Chapter 6

Conclusion and Future Works

This thesis is dedicated to the problems of synchronization between neural networks under the impact of impulses. We focused on mathematical analysis that is needed when we study synchronization problems. The major issue that comes in synchronization of neural networks is stability analysis of nonlinear differential equations. Thus, we have investigated stability analysis of error systems in effective ways and derived lesser conservative results than the existing one. Our primary focus has been to study synchronization of general neural networks' models, and to extend the stability problems for more general form of impulsive sequences. In the first chapter we elaborately introduced artificial neural networks from the origin of motivation to their modeling in mathematical equations. The basic definitions and properties of DDE and impulsive differential equations have been given. We gave physical interpretation of Hopfield, Cohen-Grossberg and BAM neural networks, and extended these models for time-delays and impulsive sequence. We finished the first chapter by introducing matrix measure theory that has been applied in most of the chapters of this thesis to investigate stability analysis.

In the second chapter, we investigated the generalized form of function projective synchronization of Cohen-Grossberg neural networks. When scaling matrix consists of distinct non-zero continuously differentiable and bounded functions then the synchronization scheme becomes modified function projective synchronization.

Since time-delays in neural network may affects their dynamics [117] so we considered Cohen-Grossberg models with mixed time-varying delays. We have shown that MFPS of neural networks with parameter mismatches can not be complete,i.e., the trajectory of error system fluctuate in a small compact domain of radius equal to synchronization error bound. Thus, we call this type of synchronization a weak MFPS.

In the third chapter, we studied impulsive effects on projective synchronization of delayed neural networks with parameter mismatches. The impulsive control consisted of continuous and discontinuous terms has been designed to control the response system at impulsive points with different ranges of impulses. We have shown that we can achieve projective synchronization between parameter mismatched neural networks up to a small error bound under the influence of different ranges of impulses,i.e., for $\tau \in (-2, 0]/-1$, and for $\tau \in (-\infty, -2]$ or $\tau > 0$. By using elementary calculus, we found optimal synchronization error bound for both cases. The impulses that we have considered in this chapter are fixed at each impulsive point of the impulsive sequence.

Thus, in the fourth chapter, we studied global exponential stability of inertial BAM neural network under the influence of hybrid impulsive sequence. We found criteria of having a unique equilibrium point, and a global exponential stability of that equilibrium point. The number of stabilizing and destabilizing impulsive points are estimated by the following inequalities.

$$\hat{N}_{\mathcal{S}}(t, t_0) \leq \frac{t - t_0}{\hat{T}_a} + N_0 \qquad \textit{and} \qquad \check{N}_{\mathcal{S}}(t, t_0) \geq \frac{t - t_0}{\check{T}_a} - N_0, \tag{6.1}$$

where $\hat{T}_a$ and $\check{T}_a$ are average impulsive interval for destabilizing and stabilizing impulses respectively. The parameters $\hat{T}_a$ and $\check{T}_a$ are always be finite. In particular, there can exist the hybrid impulsive sequence having average impulsive interval very large.

The influence of this type of impulsive sequence has been studied in the fifth chapter on synchronization problem of delayed neural networks with parameter mismatches. We studied the effects of $T_a < \infty$ and $T_a = \infty$ on quasi-synchronization. The number of impulsive points in time span (s, t) is taken as $N_\varsigma(s, t) = [\sqrt[3]{t-s}]$, where $[.]$ is greatest integer function. Using this hybrid impulsive sequence, we have found criteria under which the error system eventually fluctuate in a small domain of radius equal to synchronization error bound. Finally, we concluded that the work of this thesis devoted to investigate the effects of general type of impulsive sequence on synchronization and stability problems of delayed neural networks.

In my future research work, we will study the effects of variable time impulses on stability problems of delayed neural networks. As we have introduced about the impulsive system with variable time in (1.63), it can be observed that the solution of the system is very complicated because impulsive points depends on state. Due to this complexity, only a few works [118, 119, 120, 121] have been published in this direction. Another future research direction is to study the impact of delayed impulsive control on stability analysis of neural networks. A few research works have been published in this direction which shows that the time-delay in impulsive control has negative impact on stability analysis of neural networks [122, 123, 124, 125, 126]. Thus, we will try to investigate the following possible problems in my future research work.

(i) Extending our all results to variable time impulsive sequence.

(ii) We will focus on developing the mathematical techniques to avoid complexity in calculation.

(iii) We will try to extend our results for higher dimensional neural networks, such as complex-valued, quaternion-valued, and octanion-valued neural.

(iv) Many interesting results have been reported on delayed impulses. However, most of them considered only negative effect of time-delay on stability of nonlinear systems. How to study the positive effect for such time-delay is still difficult to handle and is tackled in only a few [122]. More methods and tools should be explored.

(v) Another increasing interest is the state-dependent-delayed impulses arising in many applications as automatic control, secure communication and population dynamics.

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