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It is certified that the work contained in this thesis entitled "Behavior of Solute Concentration in Porous Media" by "Kushal Dhar Dwivedi" has been carried out under our supervision and that it has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements, Comprehensive examination, Candidacy and SOTA for the award of Ph. D. Degree.

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Preface

The thesis contains eight chapters. It is mainly focused on developing efficient and accurate numerical methods to solve complicated unsolved fractional diffusion models, as finding an exact solution is not always possible, and also focused on observing the behavior of fractional diffusion models due to changes in various parameters.

Chapter 1 is an introductory chapter that contains some basic definitions and literature review related to the field. This chapter begins by briefly introducing porous media as in this thesis diffusion models are considered in porous media. Further, the author has given a short introduction of the advection-reaction diffusion equation as the center of the work is focused on solving this type of diffusion equation. Most of the methods are developed with the help of Fibonacci polynomial, so a brief introduction about Fibonacci polynomial and some of its important properties have been given here.

In chapter 2, the author has developed a numerical method to solve the one-dimensional space-time fractional-order advection-reaction diffusion equation in a homogeneous medium with the help of the Fibonacci operational matrix. After validation of the developed method on three examples having exact solutions, it is used to solve a fractional-order advection-reaction diffusion model. Furthermore, the effect on solute concentration is observed due to change in various parameters viz., the order of time and space fractional derivative, advection, and reaction terms.

In chapter 3, the author has taken one step ahead and has proposed the numerical method to solve the two-dimensional space-time fractional-order advection-reaction diffusion equation in a homogeneous medium with the help of the Fibonacci operational matrix. Further, it has been shown that the developed method works more efficiently than previously existing methods by applying it to certain two-dimensional diffusion models. In the last, the method is used to solve an unsolved two-dimensional advection-reaction diffusion model and observed the effect on solute concentration due to change in various parameters of the model.

In the chapter 4, the approximate solution of the fractional-order reaction advection-diffusion equation with the prescribed initial and boundary conditions is found with the help of a cubic B-spline collocation method, which is unconditionally stable and convergent. The accuracy of the scheme is validated by applying the method on four existing problems having analytical solutions and through finding the absolute errors between numerical results and the exact solutions for different particular cases. Applying the proposed method to the last two numerical problems, it is shown that the method performs better than the existing methods even for a very less number of spatial and temporal discretizations. The main contribution of the chapter is to develop an efficient method to solve the proposed fractional-order nonlinear problem and to find the effect on solute concentration graphically due to increase in the non-linearity in the diffusion term for different particular values of parameters.

In the chapter 5, a new nonstandard/standard finite difference scheme is introduced with the help of Fibonacci polynomial. The considered nonlinear fractional diffusion equation is reduced into a system linear ordinary differential equations, which are solved by using the nonstandard/standard finite difference method. The developed schemes are unconditionally stable, and the effectiveness and efficiency of the method are confirmed by applying it in two linear and one nonlinear problem and through comparison of the obtained numerical results with the existing analytical results. After validation of the efficiency of the method, the proposed method is used to solve the highly nonlinear fractional order diffusion equation. The stability of the method is also discussed. The graphical exhibitions of the effects on the solute profiles due to increasing in non-linearity of the model and order of the spatial derivative have been shown graphically for different particular cases.

In the chapter 6, the author has considered a multi-term variable-order fractional diffusion model in the heterogeneous medium and has developed a new numerical technique with the help of Fibonacci collocation and finite difference method to solve the considered model. The higher accuracy of the developed method as compared to the existing methods is also shown. After validation of the accuracy of the method, it has been used to solve the considered model. The effects on the diffusion process due to various parameters in the heterogeneous medium are shown graphically.

In the Chapter 7, overall work has been concluded and also the future scope of the related works has been furnished.