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It is further certified that the student has fulfilled all the requirements of Comprehensive Examination, Candidacy and SOTA for the award of Ph.D. degree.

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I, ***Rakesh Kumar***, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of ***Prof. Subir Das*** and the co-supervision of **Dr. Rajesh Kumar Pandey** from ***27 July, 2016*** to ***May, 2021*** at the ***Department of Mathematical Sciences, Indian Institute of Technology (Banaras Hindu University), Varanasi***. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not wilfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, thesis, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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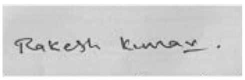

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To

My Beloved Parents & Elder Brother

Shrimati Namuna Devi

&

Shri Lalan Singh

&

Shri Ranjeet Singh

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Rakesh Kumar

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PREFACE

An artificial neural network is motivated by a human central nervous system that consists of many biological neurons interconnected as a network. An artificial neural network is designed to model the way in which human brain functions, and it is implemented in computing machines as electronic devices. In applications of neural networks, stability analysis of their mathematical models is an essential part. For example, neural networks that are designed to implement in content addressable memory must have "stable" equilibrium points, and to achieve synchronization between neural networks, error system must be "stable". Moreover, if inputs to the neurons of networks are not continuous then the states of neurons may exhibit sudden jump at discrete points. This kind of phenomena is described by the mathematical models consist of both continuous and discontinuous states that is called impulsive systems. Therefore, in the present thesis, we focused on investigating the problems of stability and synchronization of neural networks affected by impulses.

This thesis is organized into six chapters. In the beginning of this thesis, we introduced artificial neural networks from the origin of motivation to designation and modeling. We briefly discussed the architecture of distinct neural networks (Hopfield, Cohen-Grossberg, and Bidirectional associative memory neural networks) together with their modeling into mathematical equations and how to find the stability of equilibrium points. We finished the introductory chapter by introducing all the mathematical tools which are used throughout the chapters to study the problems. The first problem, studied in chapter 1, is about investigating the modified function projective synchronization between different Cohen-Grossberg neural networks. In the next chapter, we extended the problem to the projective synchronization of different Hopfield neural networks under the influence of impulses. The solution of impulsive systems is mainly affected by two factors: impulsive sequence and impulsive strength. Therefore, in the remaining two chapters, we studied stability and different types of synchronization problems under the influence of generalized impulsive sequence.

In the last chapter, we have discussed the future work that will be extension of the results obtained in this thesis.