

CERTIFICATE

It is certified that the work contained in this thesis titled “ *Problems on Stability and Synchronization of Neural Networks* ” by *Rakesh Kumar* has been carried out under our joint supervision and co-supervision, and this work has not been submitted elsewhere for a degree.

It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.



Prof. Subir Das
(Supervisor)
Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005



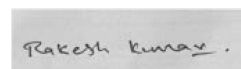
Dr. Rajesh Kumar Pandey
(Co-Supervisor)
Associate Professor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

DECLARATION BY THE CANDIDATE

I, **Rakesh Kumar**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Prof. Subir Das** and the co-supervision of **Dr. Rajesh Kumar Pandey** from **27 July, 2016** to **May, 2021** at the **Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi**. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, thesis, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

Date: 31/05/2021

Place: Varanasi



(Rakesh Kumar)

CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my/our knowledge.



(Prof. Subir Das)

Professor

Department of Mathematical Sciences

Indian Institute of Technology

(Banaras Hindu University)

Varanasi-221005



(Prof. T. Som)

Professor and Head

Department of Mathematical Sciences

Indian Institute of Technology

(Banaras Hindu University)

Varanasi-221005

COPYRIGHT TRANSFER CERTIFICATE

Title of the Thesis: *Problems on Stability and Synchronization of Neural Networks.*

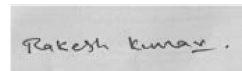
Name of the Student: *Rakesh Kumar*

Copyright Transfer

The undersigned hereby assigns to the Indian Institute of Technology (Banaras Hindu University), Varanasi all rights under copyright that may exist in and for the above thesis submitted for the award of the Ph.D. degree.

Date: 31/05/2021

Place: Varanasi



(Rakesh Kumar)

Note: However, the author may reproduce or authorize others to reproduce material extracted verbatim from the thesis or derivative of the thesis for author's personal use provided that the source and the Institute copyright notice are indicated.

To

My Beloved Parents & Elder Brother

Shrimati Namuna Devi

&

Shri Lalan Singh

&

Shri Ranjeet Singh

ACKNOWLEDGEMENTS

Life has been a long memorable unforgettable journey with ups and downs. I would not be able to express my gratitude in words for the support, love, affection. I got throughout this journey, from so many people. it is mere attempt to name a few.

First of all, I would like to express my sincere thanks to my respected supervisor and co-supervisor ***Prof. Subir Das*** and ***Dr. Rajesh Kumar Pandey*** for their excellent guidance, continuous encouragement, patience and advice during the whole span of Ph.D. I have been very lucky to have such a supervisors who cared so much about my work, and have shaped my understanding on the subject that has given me confidence to work independently. I also learnt from them which is much more than one could have expected from the supervisor.

I express my cordial thanks to ***Prof. T. Som, Head of Department, Prof. S. Das, Convener, DPGC, of the Department of Mathematical Sciences*** for their supports throughout my research work. I also express my deep sense of gratitude to all faculty members of the department, ***Prof. Rekha Srivastav, Prof. L. P. Singh, Prof. S. K. Pandey, Prof. S. Mukhopadhyay, Dr. Ashok Ji Gupta, Dr. V. K. Singh, Dr. R. K. Pandey, Dr. Anuradha Banerjee, Dr. Debdas Ghosh, Dr. Sunil Kumar, Dr. Lavanya Sivakumar, Prof. Murali Krishna Vemuri*** and RPEC member ***Prof. D. Singh, Department of Electrical Engineering***, for their constant moral supports, suggestions, and encouragement.

I am very grateful to have supportive seniors and lab-mates Dr. Vijay Kumar Yadav, Dr. Prashant Mishra, Dr. Shubham Jaiswal, Dr. Neeraj Tripathi, Dr. Pragya singh,

Dr. Anup Singh, Ms. Anuwedita Singh, during my Ph.D. journey. I would also like to thank to sincere juniors of my lab Mr. Umesh Kumar, Ms. Neha Trivedi, Mr. Shiv Shankar Chauhan, and Mr. Sunny Singh.

I have special thanks to my colleagues Mr. Rahul Kumar Maurya, Mr. Pankaj Gautam, Mr. Abhishek Singh, Dr. Anil Kumar Shukla, Mr. Sumit, Mr. Avinash Dixit, Mr. Om Namah Shivay, Dr. Swati Yadav, Ms. Manushi Gupta and all the research scholars of the department for their moral supports.

I am also grateful to my Institute, IIT(BHU), for providing necessary resources throughout my research. I express my thanks to all non teaching staff members of the department for their supports.

I gratefully acknowledge University Grants Commission, India for providing the fellowship in form of Junior Research Fellowship and Senior Research fellowship.

I express my sincere and cordial gratitude to my father ***Shri Lalan Singh***, my mother ***Smt. Namuna Devi***, my elder brother ***Shri Ranjeet Singh*** who always stood by my decisions and provided all kinds of supports, moral as well as financial. It was their love, care and patience which encouraged me to move on.

This acknowledgement would be incomplete if the name of great visionary ***Pt. Madan Mohan Malaviya*** is not mentioned, who made this divine centre of knowledge. Deepest regards to him.

Above all, praises and thanks to the God, the Almighty, for His showers of blessings throughout my research work, who has made everything possible.

Rakesh Kumar

Contents

List of Figures	ix
List of Tables	xi
Preface	xiii
1 Introduction	1
1.1 Biological neurons	2
1.1.1 Biological model	5
1.2 Artificial Neural Networks	9
1.2.1 Basics of electrical circuit	19
1.2.2 Additive model	20
1.2.3 Hopfield neural network	22
1.2.4 Cohen-Grossberg neural network	27
1.2.5 Bidirectional Associative Memory	29
1.3 An Overview of Mathematical Concepts	31
1.3.1 Delay Differential Equations	32
1.3.2 Impulsive Differential Equation	37
1.3.3 Matrix measure theory	39
1.4 Synchronization	42
2 Weak, modified, and function projective synchronization of Cohen-Grossberg neural networks	47
2.1 Introduction	47
2.2 Model description and mathematical preliminaries	48
2.3 Main results	50
2.4 Results and discussion	63
2.5 Conclusion	65
3 Projective synchronization of delayed neural networks with mismatched parameters and impulsive effects	67

3.1	Introduction	67
3.2	Problem formulation and preliminaries	68
3.3	Main Results	73
3.4	Optimization of error bound	84
3.4.1	Optimized error bound for $-2 < \tau < 0$ except $\tau = -1$	84
3.4.2	Discussion about optimal error bound for $-\infty < \tau < -2$ or $\tau = 0$	86
3.5	Numerical computation and discussion	87
3.6	Conclusion	91
4	Exponential stability of inertial BAM neural network with time-varying impulses	93
4.1	Introduction	93
4.2	Model description and preliminaries	94
4.3	Global exponential stability criteria of inertial BAM neural network .	104
4.4	Numerical simulations and discussions	113
4.5	Conclusion	117
5	Effects of infinite occurrence of hybrid impulses on quasi-synchronization of parameter mismatched neural networks	119
5.1	Introduction	119
5.2	Problem formulation and some preliminaries	120
5.3	Main Results	126
5.3.1	Quasi-synchronization criteria for $T_a < \infty$	126
5.3.2	Quasi-synchronization criteria for $T_a = \infty$	133
5.4	Numerical simulation and discussions	139
5.5	Conclusion	142
6	Conclusion and Future Works	145
	Bibliography	149
	List of Publications	165

List of Figures

1.1	Schematic structure of a typical neuron [1].	3
1.2	Schematic diagram of a network of neurons. Neuron N_i sends a signal pulse S_{ij} through its axon to hits the target neuron N_j with coupling strength $Z_{ij}(t)$. The external inputs I_i are stimuli for the neuron model.	6
1.3	A diagram of an artificial neuron of the network.	10
1.4	(a) A threshold function, (b) Piecewise linear function, and (c) Sigmoid activation function for $\beta = 2, 3$, and 4.	12
1.5	A feedforward network without hidden layer.	14
1.6	Multilayer feedforward neural network with one hidden layer.	15
1.7	A recurrent network with one hidden layer of neurons.	16
1.8	A recurrent neural network without a hidden layer and a self-feedback loop.	17
1.9	Partially connected neural network.	17
1.10	RC-circuit with a source of voltage.	19
1.11	Additive model of a neuron.	21
1.12	An architecture of Hopfield neural network consisting of $n = 4$ neurons.	23
1.13	Encoding-Decoding illustration between the space of fundamental memories ξ_μ and the space of stored vectors x_μ	24
1.14	Bidirectional associative memory neural network.	30
2.1	Periodic attractors of neural networks without coupling terms.	64
2.2	Time evolution of the error system is shown in (a) for $\ \Lambda(t)\ _1 = 2.3$, and in (b) for $\ \Lambda(t)\ _1 = 0.5$	65
2.3	Time evaluation of the error system for $\omega_i/\omega_j \approx 0.000001, i \neq j$	65
3.1	The chaotic attractors of master system (3.53) and response system (3.54).	90
3.2	(a) and (b) presents error bound for $\tau = -0.5$ and $\tau = 0.2$, when $\gamma_i = 0$ and $\gamma_i = 20$ respectively.	90
3.3	(c) demonstrate the error curve when $\tau = 0.2$ and $\gamma_i = 0$. (d) plot of nonuniform distribution of impulses.	90

4.1	(a) and (b) demonstrate time evolution and phase portrait of states' trajectories for the system (4.44), respectively	114
4.2	(a) and (b) demonstrate time evolution of $\ e(t)\ _2$ for stabilizing and destabilizing impulses with $\hat{T}_a = 0.071$, respectively.	114
4.3	(a) demonstrate time evolution of $\ e(t)\ _1$ for stabilizing with $\hat{T}_a = 0.03$. (b) demonstrate time evolution of $\ e(t)\ _1$ for destabilizing impulses with $\hat{T}_a = 0.04$	114
5.1	Phase portrait of hybrid impulses with $T_a = \infty$ for $N_\zeta(t, s) = [\sqrt[3]{t - s}]$ in Example 5.4.2.	140
5.2	Chaotic attractors of equations (5.2) and (5.4) are shown in (a) and (b), respectively.	140
5.3	Hybrid impulsive sequence with $\mu = 1.1$ and the corresponding time evolution of controlled error neural network (5.12) for $T_a < \infty$ are shown in (a) and (b), respectively.	140
5.4	Time-evolution of error neural network (5.12) with $\mu = 1.15$ and $\mu = 0.35$ for $T_a = \infty$ are shown in (a) and (b), respectively.	142

List of Tables

1.1	Comparison between a human brain and an artificial neural network.	10
-----	--	----

PREFACE

An artificial neural network is motivated by a human central nervous system that consists of many biological neurons interconnected as a network. An artificial neural network is designed to model the way in which human brain functions, and it is implemented in computing machines as electronic devices. In applications of neural networks, stability analysis of their mathematical models is an essential part. For example, neural networks that are designed to implement in content addressable memory must have "stable" equilibrium points, and to achieve synchronization between neural networks, error system must be "stable". Moreover, if inputs to the neurons of networks are not continuous then the states of neurons may exhibit sudden jump at discrete points. This kind of phenomena is described by the mathematical models consist of both continuous and discontinuous states that is called impulsive systems. Therefore, in the present thesis, we focused on investigating the problems of stability and synchronization of neural networks affected by impulses.

This thesis is organized into six chapters. In the beginning of this thesis, we introduced artificial neural networks from the origin of motivation to designation and modeling. We briefly discussed the architecture of distinct neural networks (Hopfield, Cohen-Grossberg, and Bidirectional associative memory neural networks) together with their modeling into mathematical equations and how to find the stability of equilibrium points. We finished the introductory chapter by introducing all the mathematical tools which are used throughout the chapters to study the problems. The first problem, studied in chapter 1, is about investigating the modified function projective synchronization between different Cohen-Grossberg neural networks. In the next chapter, we extended the problem to the projective synchronization of different Hopfield neural networks under the influence of impulses. The solution of

impulsive systems is mainly affected by two factors: impulsive sequence and impulsive strength. Therefore, in the remaining two chapters, we studied stability and different types of synchronization problems under the influence of generalized impulsive sequence.

In the last chapter, we have discussed the future work that will be extension of the results obtained in this thesis.