$.	70
5.1	The behavior of exact solution and approximate solution at $n = 3$ for Example 1.	83
5.2	The behavior of exact solution and approximate solution at $n = 3$ for Example 2.	84
5.3	Plots of solute profile vs. x for various values of β at $q = 1$ at $t = 0.5$.	86
5.4	Plots of solute profile vs. x for various values of β at $q = 2$ at $t = 0.5$.	87
5.5	Plots of solute profile vs. x for various values of β at $q = 4$ at $t = 0.5$	87
5.6	Variations of solute concentration for different values of q at $\beta = 2$, $t = 0.5$.	87
6.1	Plots of solute profile for $q = 0$, $d = v = k = 1$ and $\beta(x, t) = 2$ at $t = 0.5$.	100
6.2	Plots of solute profile for $d_1 = d_2 = v = k = 1$, $q = 0$ and $\beta(x, t) =$	
	$1.5 + 0.5sin(0.5\pi x)$ at $t = 0.5$.	100
6.3	Plots of solute profile for $d_1 = d_2 = d = v = k = 1, q = 0$ at $t = 0.5, \ldots$	100
6.4	Plots of solute profile for $d_1 = d_2 = d = v = k = 1$, and $\beta(x, t) = 1.5 +$	
	$0.5 \sin(0.5\pi x)$ at $t = 0.5$	101
6.5	Plots of solute profile for $d_1 = d_2 = d = k = 1$, $q = 0$ and $\beta(x, t) =$	
	$1.5 + 0.5 \sin(0.5\pi x)$ at $t = 0.5$.	101
6.6	Plots of solute profile for $d_1 = d_2 = d = v = 1$, $q = 0$ and $\beta(x,t) =$	
	$1.5 + 0.5 \sin(0.5\pi x)$ at $t = 0.5$	101

List of Tables

2.1	Absolute error of t^5 for different fractional order derivatives	20
2.2	Maximum absolute error at $t = 0.5$ for Example 2	24
2.3	Maximum absolute error at $t = 0.5$ for example 3	25
3.1	Maximum absolute error at $t = 1$	42
3.2	Maximum absolute error at $t = 1$.	42
3.3	Maximum absolute error at $t = 1$.	44
3.4	Maximum absolute error at $t = 1$.	44
3.5	Maximum absolute error at $t = 1$	46
3.6	Maximum absolute error at $t = 1$	46
4.1	Comparison of results for Example 1	64
4.2	Calculated error of approximate solution in l^2 and l^{∞} -norms for Example 2.	65
4.3	Comparison of error obtained from Example 3 at $\alpha = 0.5$	66
5.1	Comparison of absolute errors among our method and the methods given in [206, 207] at $t = 1$ for Example 1.	82
5.2	Comparison of absolute errors among our method and the methods given in [206, 207] at $t = 2$ for Example 1.	82
5.3	Comparison of absolute errors among our method and the methods given in [206, 207] at $t = 10$ for Example 1.	83
5.4	Comparison of maximum errors among different methods and our proposed method for Example 2.	84
5.5	Maximum absolute error for $n = 5$, $\phi = h$ and $\delta t = 0.0001$	85
6.1	Comparison of maximum absolute error of previous method with our proposed method when $\alpha_1 = 0.15, \alpha_2 = 0.95, d_0 = d_1 = d_3 = 1$ at $T = 1.5$.	
6.2	Comparison of maximum absolute error of previous method with our pro- posed method when $\alpha_1 = 0.2$ $\alpha_2 = 0.8$ $d_0 = 0$ $d_1 = d_2 = 1$ at $T = 1.5$	98
	$\begin{array}{c} p = 1 \\ p = 1 \\$	98
6.3	Comparison of maximum absolute error of previous method with our proposed method when $\alpha_1 = 0.2, \alpha_2 = 0.5, d_0 = d_1 = d_3 = 1$ at $T = 1.5$.	00
		99