*CHAPTER 3*

## ANALYSIS OF FIBRE LAMINATED GIRDER

### 3.1 General

Fiber-reinforced polymer (FRP) composite materials have shown the great potential of alternative structural strengthening material to conventional one, especially in repair and rehabilitation of existing girders of the bridges. The popularity of FRP increased rapidly in the bridge industry due to their easy handling, less complication application procedure, lightweight, high strength, long-term durability, excellent anti-corrosive behavior, and fatigue resistance. The production and installation are very convenient due to their lightweight.

The aim of this chapter is to quantify the effect of the different retrofitting combinations of FRP on the various bridge span and to conclude an appropriate layer of FRP for each length of girder. The analytical analysis of a standard girder is carried out with and without strengthening. For the strengthening, the beam is retrofitted with glass-reinforced polymer in shear and carbon-reinforced polymer in flexure and analysed this combination by varying girder lengths and changing the layers of FRPs and analysed the static and modal behavior of a simply supported beam. For this purpose, both static structural analysis and modal analysis are performed on different variations of the same beam in a 3D simulation environment of ANSYS Workbench 18.1. This software uses FEA (Finite Element Analysis) to analyze complex real-world problems by simulating the same virtually. For this study, around 90 beam models were simulated to generate a wide array of data. This data is further used to develop the relevant trends and relations with the aid of graphs and statistical techniques.

The parametric study included the effect of change in width to depth ratio, percentage of steel and loading intensity in a simply supported reinforced concrete (RC) beam on the maximum deflection, stress, strain energy, natural frequencies and deflections of various modes of free vibration till the failure of RC beam and a comparison is sought between the original beam and a failed beam retrofitted with one, two and three layers carbon and glass fibre sheets.

### 3.2 Analysis of the girder

A standard beam of fixed dimensions (Figure 3.2 a )and a two-point loading was first simulated in the software while comparing the results with the manual calculations of maximum deflections to ascertain the most appropriate simulation settings in the three-dimensional analysis environment of the software. Various beams were then analysed altering the width to depth ratio, the percentage of tension steel and the loading intensity over the beam and the results of the static as well as modal (frequency) analysis were noted including the deflections, stresses, strains, strain energies, natural frequencies of first 5 modes of free vibrations and their respective displacements. The identical beams were then retrofitted with layers glass fibres sheets with their principal axis inclined at an angle of zero degrees with the span of the beam. The same analysis was done on the beams and the results were noted. These results are compared and multivariate regression analysis is performed over the results to obtain equations to testify the observations and the trends. This whole process concludes that the effect of retrofitting is insignificant for single layer FRP while the most effective results were obtained for the case of triple-layer FRP. Deflections, stress and strain energy with their relative change across various cases are accurately related to the varying parameters through regression analysis. Frequency is also accurately related but the relative change in frequency is not well established.

RC beams strengthened with FRPs suffer failure in three ways i.e., flexure, shear and de-bonding with the de-bonding of the FRP sheet being the dominant failure mechanism. The plates can be provided on the tension side of the beams to enhance flexural strength and on the lateral sides to enhance its shear resisting properties as shown in Fig. 3.1.

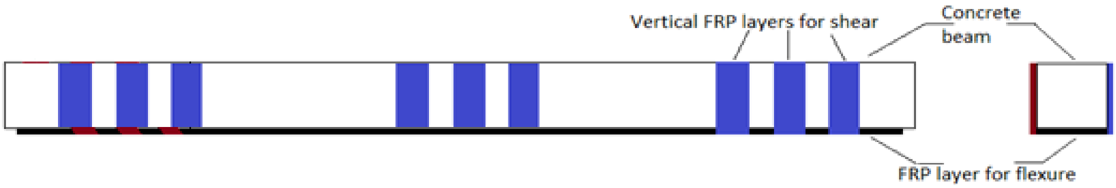



**Figure 3. 1** Schematic diagram of RC beam retrofitting using FRP

There are significant numbers of studies on the static performances of the structural entities but not many incorporate the modal analysis and effect of these techniques on the natural frequencies and mode shapes of a structure. The knowledge of natural vibrations of a structure is indispensable today for seismic design. In 1985 Mexico City earthquake, most damaged buildings were between 6 to 15 storeys, which collapsed as their natural mode of vibration was close to that of the earthquake. Hence, a study of the effect of retrofitting on these properties of the structure is essentially required.

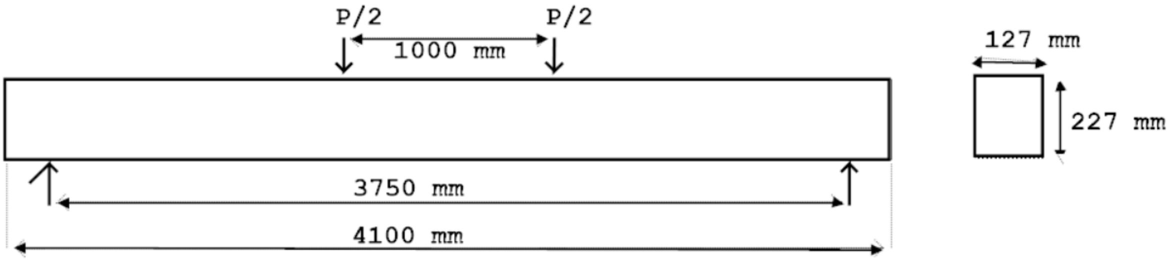
In most of the reinforced structures, only two types of girder have been used worldwide- 1) I-girder and 2) prismatic girder. In this chapter analysis for both kinds of girders has been performed.

### 3.2.1 Numerical analysis of girder

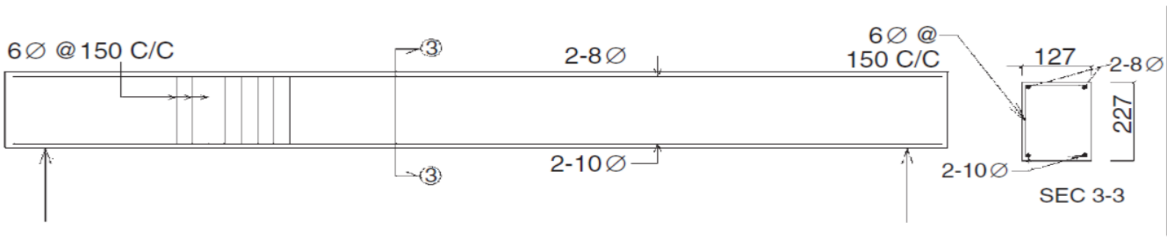
Numerical analysis of girder using ANSYS workbench 18.1 is executed. The above dimension girder is modeled using the finite element method and the parametric study is carried out. The characteristics of girder, convergence strategy and loading intensity have been discussed below.

### 3.2.2 Geometric and elastic properties of benchmark problem

A study of the effect of retrofitting of FRP sheets for flexure on a simply supported beam in terms of static analysis and frequency analysis with the help of ANSYS Workbench is been performed to develop trends based on the observations and relations between the effect of retrofitting and the configuration of beam.




(a) Loading arrangement




(b) Longitudinal and transverse sections

**Figure 3. 2** Benchmark beam used for verification (Prem pal et. al.)

A standard beam studied by pal et al. with a predefined loading pattern as shown in Fig. 3.3 a is chosen with reinforcement details as shown in fig 3.2 b. This beam is 4100 mm in length and is simply supported having a cross section of 127mm×227 mm. The supports are at a distance of 3750 mm, which is the effective span of the beam. The beam has 2-10mmφ Fe 250 bars in the tension side and 2-8mmφ bars in the compression side. Shear stirrups of 6mmφ are provided at a distance of 150mm c/c. M 20 concrete is used as the primary material for beam. This beam is solved analytically for a value of load P=13.3 kN by the methods mentioned in Sec. 3.1 and then analyzed in ANSYS to obtain the value of deflection at the midpoint in both cases.

For the given beam : $E_c = 3 \times 10^4\ Nmm^{-2}$

$$I_u = 1.341 \times 10^8\ mm^4$$

The value of flexural rigidity $E_cI_u$ in the transformed area approach is obtained to be 4.0235E+12 Nmm$^2$ and the deflection $v$ as a function of span comes out to be:

$$E_cI_uv = -0.1417x^4 + 5.98 \times 10^9 x - 1.046 \times 10^{12} + 1108.33\{\langle x - 175\rangle^3 - \langle x - 1550\rangle^3 - \langle x - 2550\rangle^3 + \langle x - 3925\rangle^3\}$$

This equation gives the deflection at the midpoint as $v_{x=2050}$ =3.945mm. Analysing the same beam in ANSYS, as shown in Figure 3.4 presents the displacement at the midpoint to be 3.653mm. Support used in the simulation is a prism with a (40*40) mm$^2$ cross-section with chamfer of 19 mm on both top edges and the load bearing block used in the simulation is a semi-circular prism of radius 40 mm, as shown in fig 3.5. This process completes with an error of approximately 7%. Hence, it can be concluded that the simulation of the benchmark beam is correct. The same settings, i.e. geometrical setup, meshing and analysis settings etc. can be used to analyse other cases.

## 3.3 Modeling of prismatic girder

Analysing the same beam in ANSYS as shown in Fig 3.4 a gives the displacement at the midpoint to be 3.653mm. Support used in the simulation is a prism with a (40*40) mm$^2$ cross-section with chamfer of 19 mm on both top edges fig 4b and the load bearing block used in the simulation is a semi-circular prism of radius 40mm as shown in fig 4c . This process completes with an error of approximately 7%. Hence, it can be concluded that the simulation of the benchmark beam is correct. The same settings, i.e. geometrical setup, meshing and analysis settings etc. can be used to analyse further cases.

A beam is selected as a benchmark and it is analysed analytically. The same beam is simulated in ANSYS and the results are compared to verify that the simulation is correct. The methods used are described in Sec. 4.1 and 4.2.

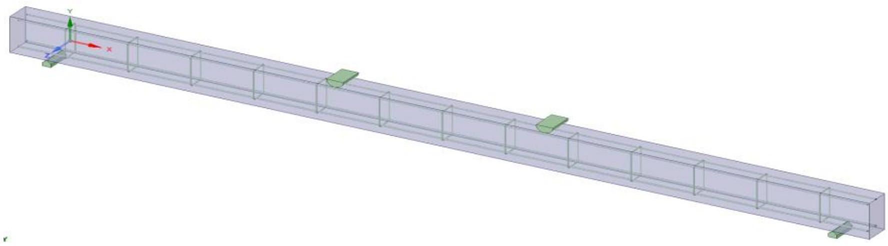
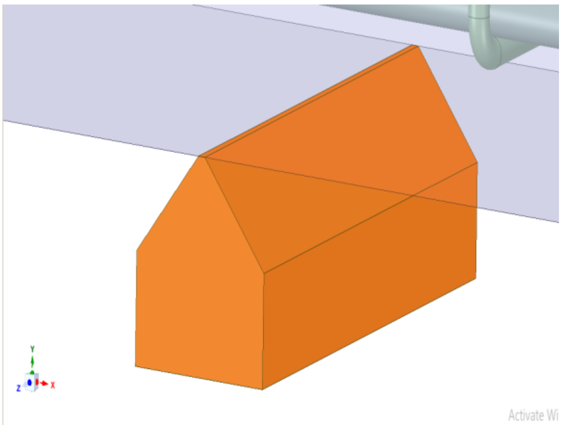
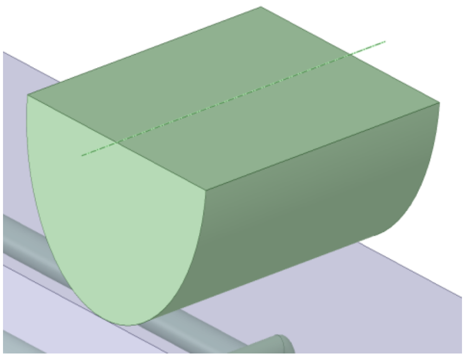


**Figure 3. 3** Model of simulated beam



a) Support



b) Load bearing block

**Figure 3. 4** Support and load bearing block

### 3.3.1 Convergence study

The analysis of this girder has been carried out using finite element method analysis (FEM). In the FEM analysis the convergence study is very crucial to adjudge the correct mesh size in analysis. The coarser element generally produces poor results, while the finer element gives the best results but takes a long time to analyze. So to decide the right amount of mesh size three significant steps have been taken to determine the compatible mesh size –

1. Analysis of girder for different mesh size
2. Recognition of the area of maximum deformation and stress in girder and analyse the result of that area only with finer mesh size.
3. Keep refining the mesh until no difference in FEA results Analys observed (for e.g. check force displacement curve or stress strain distribution or crack growth etc). Analyse the time taken and error for each mesh size and find the suitable size of mesh which have suitable analysis time and less error.

If it intends to use the local damage model, then it will never converge. Choosing the element size infinitely small will make the energy dissipated in the damage zone = 0. In this case, it needs to regularize the given model for mesh dependency OR use a nonlocal formulation.

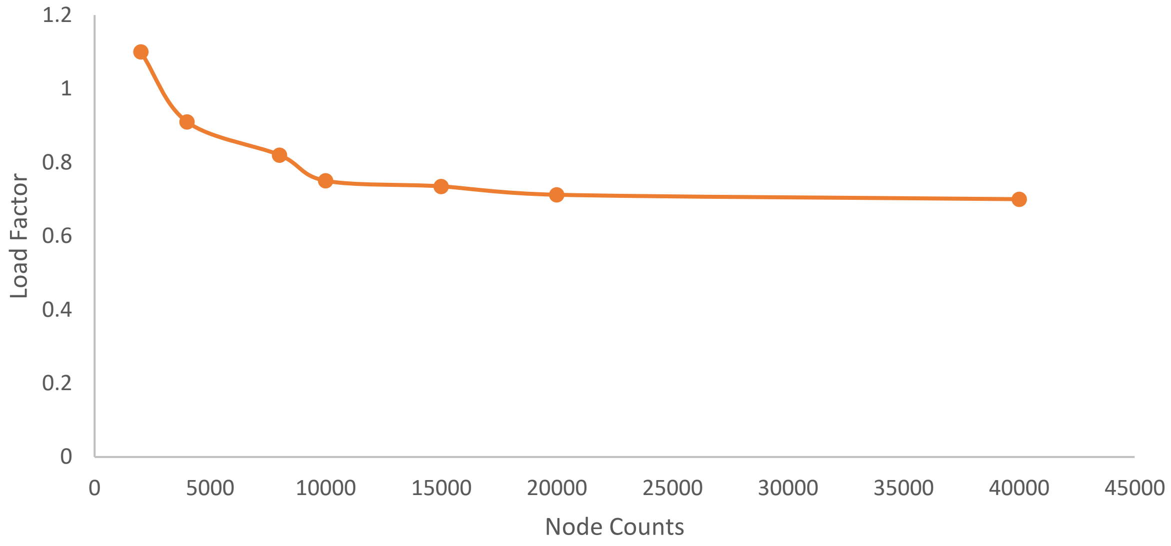



**Figure 3. 5** Analysis of load factor in comparison to node counts

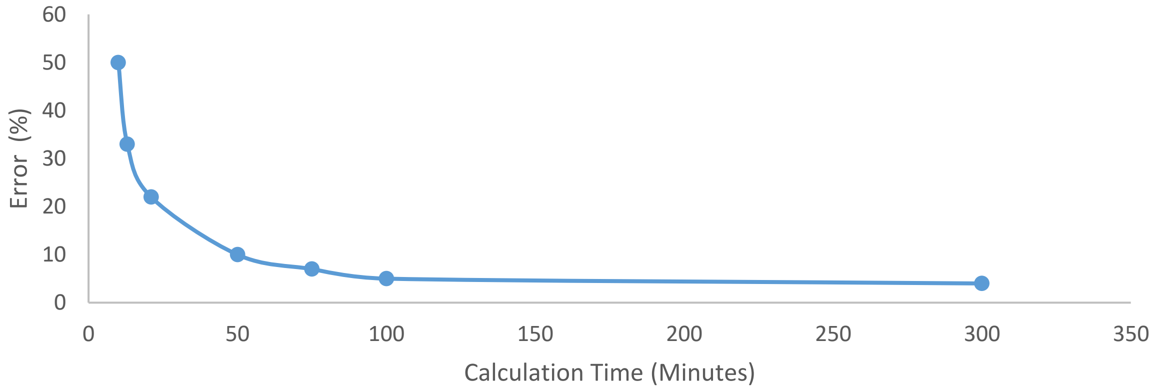



**Figure 3. 6** Relation between calculation times with respect to reduction in error

The analysis observed that if the accuracy increased by 1-2%, then computing time increased by nearly 2 times (Figure 3.7). The different mesh size have been worked out with an analytical solution and accurate result are obtained after analysing different mesh analysis

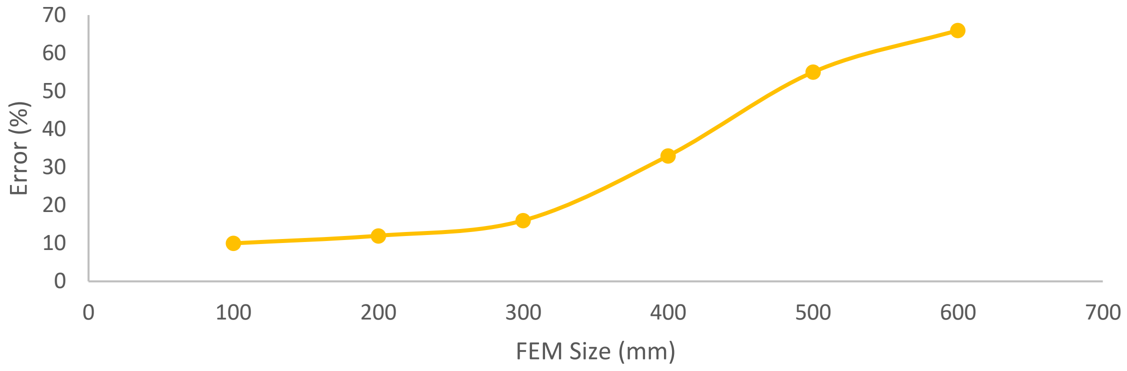



**Figure 3. 7** Relation of reduction of error with respect to finer mesh size

### 3.3.2 Properties of fibre laminates used in lamination

Table 3.1 and Table 3.2 listed property of carbon and glass fibre we have used in ANSYS. These properties are obtained by laboratory testing of these material, the type of glass fibre is used here is S-Glass UD.

**Table 3. 1** Properties of carbon fibre laminates used

| Property | Value |
|---|---|
| Density (gm/cm$^3$) | 1.85 |
| Elongation (%) | 1.5 |
| Young's Modulus (GPA), longitudinal | 350 |
| Young's Modulus (GPA) in lateral direction | 31 |
| Tensile Strength, Longitudinal (MPa) | 2800 |
| Tensile Strength, Lateral (MPa) | 90 |
| Compressive Strength, Longitudinal (MPa) | 950 |
| Compressive Strength, Lateral (MPa) | 410 |

**Table 3. 2** Properties of glass fibre laminates used (S Glass)

| Property | Value |
|---|---|
| Density (gm/cm$^3$) | 2.35 |
| Elongation (%) | 2.2 |
| Young's Modulus (GPA), longitudinal | 90 |
| Young's Modulus (GPA) in lateral direction | 20 |
| Tensile Strength, Longitudinal (MPa) | 1442 |
| Tensile Strength, Lateral (MPa) | 73 |
| Compressive Strength, Longitudinal (MPa) | 560 |
| Compressive Strength, Lateral (MPa) | 296 |

### 3.3.3 Parametric study

Analytical analysis of prismatic and I- girder of standard dimension have been analysed in all the three possible combinations of fibre reinforced plastic for the five most used length- 10 meter, 15 meter, 20 meter, 25 meter and 30 meter.

After the analysis methodology was verified, different cases of beams are generated (approximately 90 in numbers). These beams are very similar to the benchmark beams except for variation in the following criteria:

a) Aspect ratio (b/d)

b) Percentage of steel (P)

c) Uniformly distributed loading intensity (w)

d) Number of layers of CFRP and GFRP used

The different combination of fibre has been used here to find the appropriate combination of CFRP and GFRP for both in static and dynamic analysis. The following combination of fibre have been used:

a. Glass fibre reinforced plastic (GFRP)
b. Carbon fibre reinforced plastic (GFRP)
c. Composite fibre reinforced plastic (GFRP and CFRP)

The glass fibre has used in the shear zone and carbon fibre in flexure zone of the girder. The analysis has been concluded on the various parameter:

## 3.4 Result and discussions

The prismatic analysis of beams using finite element modeling has been performed. The variations in beams in this category range from Without FRP, Single layer CFRP to triple layer CFRP, Single layer GFRP to triple layers of GFRP and their combination has been used for retrofitting of girder. These sheets were bonded to the beam so that the major principal axis of the fibres makes an angle of $90^{o}$ with the length of the beam as shown in. This ensures maximum effective use of the FRP retrofitting.

The analysis in ANSYS Workbench includes static structural analysis and modal analysis (frequency analysis) as covered in the Sec 3.2. The static analysis is used to generate total deformations (e.g. Fig. 3.8)), equivalent Von-Mises strains, equivalent Von-Mises stresses and total strain energies. For modal analysis, 5 modes of vibrations are extracted and their natural frequencies and total deformations (Figure 3.9) for Mode IV) are obtained with the different mode shapes.

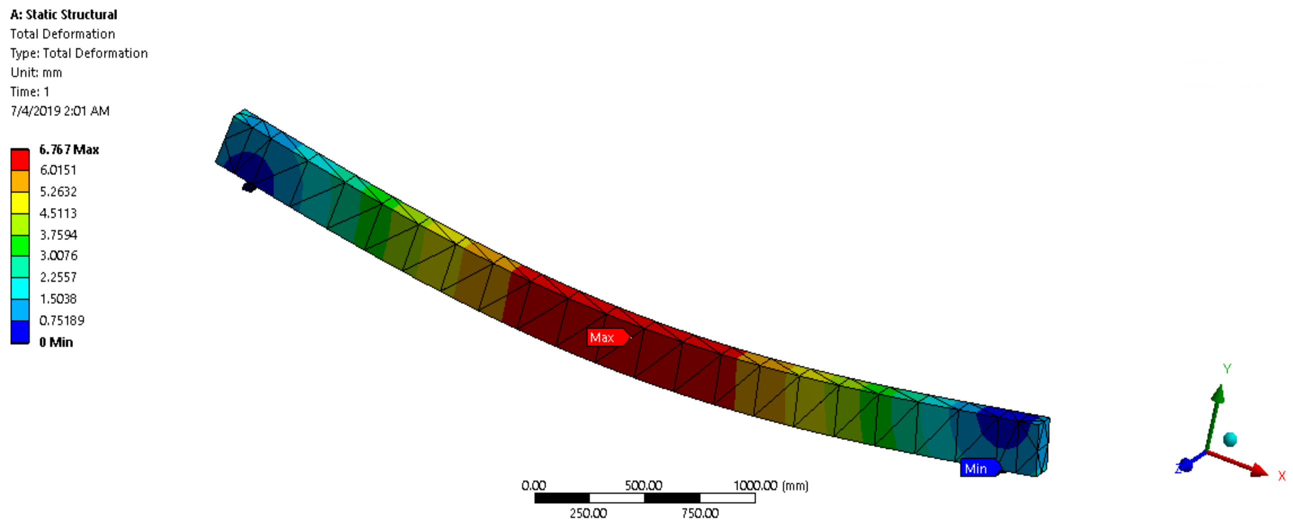



**Figure 3. 8** Static analysis for total deformation

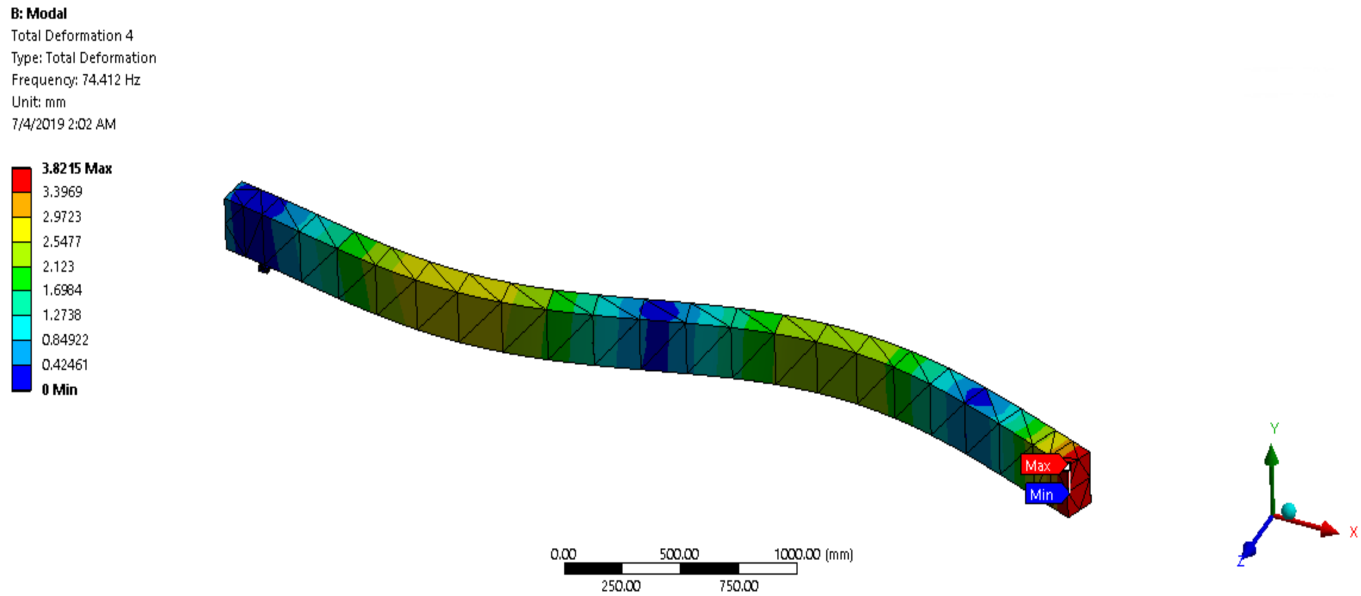



**Figure 3. 9** Modal analysis for mode shape and total deformation of Mode 4

The variation of static deflection, stress, strain energy, frequency and deflection of various modes with respect to aspect ratio, percentage of steel and loading intensity are discussed here:

### 3.4.1 Variation in deflection, tensile stress and strain energy of beams with different aspect ratio (b/d):

The non-dimensional deflection, tensile stress and total strain energy have been observed in Fig. 3.10, 3.11 and 3.12, to increase with the aspect ratio of the beams. However, the rate of increase is significantly larger for the case of without GFRP and single layer GFRP. The maximum reduction in deflection, stress and strain energy is observed in case of triple layer GFRP at the maximum aspect ratio of the dataset.

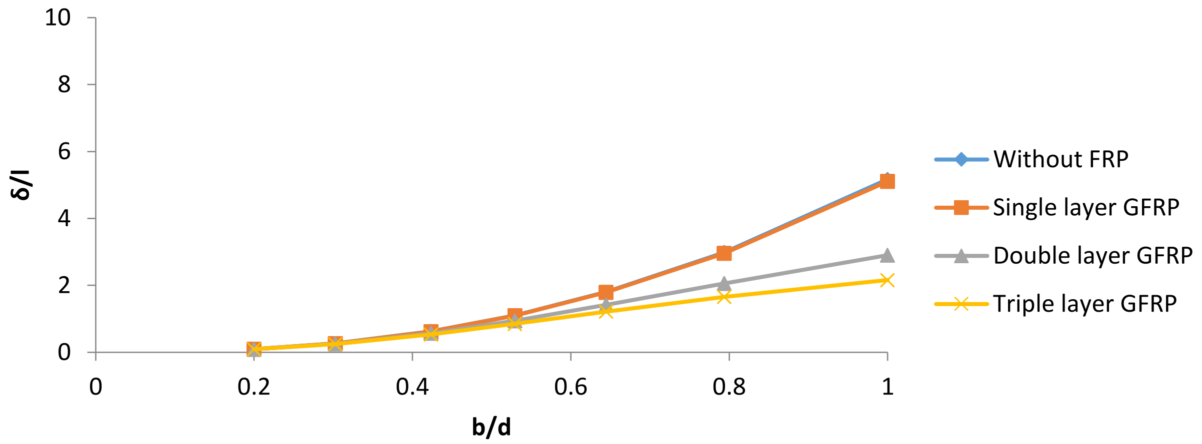



**Figure 3. 10** Variation of non-dimensional deflection with respect to aspect ratio

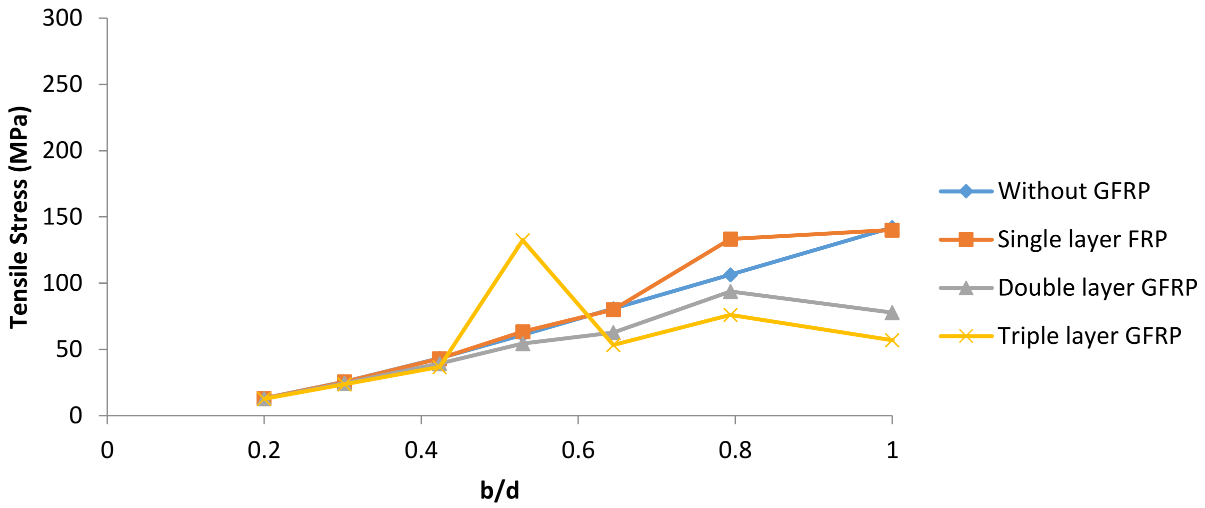



**Figure 3. 11** Variation of tensile stress with respect to aspect ratio

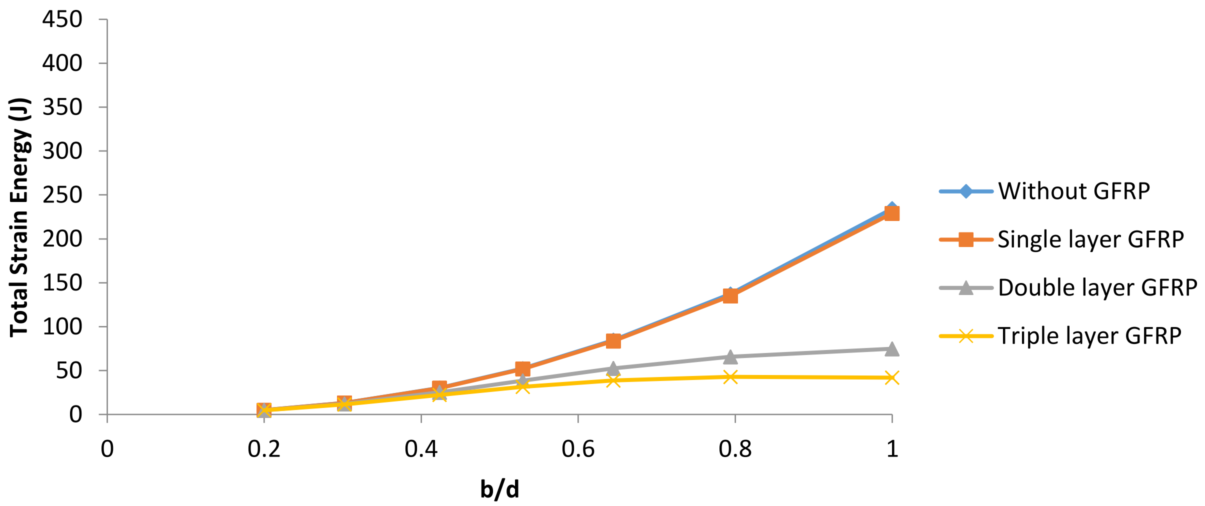



**Figure 3. 12** Variation of total strain energy with respect to aspect ratio

### 3.4.2 Variation in frequency with respect to aspect ratio for different modes

The variation of frequency with respect to the aspect ratio as shown in Fig. 3.13 3.14, 3.15 3.16, and Fig. 3.17, are not monotonous for any mode except Mode 1. However, as the number of layers of GFRP increases to three the curves tend to become smooth even achieving monotonous increase in Mode 2 and 4 and a monotonous decrease in Mode 3 for the case of triple layer GFRP. For Mode 5 a local maxima is observed for all cases when the aspect ratio is around 0.5. The increase in frequency between the cases of without GFRP and triple layer GFRP for Mode 2 is the highest at the maximum aspect ratio of the dataset.

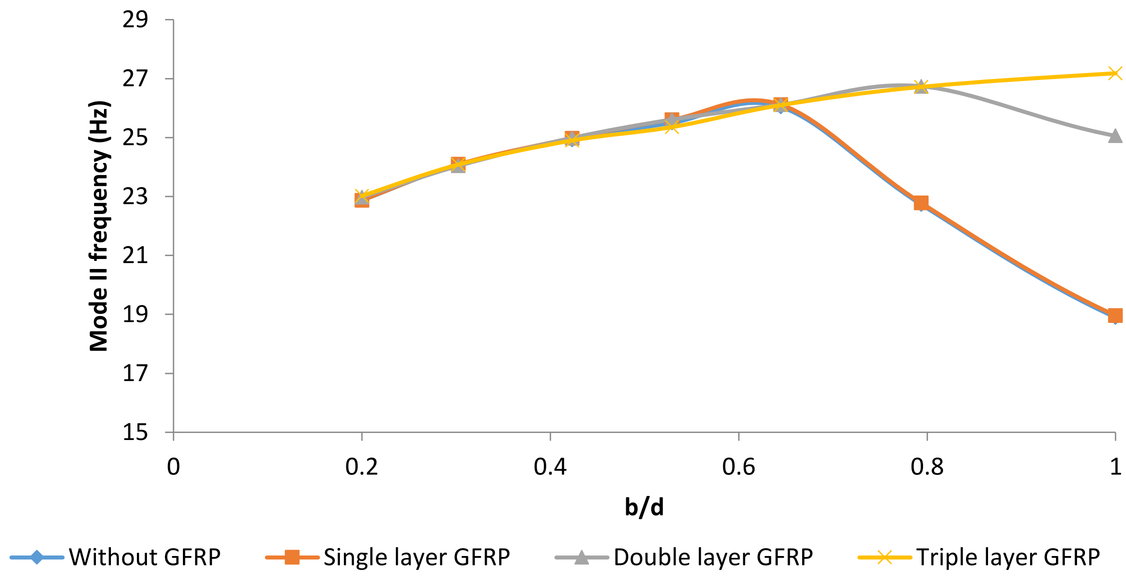



**Figure 3. 13** Variation of mode 1 frequency with respect to aspect ratio

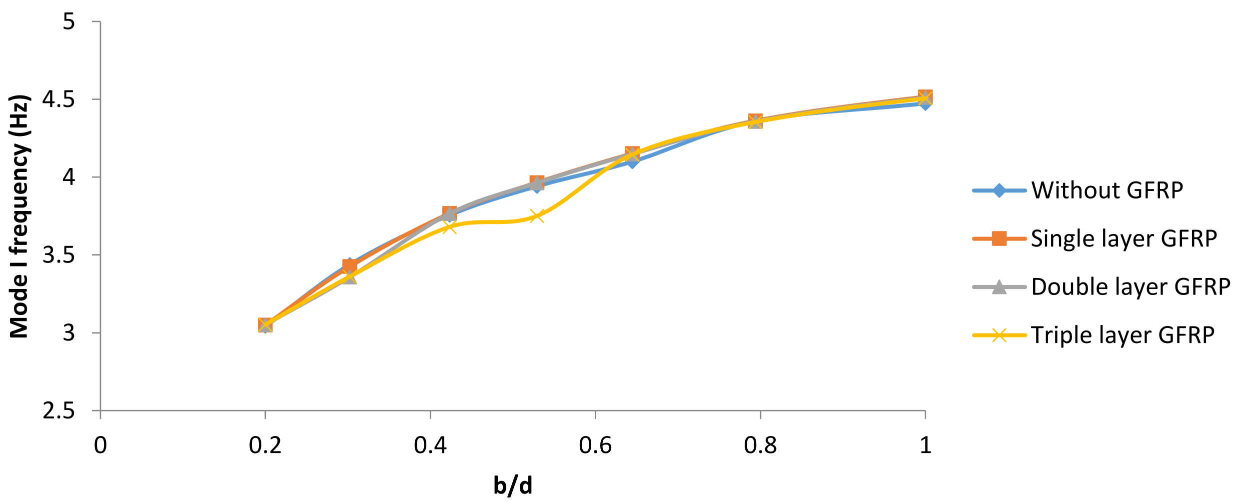



**Figure 3. 14** Variation of mode 2 frequency with respect to aspect ratio

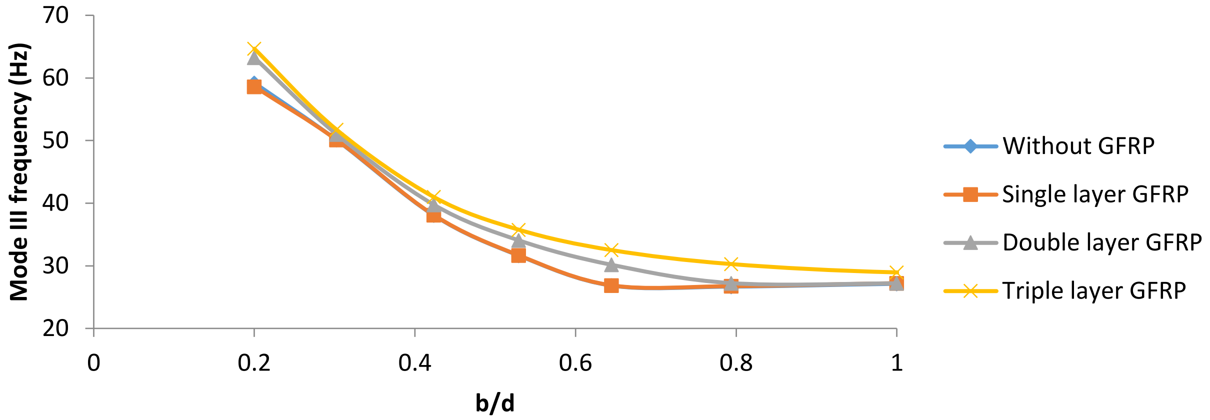



**Figure 3. 15** Variation of mode 3 frequency with respect to aspect ratio

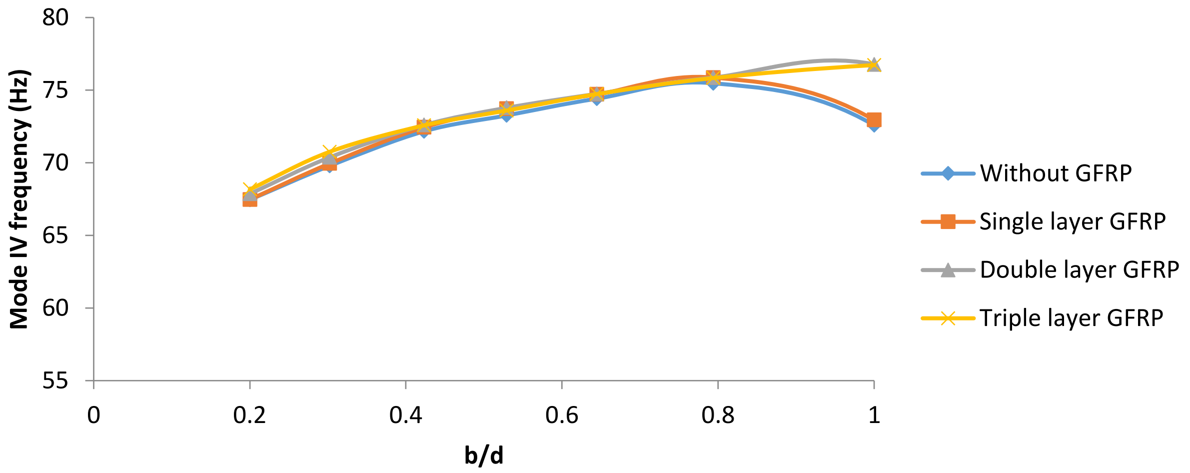



**Figure 3. 16** Variation of mode 4 frequency with respect to aspect ratio

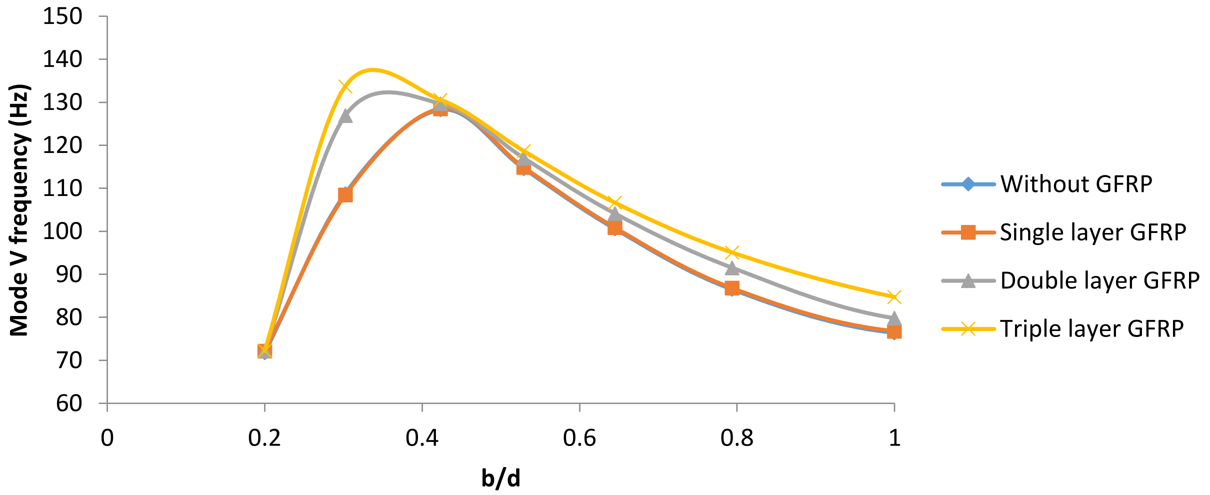



**Figure 3. 17** Variation of mode 5 frequencies with aspect ratio

### 3.4.3 Variation in deflection with respect to aspect ratio for different modes

The variation of deflection due to vibration with respect to aspect ratio as shown in Fig.3.18 3.19, 3.20, 3.21, and Fig. 3.22 are most consistent for Mode 1showing an increase with the aspect ratio for all cases. For Mode II, III and IV the variations are inconsistent. The case of triple layer GFRP experiences the maximum deflection for some aspect ratio in Mode 2 and IV while for Mode 3 and V, it is the single layer GFRP experiencing the maximum deflection for some aspect ratios. Failure due to de-bonding is observed in Mode 5 at lower aspect ratios thus rendering the deflections to go off charts.

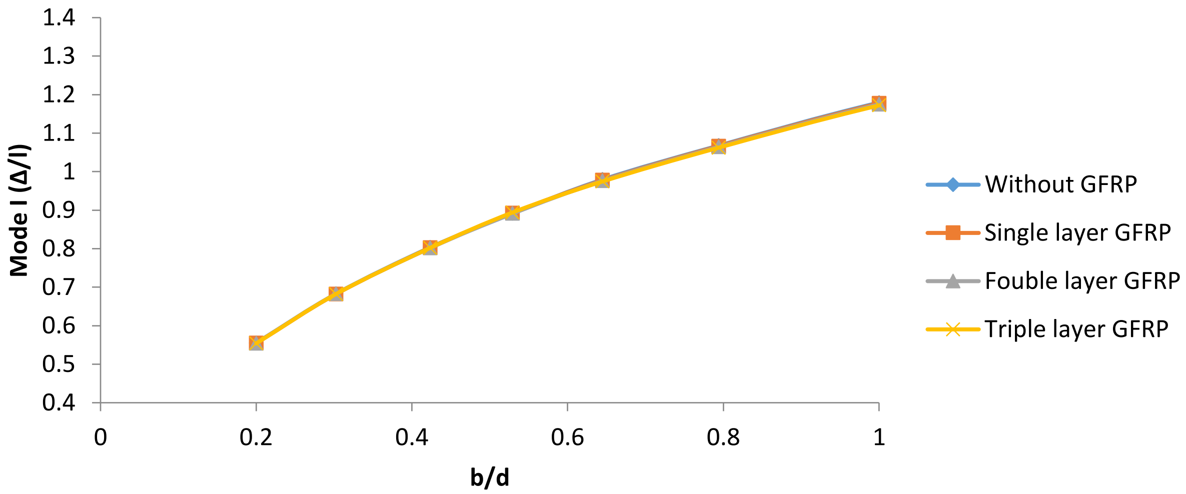



**Figure 3. 18** Variation of Mode I deflection with aspect ratio

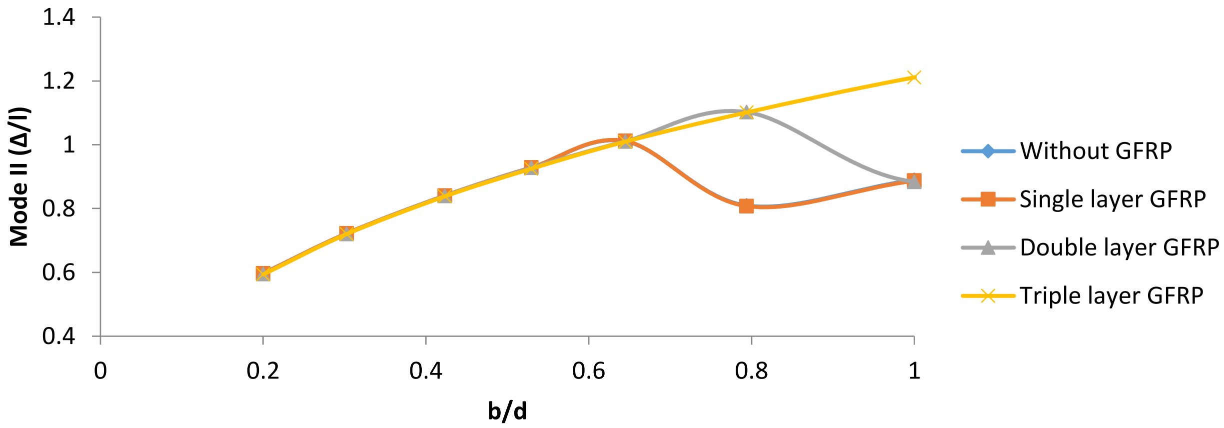



**Figure 3. 19** Variation of Mode 2 deflection with aspect ratio

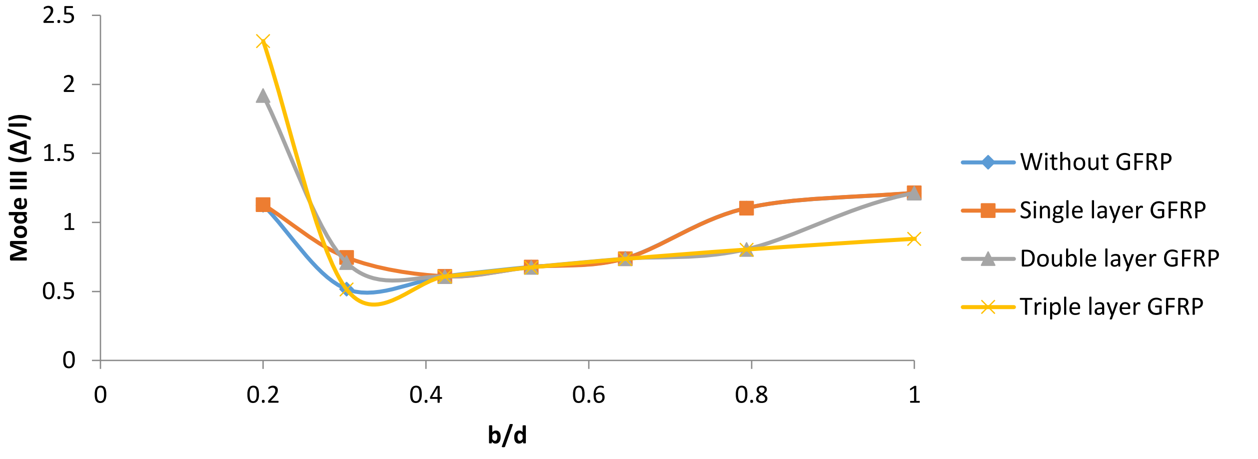



**Figure 3. 20** Variation of Mode 3 deflection with aspect ratio

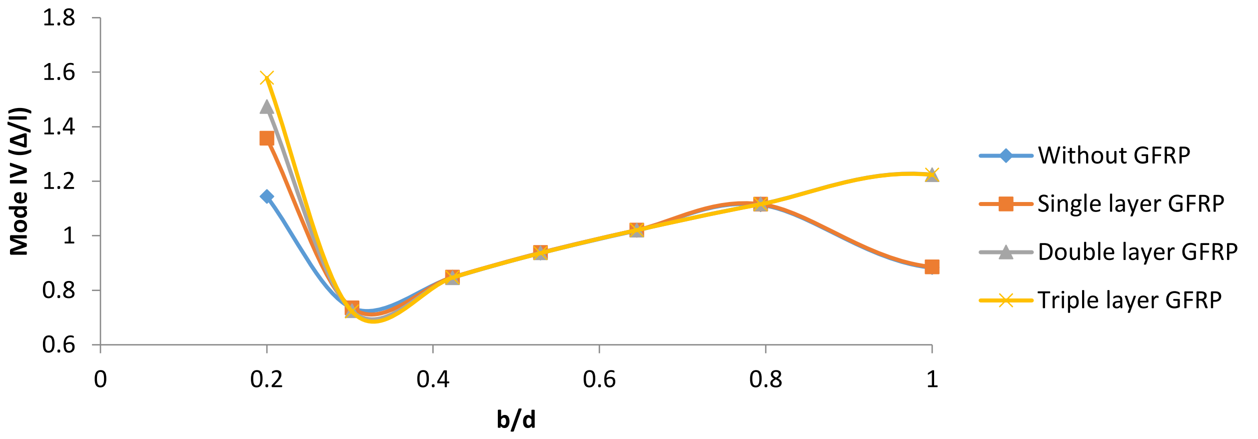



**Figure 3. 21** Variation of Mode 4 deflection with aspect ratio

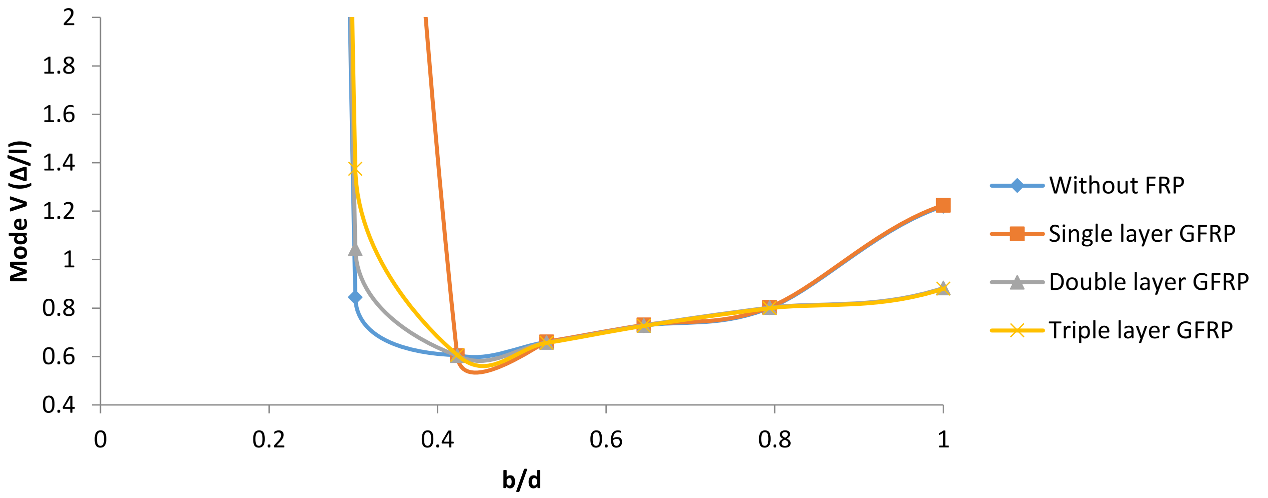



**Figure 3. 22** Variation of Mode 5 deflections with aspect ratio

### 3.4.4 Variation in deflection, tensile stress and strain energy of beams with different percentage of steel (p %)

The Non-dimensional deflection and total strain energy as shown in Fig. 3.23 3.24, and 3.25 are showing a gradual but steady decline with the increase in the percentage of steel. The tensile stresses remain uniform throughout with any number of layers of GFRP. Without GFRP the tensile stresses are inconsistent and have an ultimate rise with the highest percentage of steel in the dataset. Overall, the non-dimensional deflection, tensile stress and total strain energy are minimum and significantly lesser for the case of triple layer GFRP.

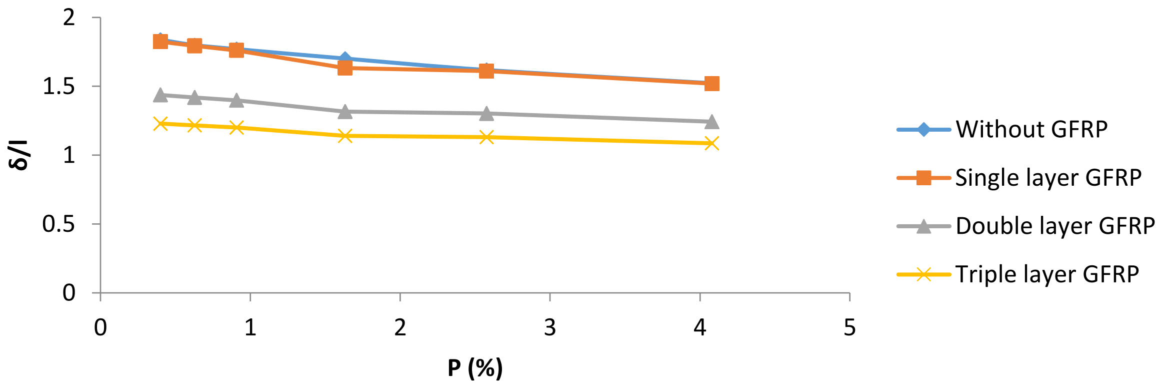



**Figure 3. 23** Variation of non-dimensional deflection w.r.t. % of steel

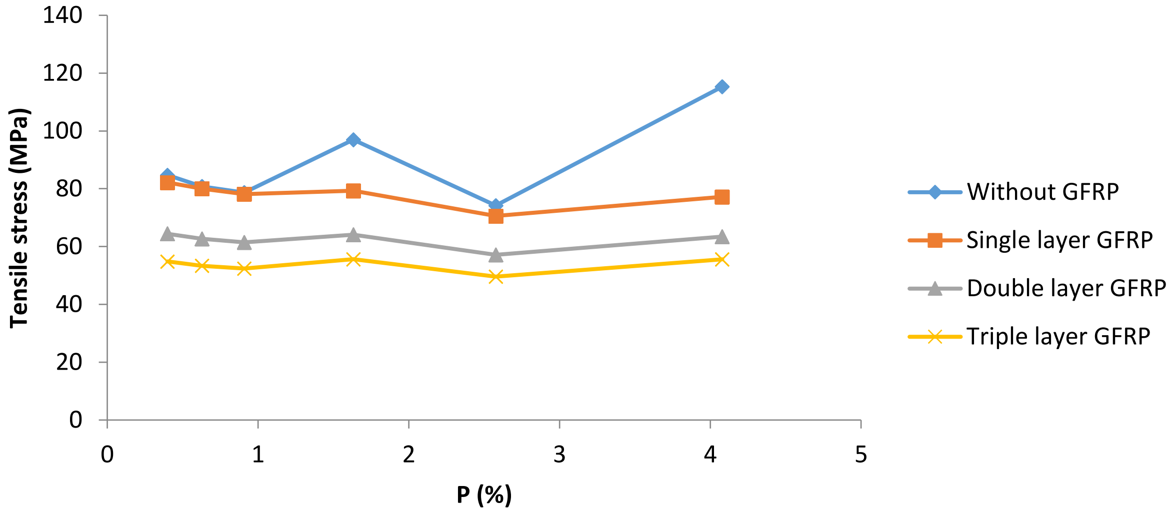



**Figure 3. 24** Variation of tensile stress with respect to % of steel

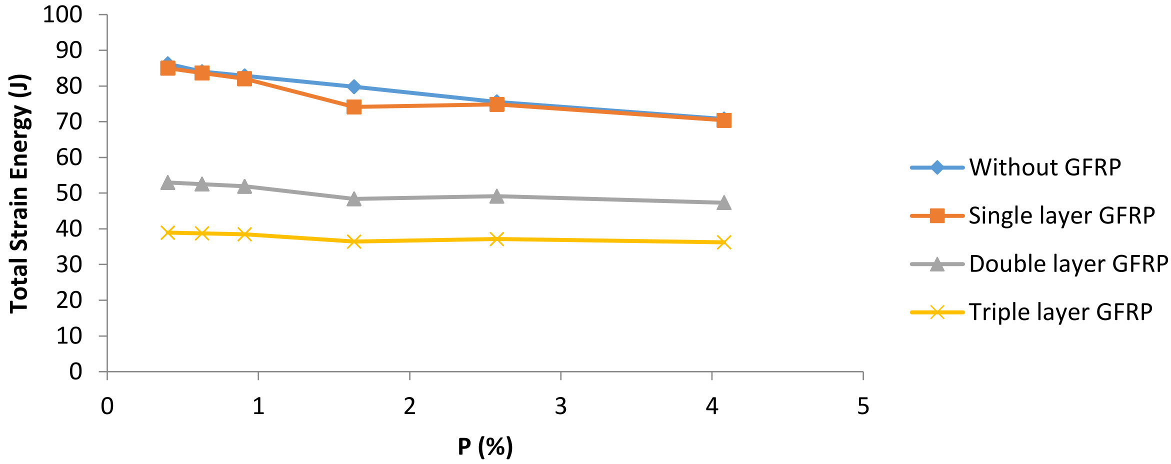



**Figure 3. 25** Variation of total strain energy with respect to percentage of steel

### 3.4.5 Variation in frequency with respect to percentage of steel for different modes:

The variation of frequency with respect to (w.r.t.) percentage of steel as shown in Fig. 3.26, 3.27 3.28, 3.29, and 3.30 are very similar for the cases in which retrofitting is done. The behaviour of beams without GFRP is significantly different from the other cases which have so much similarity that even their curves are almost same for Mode 1, 2 and 4. For Mode 3 and V the variations are similar for all cases and experience a monotonous increase in frequency with significant difference in frequencies of all the cases. There is a local maxima in Mode 1, 2 and 4 at 1.6 % of steel. For these modes the retrofitted beams have almost equal frequencies.

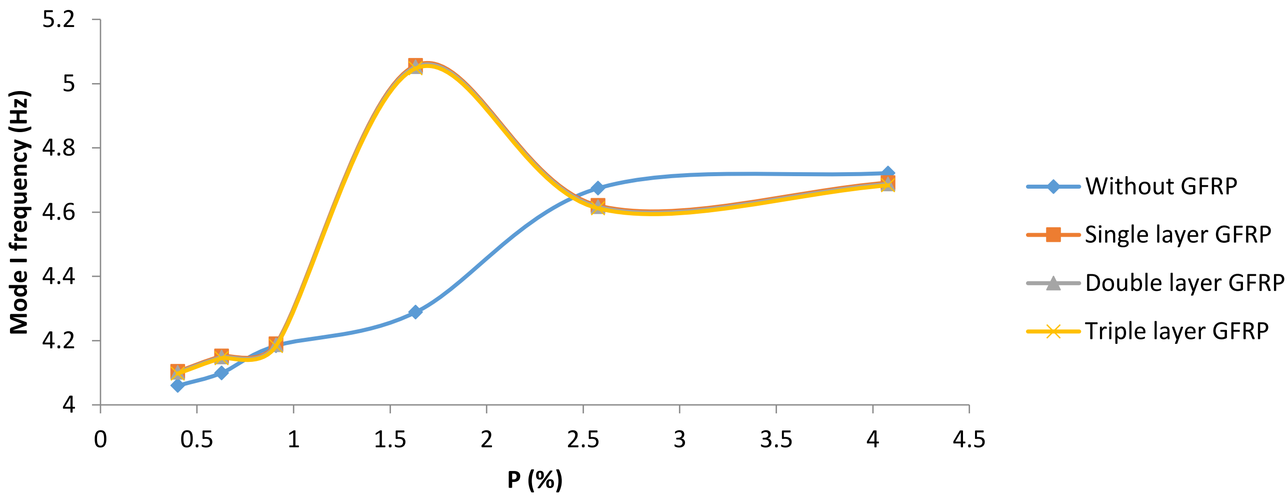



**Figure 3. 26** Variation of mode 1frequency w.r.t. percentage of steel

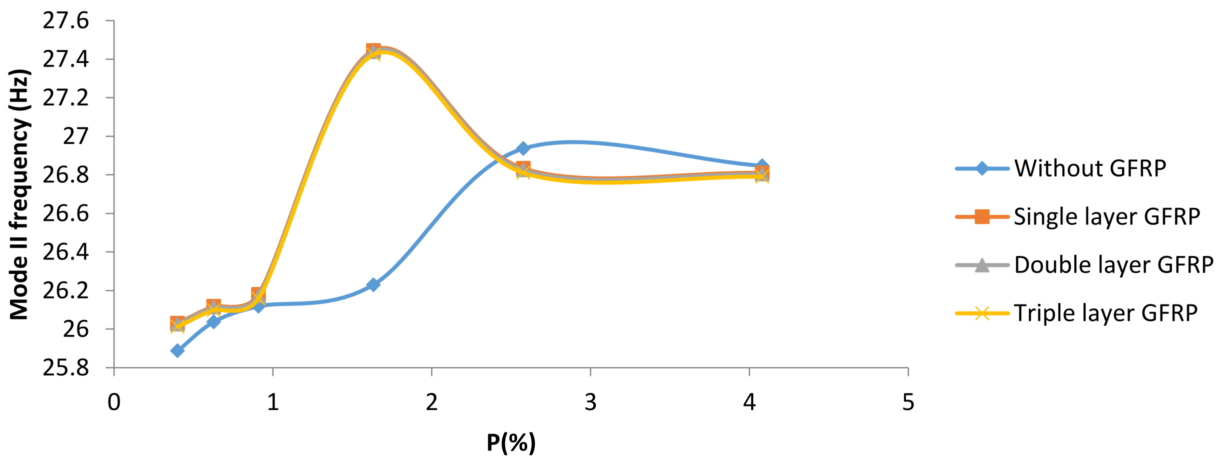



**Figure 3. 27** Variation of Mode 2frequency w.r.t. percentage of steel

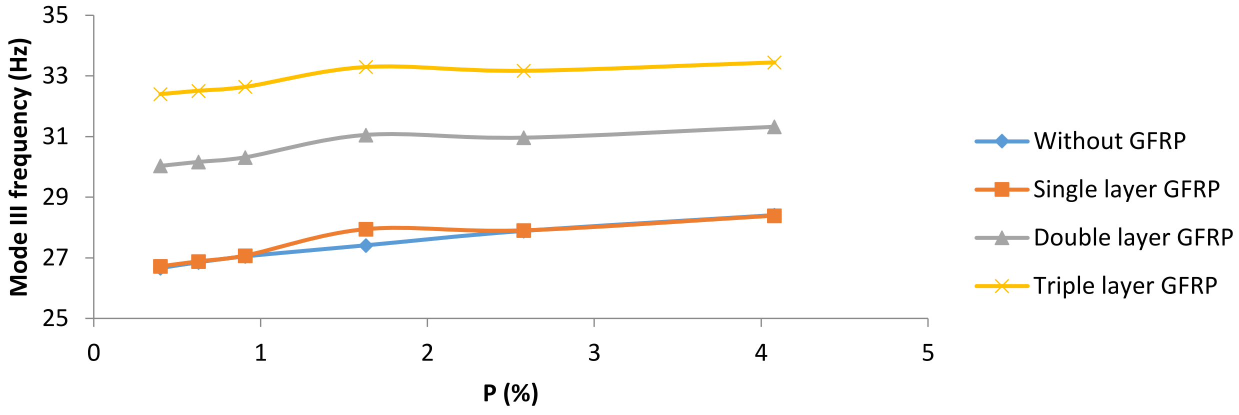



**Figure 3. 28** Variation of mode 3 frequency w.r.t. percentage of steel

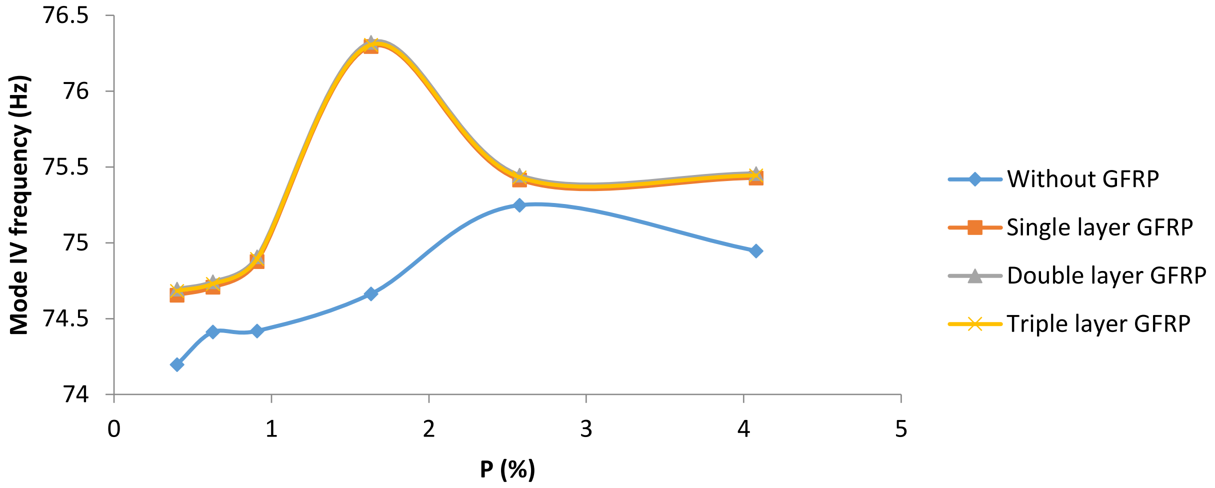



**Figure 3. 29** Variation of mode 4 frequency w.r.t. percentage of steel

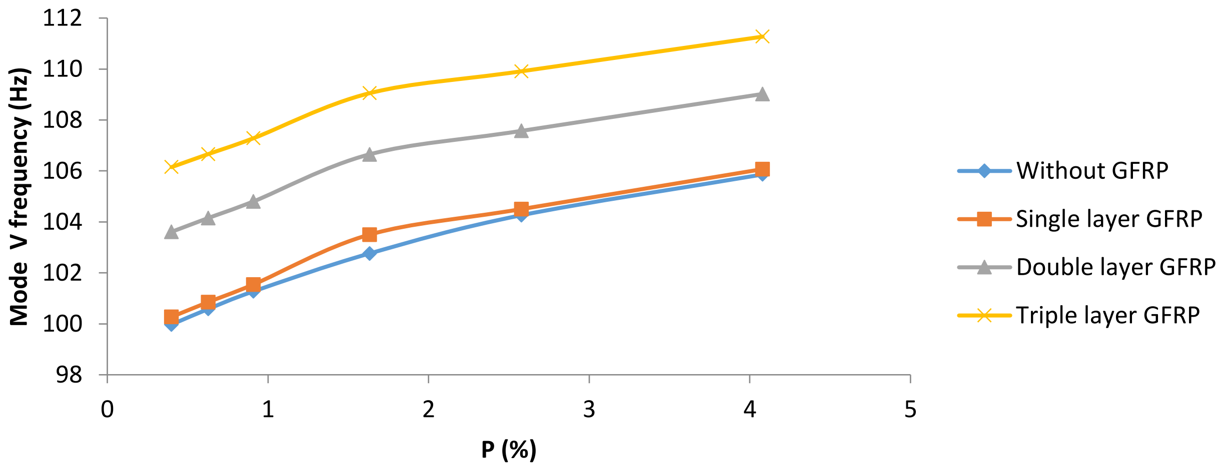



**Figure 3. 30** Variation of Mode 5 frequency w.r.t. percentage of steel

### 3.4.6 Variation in deflection with respect to percentage of steel for different modes:

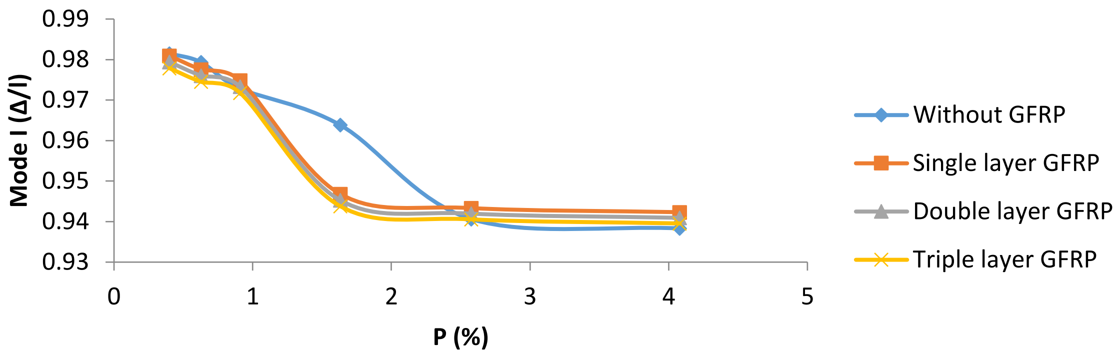



**Figure 3. 31** Variation of mode 1 deflection w.r.t. percentage of steel

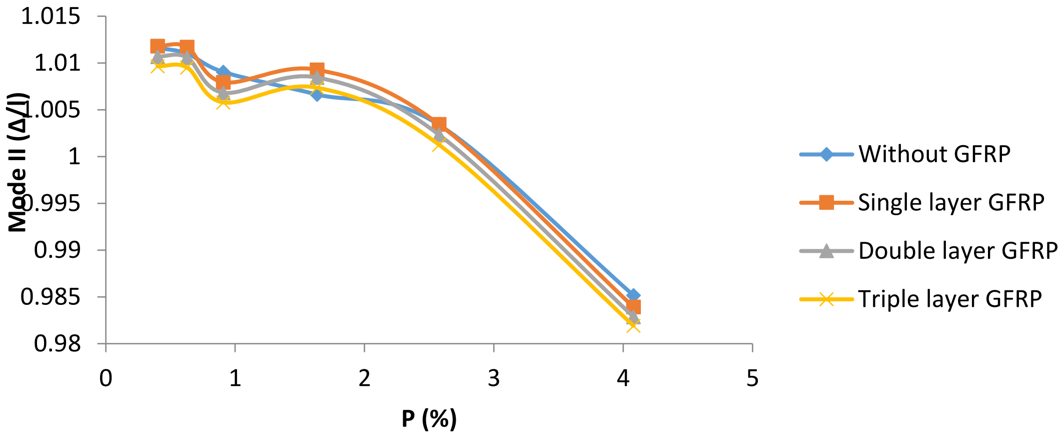



**Figure 3. 32** Variation of mode 2 deflection w.r.t. Percentage of steel

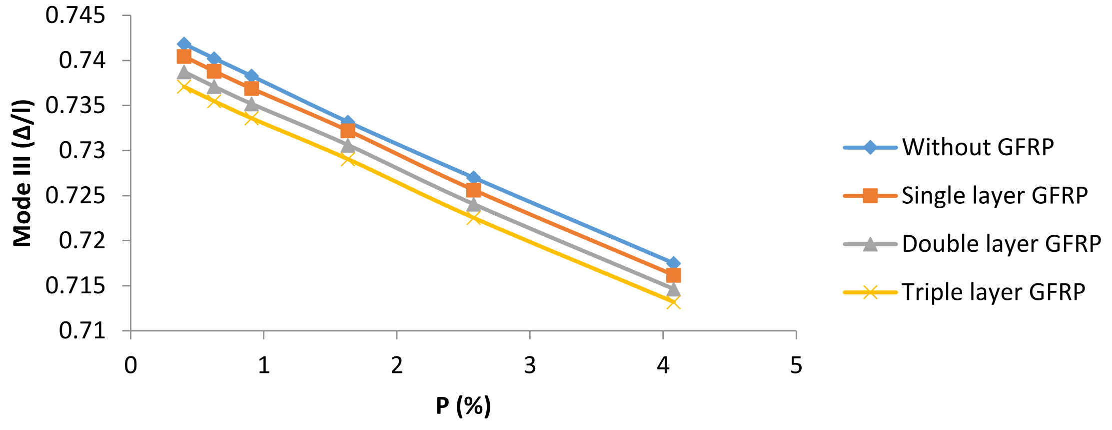



**Figure 3. 33** Variation of mode 3 deflection w.r.t. percentage of steel

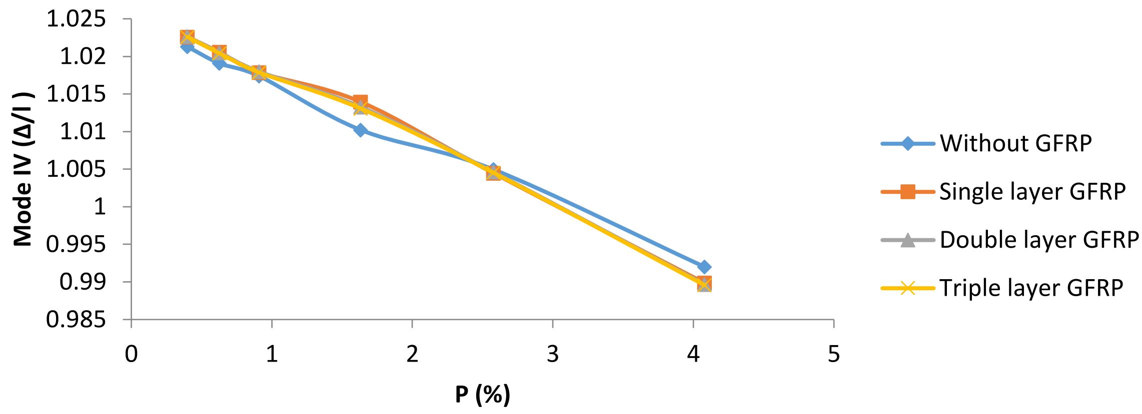



**Figure 3. 34** Variation of mode 4 deflection w.r.t. percentage of steel

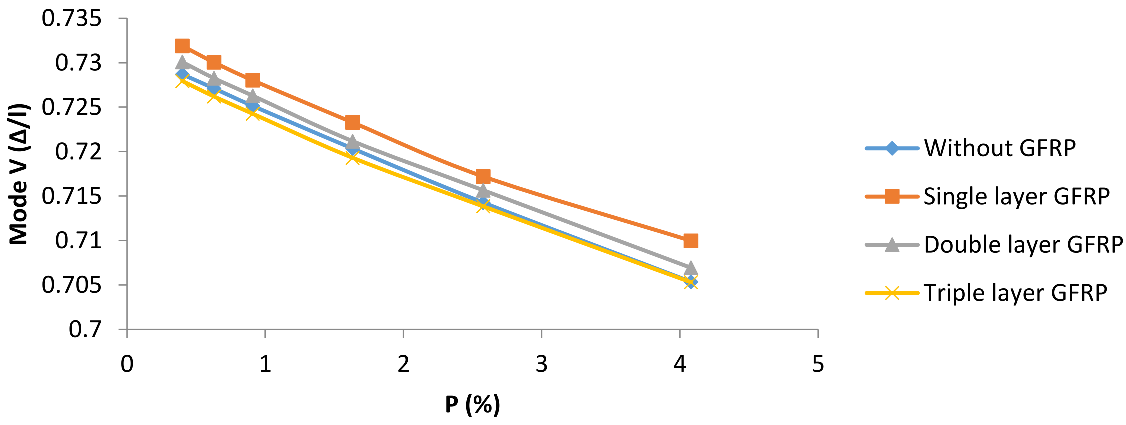



**Figure 3. 35** Variation of mode 5 deflection w.r.t. percentage of steel

Deflection due to natural vibrations varies the most consistently with percentage of steel out of all the parametric studies performed as can be observed from Fig.3.31, 3.32, 3.33, 3.34, and Fig 3.35  The behaviours of retrofitted beams show a similar result which is slightly different than beam without GFRP in Mode I, II and IV. The deflection in every mode decreases with an increase in the percentage of steel. While in Mode 1and II the decrease is not linear, Mode 3 has almost linear decrease and Mode 4 and V experience a linear, monotonous decrease in the deflections. For all cases it is observed the deflection is the minimum for triple layer GFRP with exception of a few points where beams without retrofitting have the minimum deflection.

### 3.4.7 Variation in deflection, tensile stress and strain energy of beams with different uniformly distributed loading intensities (w):

The non-dimensional deflection and tensile stress are showing a linear increase with respect to the loading intensity while the total strain energy is showing a quadratic increase as can be observed in Fig.3.36, 3.37, and 3.38 Similar to the earlier studies the beams without GFRP and those with a single layer of GFRP are having almost similar values of deflection, stress and strain energy at different points. The triple layer GFRP is the most effective in reducing the deflection, stress and strain energies at every value of loading intensity. The reduction is maximum at higher loading intensities.

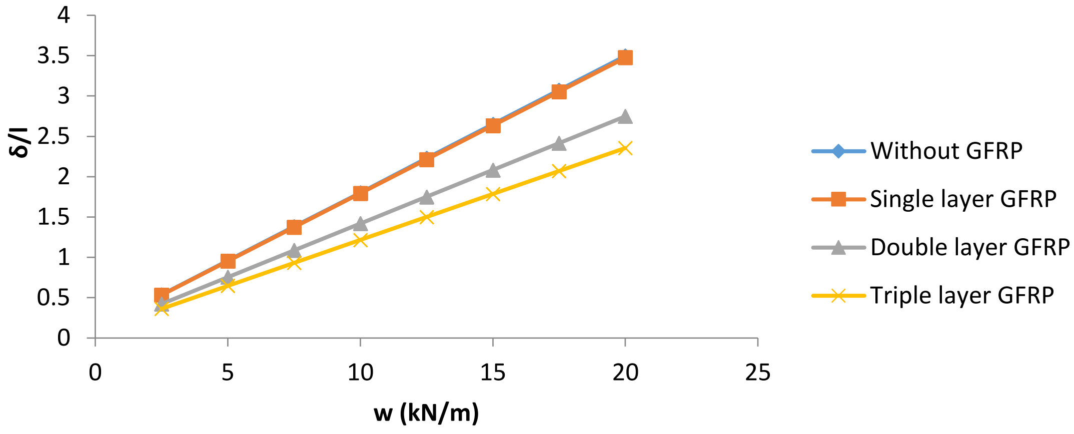



**Figure 3. 36** Variation of deflection w.r.t loading intensity

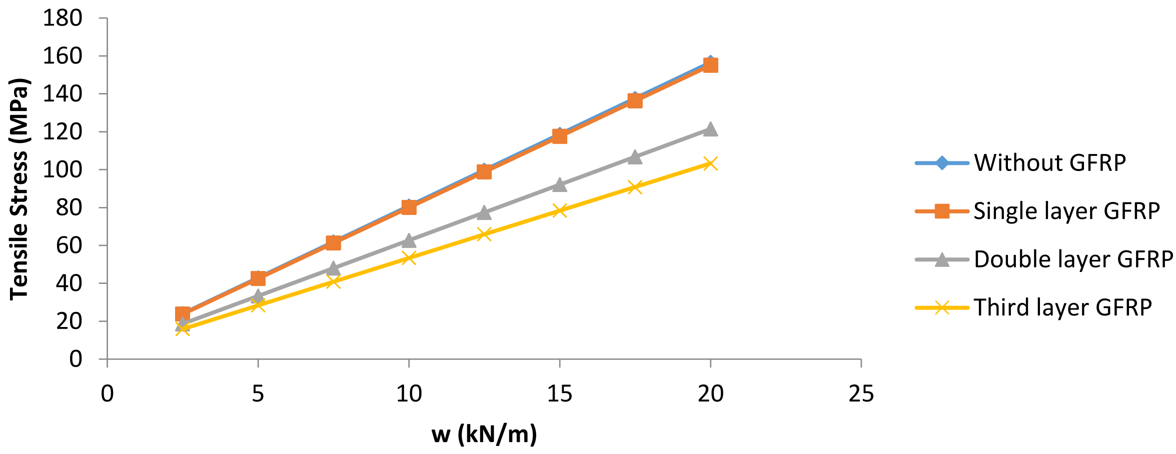



**Figure 3. 37** Variation of Tensile w.r.t loading intensity

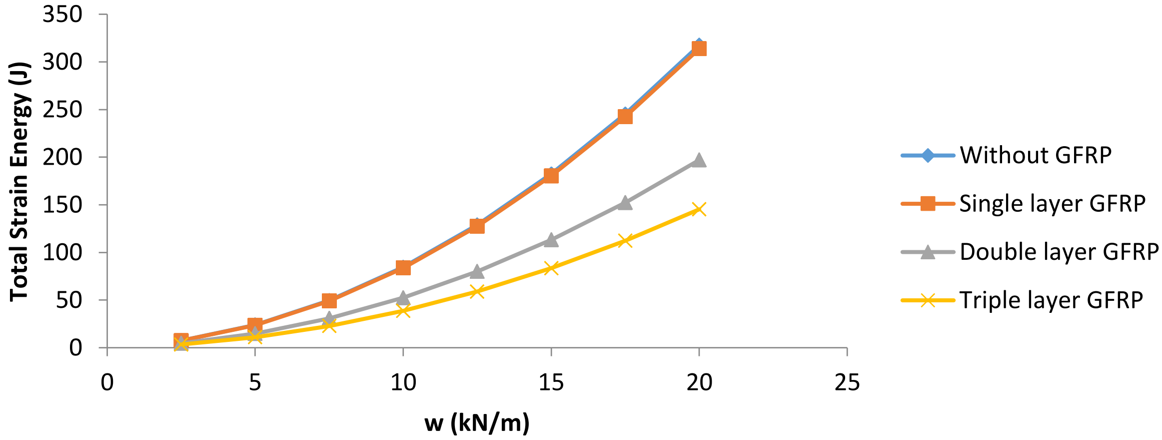



**Figure 3. 38** Variation of total strain energy with respect to loading intensity

### 3.4.8 Variation in frequency with respect to loading intensity for different modes

The frequencies don't appear to vary much with the loading intensity as can be seen in Fig.3.39, 3.40, 3.41, 3.42, and 3.43. For Mode 1and II, the frequency decreases slightly with an increase in the loading intensity. For Mode 3, this decrease is more subtle. For other modes, the frequency remains constant. There is no particular case which experiences the maximum or minimum frequency values across all modes but overall, it can be implied that frequency may increase slightly with number of layers of GFRP. Except for Mode 3, the frequencies of the extreme cases don't vary by more than 6% (Mode 4).

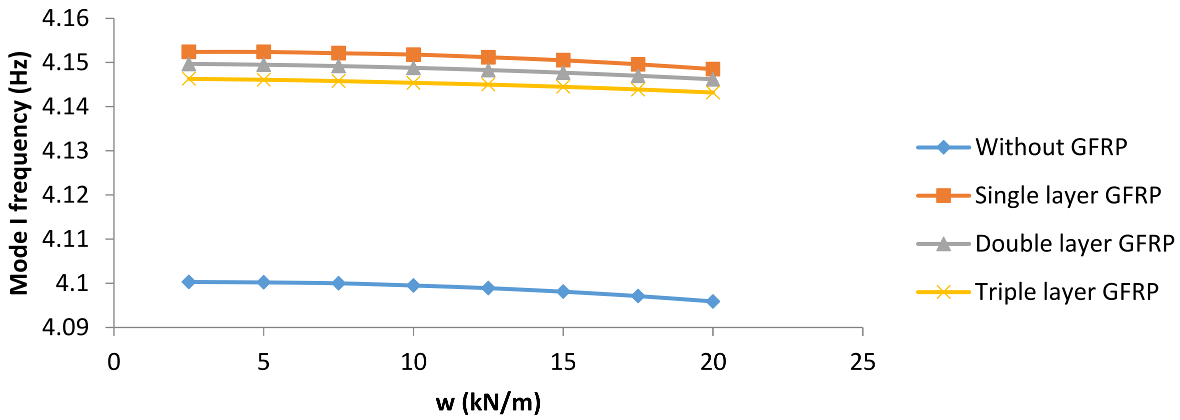



**Figure 3. 39** Variation of Mode 1 frequency w.r.t. loading intensity

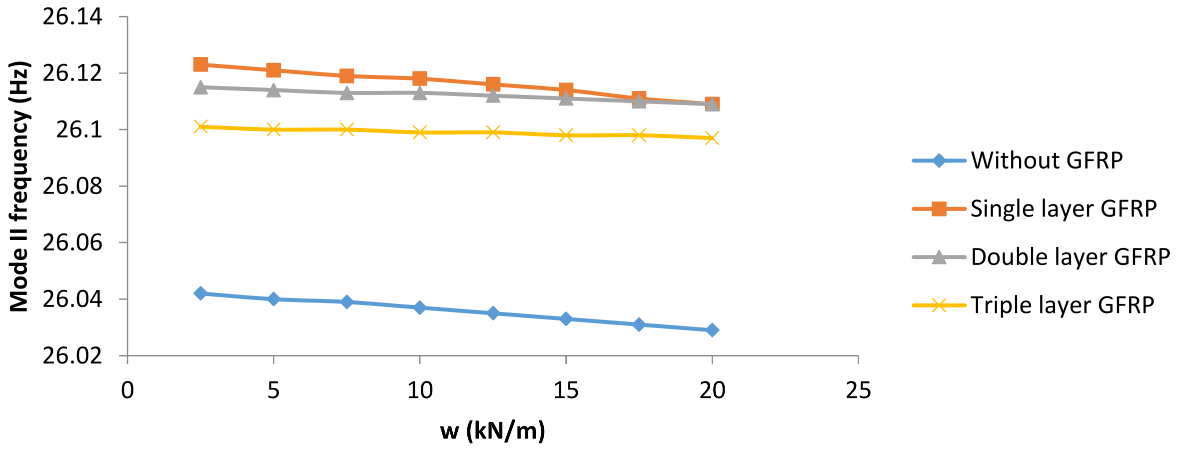



**Figure 3. 40** Variation of mode 2 frequency w.r.t. loading intensity

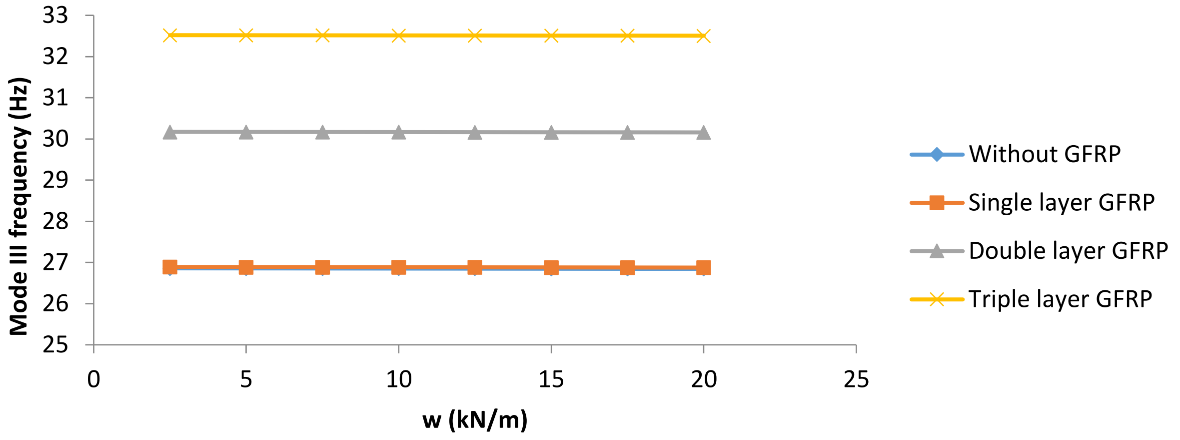



**Figure 3. 41** Variation of mode 3 frequency w.r.t. loading intensity

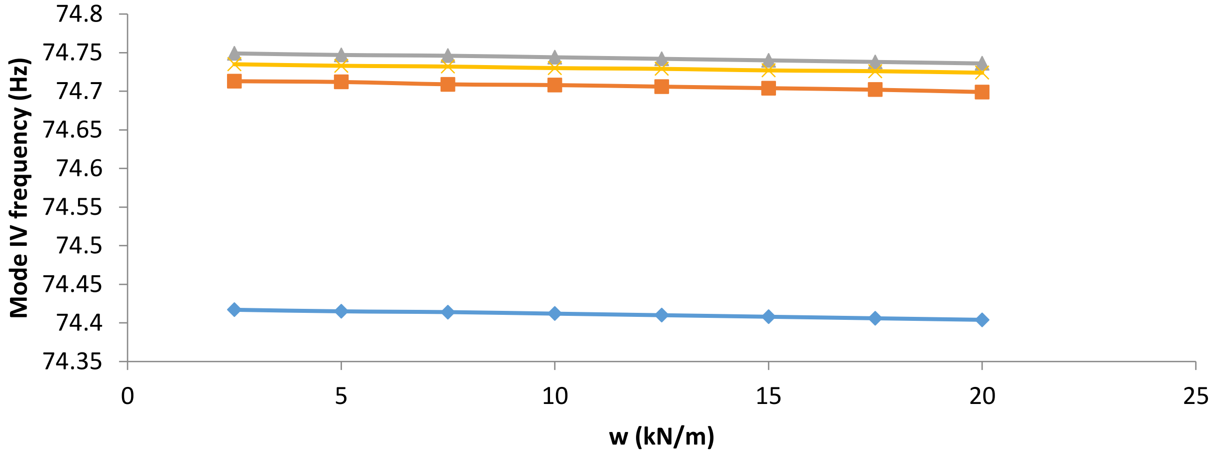



**Figure 3. 42** Variation of mode 4 frequency w.r.t. loading intensity

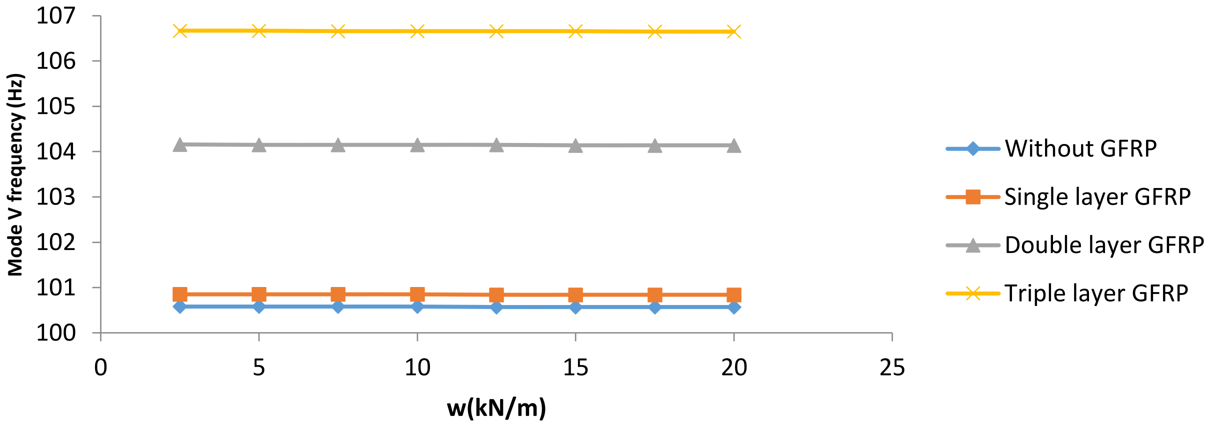



**Figure 3. 43** Variation of mode 5 frequency w.r.t. loading intensity

### 3.4.9 Variation of deflection with respect loading intensity for different modes

Similar to the variations observed in fig 3.44, 3.45, 3.46, 3.47, and 3.48, deflections due to natural vibrations don't appear to vary much with the loading intensity. For Mode 2 and 3, the deflections are constant. For Mode 1 the deflection curves have a gradual upward slope but the increase is not very significant. For Mode 4, the deflection for some cases increase while for some cases it decreases uniformly implying no particular trend. For Mode 5, the deflections are approximately equal and constant for all loading intensities.

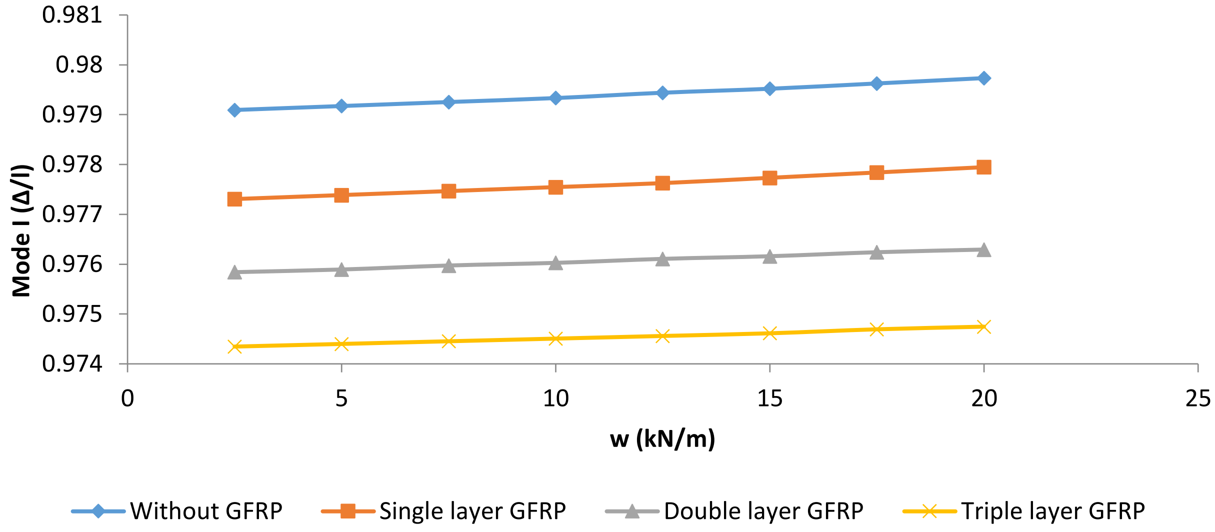



**Figure 3. 44** Variation of mode 1 deflection w.r.t. loading intensity

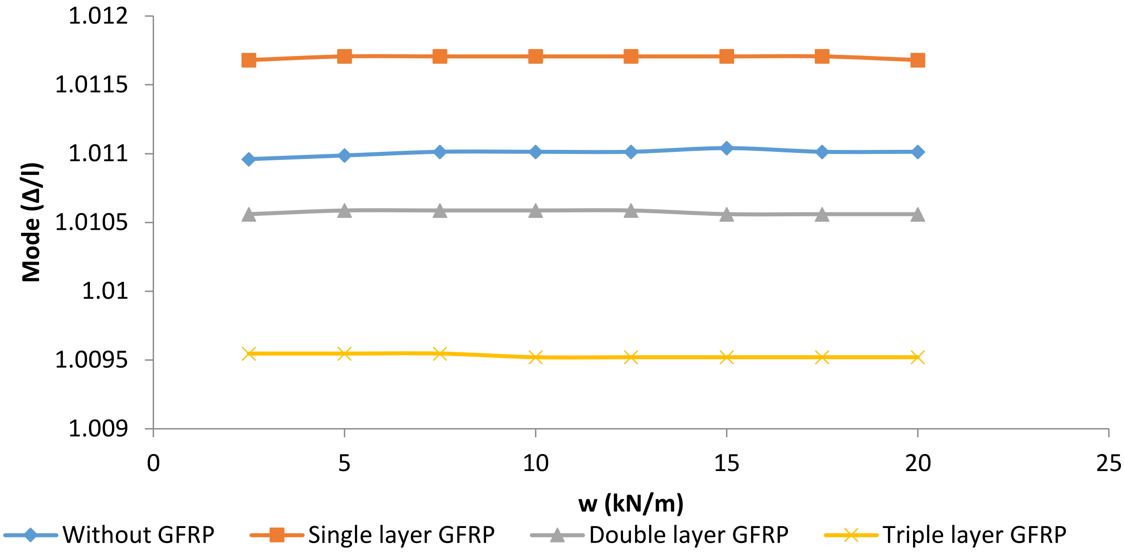



**Figure 3. 45** Variation of mode 2 deflection w.r.t. loading intensity

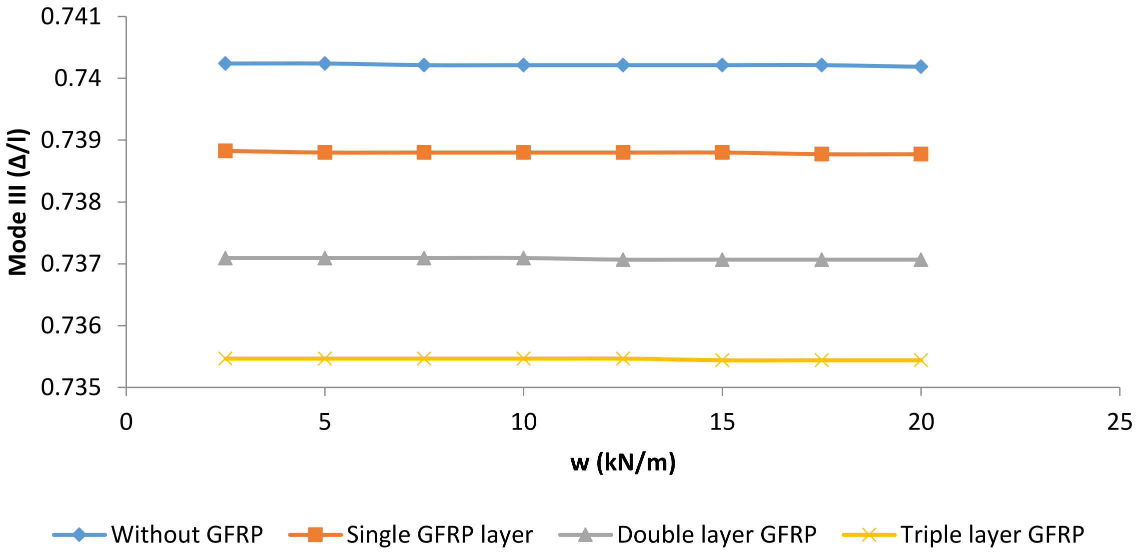



**Figure 3. 46** Variation of mode 3 deflection w.r.t. loading intensity

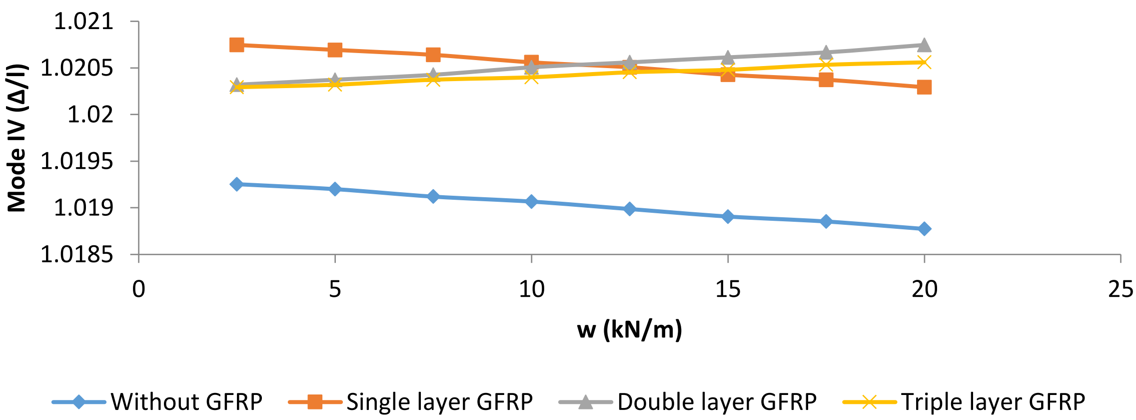


**Figure 3. 47** Variation of mode 4 deflection w.r.t. loading intensity

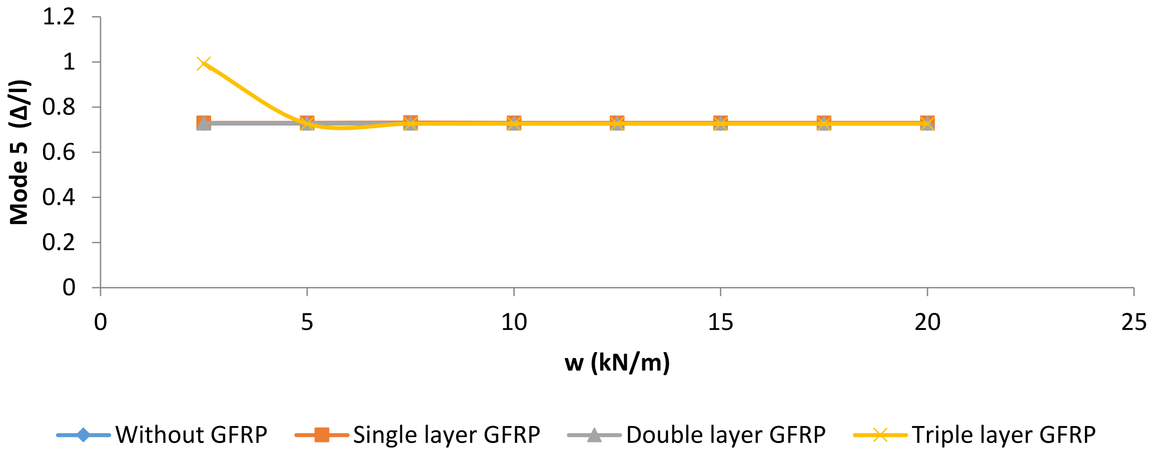



**Figure 3. 48** Variation of mode 5 deflection w.r.t. loading intensity

### 3.5 Analysis of prismatic girder with varying length

Five prismatic girder of span length 10,15,20,25 and 30 meter have been analysed using ANSYS. The parametric study have been carried out using different combination of fibre and varying length of girder. the width and depth (b/d) ratio is kept constant throughout the study as shown in Figure no 3.4 a and 3.4 b

### 3.6 IRC Loading of prismatic girder

The prismatic girder has been loaded symmetrically with 6 axles of 7.8t, 11.5t, 11.5t, 11.55t, 11.55t 7.9 t on the lane. The contact area of tires with the girder surface has been considered according to IRC 6: 2016, Class A loading as 250mm X 500mm for 11.5t and 11.55t axles, and 200mm X 380mm for 7.8t and 7.9t axles. Self-weight of the girder has also been considered by applying standard earth gravity in ANSYS environment. The pavement weight has been applied as a pressure load of intensity 1875 Pa (considering unit weight of the pavement as 25kNm$^{-3}$), distributed on the top surface of the girder.

### 3.6.1 Maximum deformation of prismatic girder

The deformation due to live load has been displayed after finite element analysis of girder for different IRC loads. Few apparent changes have observed when the layers of composite fibre were increased. Figure 3.49 it is clearly shows that for the longest span of bridge i.e. 30 m, the deformation was vastly reduced when a layer of composite fibres CFRP and GFRP were tripled. In the case 1 when the length of the girder was 30 meter and the single layer of composite fibre were glued then the deformation in the girder was noticed 21.27 mm while this layer increased by one more layer then the deformation was found 12.12 mm and in the case of triple layer it decreased and reached to 8.03 mm. In the case 2 when the length of girder was 25 meter, the deformation in the single layer fibre 11.29 mm, in the double layered fibre it was 9.23 mm and in the triple layered fibre it came to 7.20 mm.

In the case 3 when the length of the girder was 20 meter, then in the single layer composite fibre the deformation was 5.49 mm, as soon as the layer of fibres were doubled the deformation was decreased and reached up to 4.47 mm and in the next case when the layers of fibres were tripled the deformation was reduced up to 3.46 mm.

The girder was again modeled without glass fibres (Figure 3.50) with the increasing number of layer of carbon fibres for all the three girder of 20mm, 25 mm and 30mm. The span of 10m, 15, and 20 m also does not impact the results after increasing the layer of fibre. In the case of 20 meter the deformation of the girder when there was no fibre, was found 5.49 mm. with the single layer of carbon fibre it was 5.04 mm, as doubled the layers for this girder the deformation came to 4.97 mm and in the three-time CFRP wrapped girder the deformation was found 4.16 mm. for the second girder whose length was 25 meter the deformation was 11.30 in no fibre strengthening case. The deformation was reduced and reached to 9.30 mm when single layer CFRP was applied similarly it further sink to 8.23 mm and 8.20 mm for two and three layer lamination of girder. The third girder with span length 30 meter was examined wrapped similarly by one, two and three layer of carbon fibre and the deformation was noticed 20.28 mm, 17.12 mm and 15.04 mm respectively.

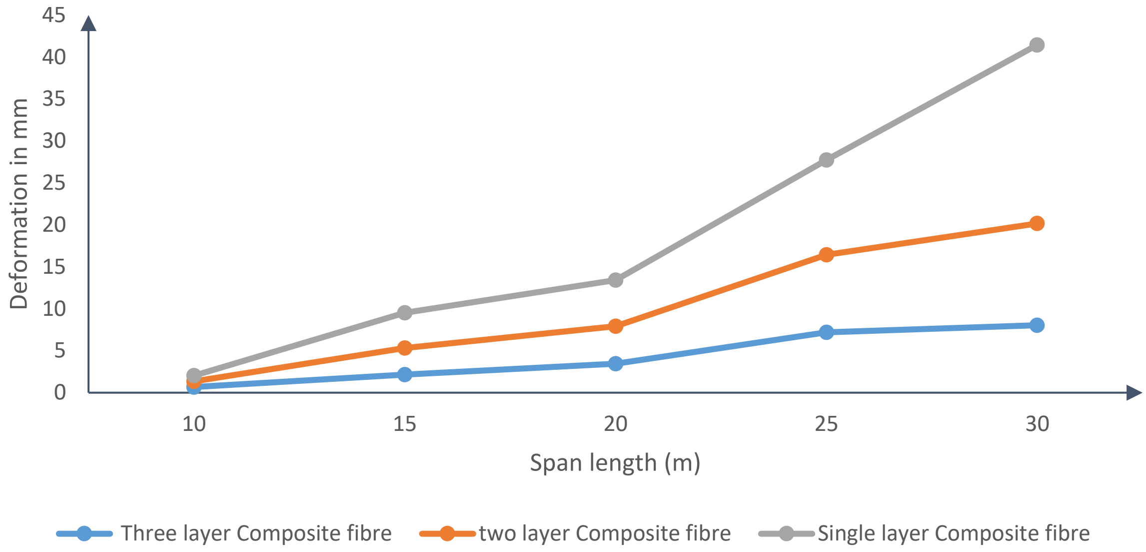



**Figure 3. 49** Maximum deformation vs span length in composite laminate girder

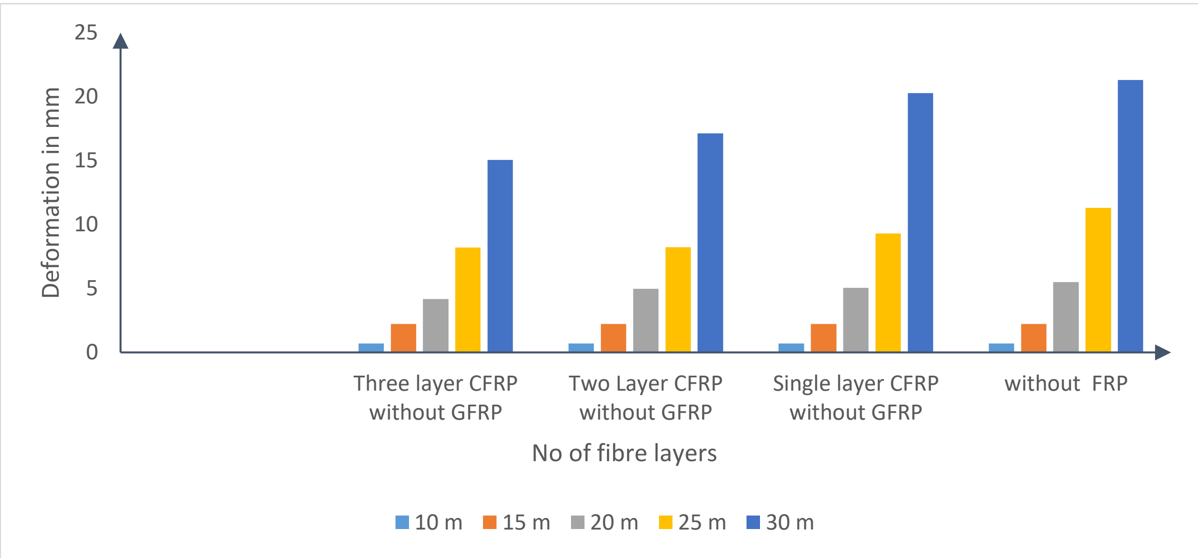



**Figure 3. 50** Maximum deformation vs no of layer of carbon laminate girder

### 3.6.2 Maximum Von Mises strain of prismatic girder

Figure 3.51 and Figure 3.52 confirmed that every laminated bridge girder experiences the yielding before going in failure mode as their Von Mises strain is greater than their simple deformation in each case. In Figure 3.52 the single, double and triple layer composite fibre stratum on all the three girder of sample. In the single layer and longest girder the strain was found 1.23 mm while it got reduced by 49 percent in the case of triple layer composite fibre on the same length of girder. The Von misses strain on the single layer laminated girder of 25 meter and 20 meter length is 0.53 mm and 0.76 mm. when the composite fibre layer tripled the strain found as 0.4 mm, 0.55 mm and 0.6 mm for 30m, 25m, and 20m girder.

Figure 3.53 consist the von mises strain of girders which was laminated only using the carbon fibre only. The single layered girder of 30 meter length showed von misses strain of 1.03 mm while other two girder of comparatively smaller length are showing 0.6 mm and 0.47 mm for 25 meter and 20 meter respectively. The similar trend observed in the double and triple layered girder of all length.

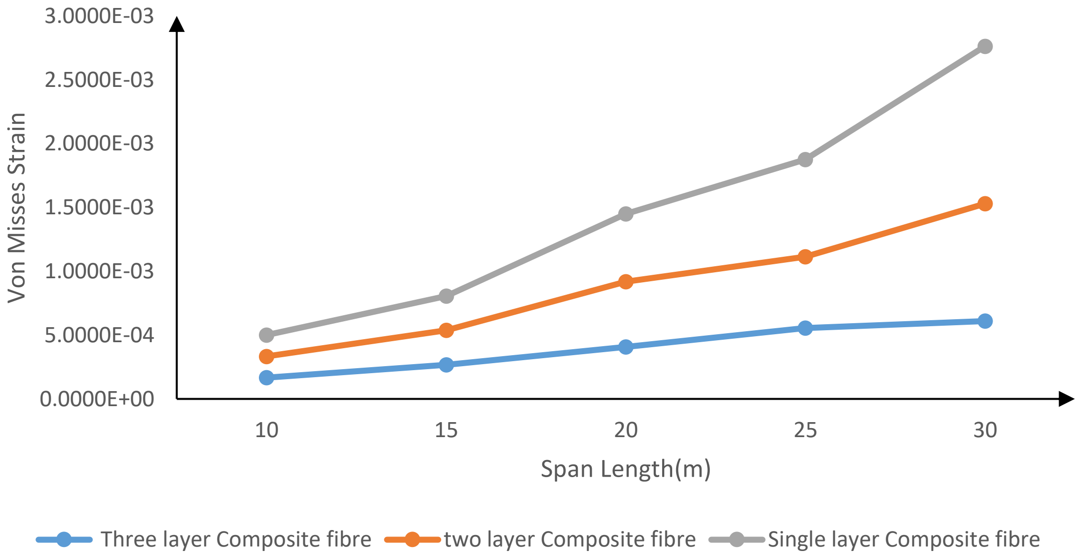



**Figure 3. 51** Maximum Von Mises strain vs span length in composite laminate girder

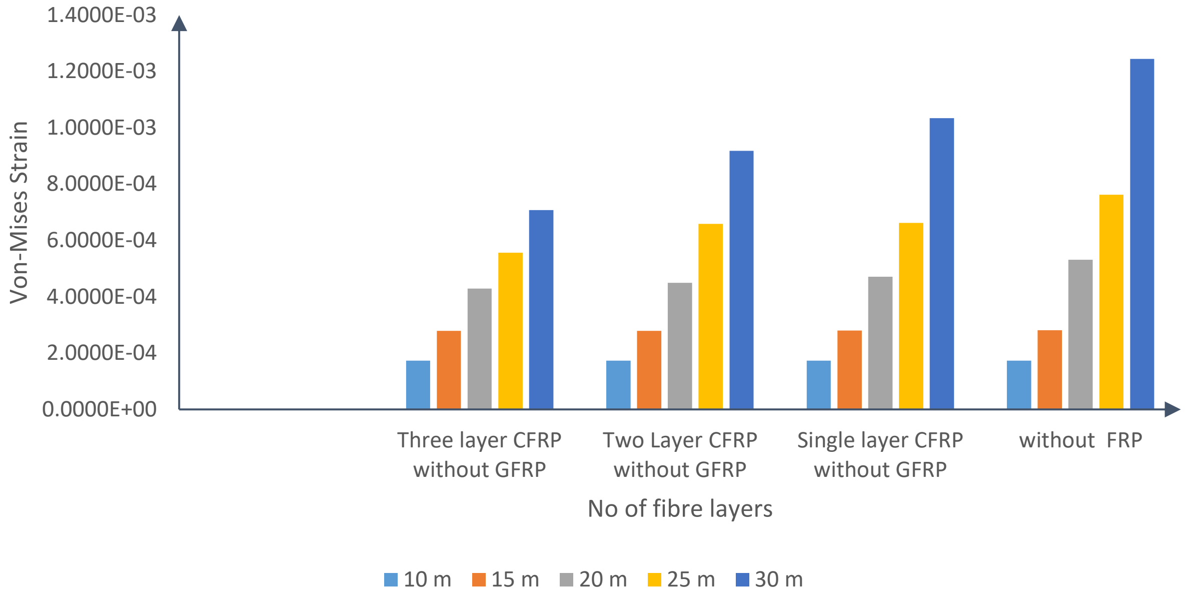



**Figure 3. 52** Maximum Von Mises strain vs no of layer of carbon laminate girder

### 3.6.3 Maximum compressive Stress of prismatic girder

When all the five girder were retrofitted by both carbon and glass fibre the maximum compressive stress was found 6.03 MPa for 30 meter span in single layer fibre and reduced this by approximately 40 percent when the layers were tripled in the same beam. For the girder of 25 meter length it was 4.62 MPa for single layer and reduced to 3.09 when increased the layer of girder to three. For the 20 meter span and single layer lamination the stress was 3.40 MPa and decreased to 3.18 MPa when it was wrapped by triple layer. In the other two beam of 5m and 10 m, the reduction was prolonged so it should be avoided for economical purpose. (Figure 3.53)

In the case of carbon laminated girder of length 30 meters, the compressive stress was found 6.02 MPa for single layer and reduced it to 4.96 MPa for triple layer lamination. When the girder became slightly smaller of length 25 m, the compressive stress for a single layer was found 4.62 MPa and reduced to 4.59 MPa for three layer laminated girders. For the 20 meter long girder, the compressive stress was 3.00 MPa in single layer laminated and 2.38 for triple layer lamination. (Figure 3.54)

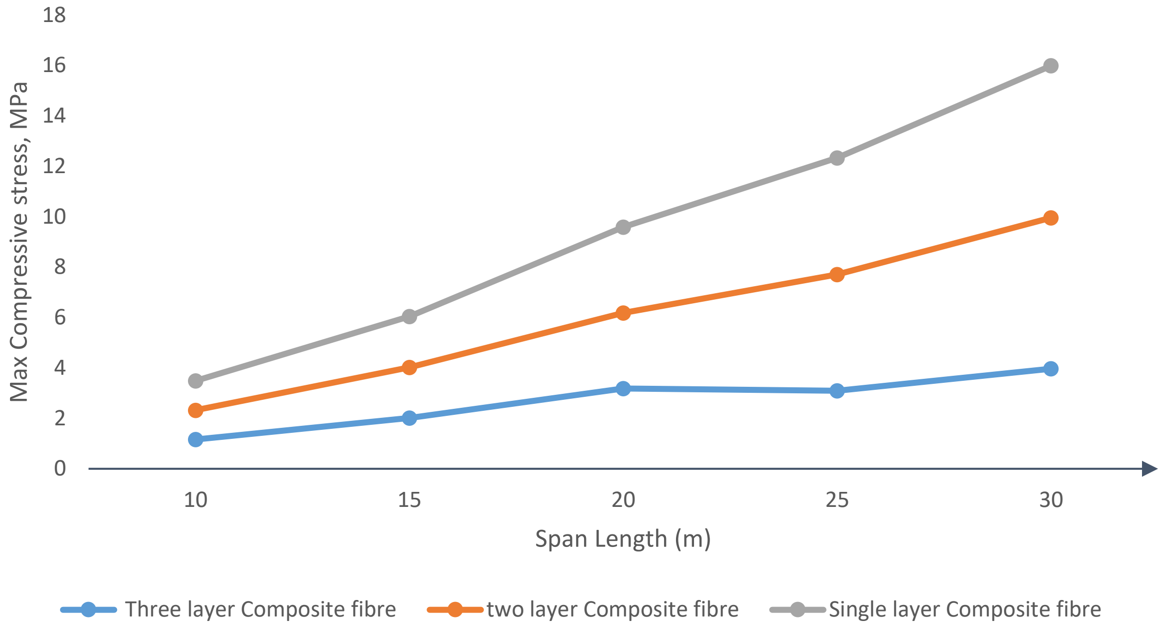



**Figure 3. 53** Maximum compressive stress vs span length in composite laminate girder

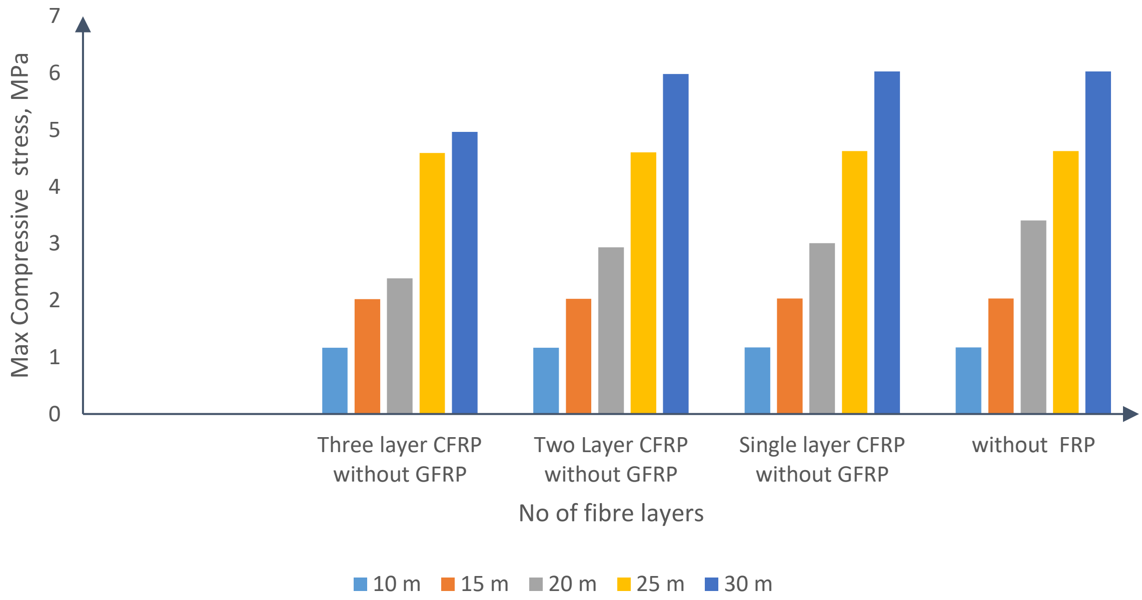



**Figure 3. 54** Maximum compressive stress vs. no of the layer of carbon laminate girder

### 3.6.4 The maximum tensile stress of prismatic girder

It is well known that the concrete is weak in tension especially simple reinforced girder of all the structures. So it should ensure that if the load is traveling in the tension zone due to any reason, then the tension zone's strengthening is constitutive. Figure 3.55 assured that the effect of multiple layer FRP increased the strength of concrete considerably in tension zone. The maximum tensile stress in singly, doubly, and triple layer wrapped girder of three different length is being analysed in this figure. The first case of the girder length 30 meter was singly wrapped with stress of 9.55 MPa, 7.49 MPa for the 25 meter long girder and 5.24 MPa for 20 meter girder. As the strengthening of all girder were completed and tested again the maximum reduction was observed in the longest girder about 85 percent. In double-coated lamination, the stress in the longest girder of 30 meter length it is 8.47 MPa, in 25 meter long girder it is 7.36 MPa, and in the shortest girder it is 4.86 MPa. In the three-layered laminated girder case the maximum tensile stress is 5.43 MPa, In 25 m span, it is 4.34 MPa, and in the 20-meter span it is 4.70 MPa

Figure 3.56 proved that without glass fibres there will be no such increment in tensile strength of concrete. The reduction in the tensile stress in all the girder was not as found in composite fibres.

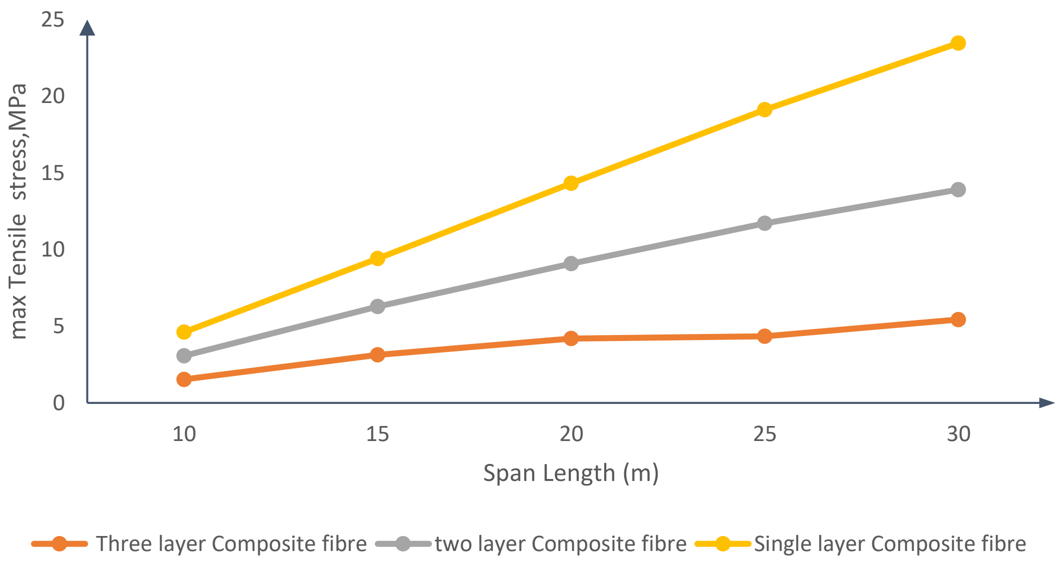



**Figure 3. 55** Maximum tensile stress vs span length in composite laminate girder

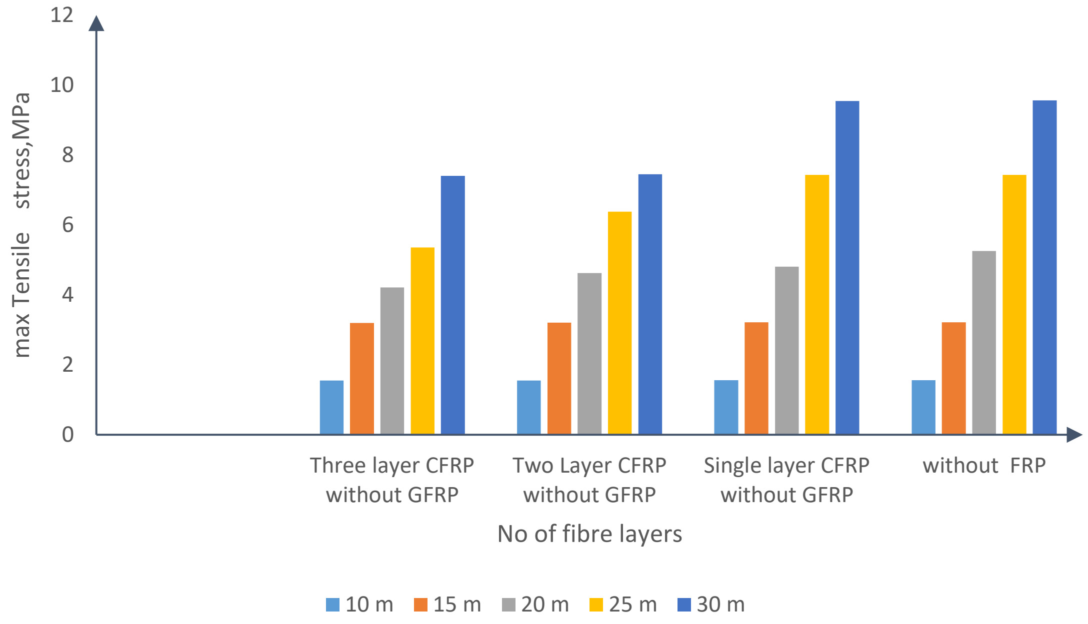



**Figure 3. 56** Maximum tensile stress vs no of layer of carbon laminate girder

### 3.6.5 Maximum Von Mises stress of prismatic girder

Figure 3.57 the von Mises stress of any structure describes its ability to yield before discomfiture. The Figure 3.57 consist the behaviour of composite laminated girders. The stress in single layered laminated girder the von Mises stress was found 36.68 MPa for the longest span which was 30m. Respectively for the double and triple layer it was 25.27 Mpa and 15.99 MPa. For shorter span length i.e. 25m and 20m the von Mises stress was reported 18.39 MPa and 12.82 MPa when girder was double coated by composite fibre and 16.33 MPa and 11.82 MPa for triple coated composite layer.

In the Case where glass fibre were not attached with girder and only strengthened by one, two and three layers of carbon fibre is being discussed in Figure 3.58. In the case of triple layered wrapping of fibre the von mises stress on the girder length 30 m, 25 m and 20 m was 25.92 MPa, 17.36 MPa and 10.83 MPa, respectively. If the strengthening was done using double layered carbon fibre then the stress was observed 33.24 MPa, 21.42 MPa and 11.86 MPa for descending length of last three girders. When the layers of carbon downed to a single layer, the stress in the longest girder was 34.7 MPa while in the girders of 25 meter and 20 meter, the stress was 21.50 MPa and 13.9 respectively.

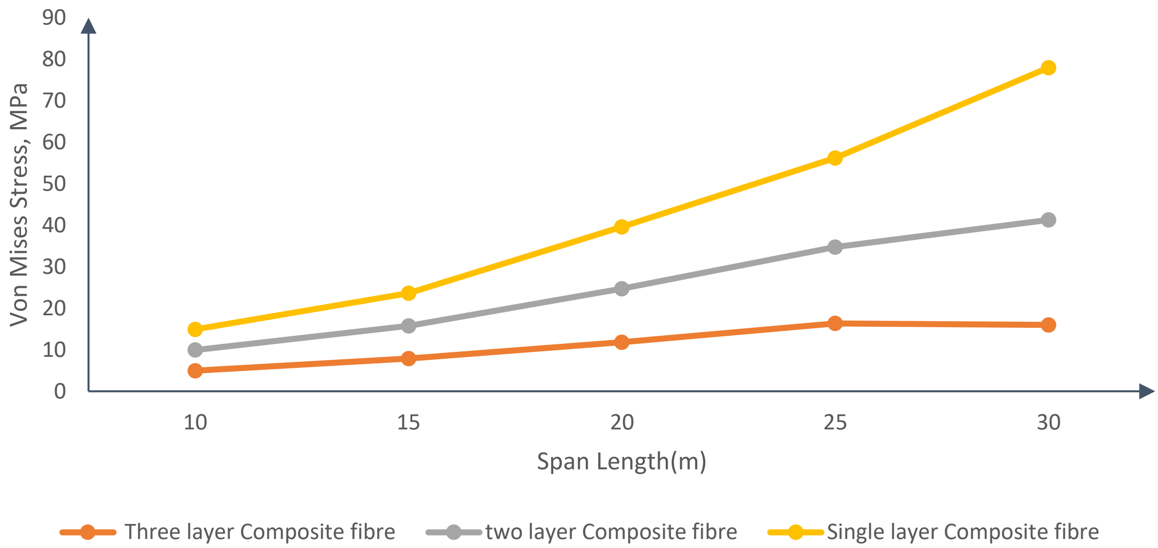



**Figure 3. 57** Maximum Von-Mises stress vs span length in composite laminate girder

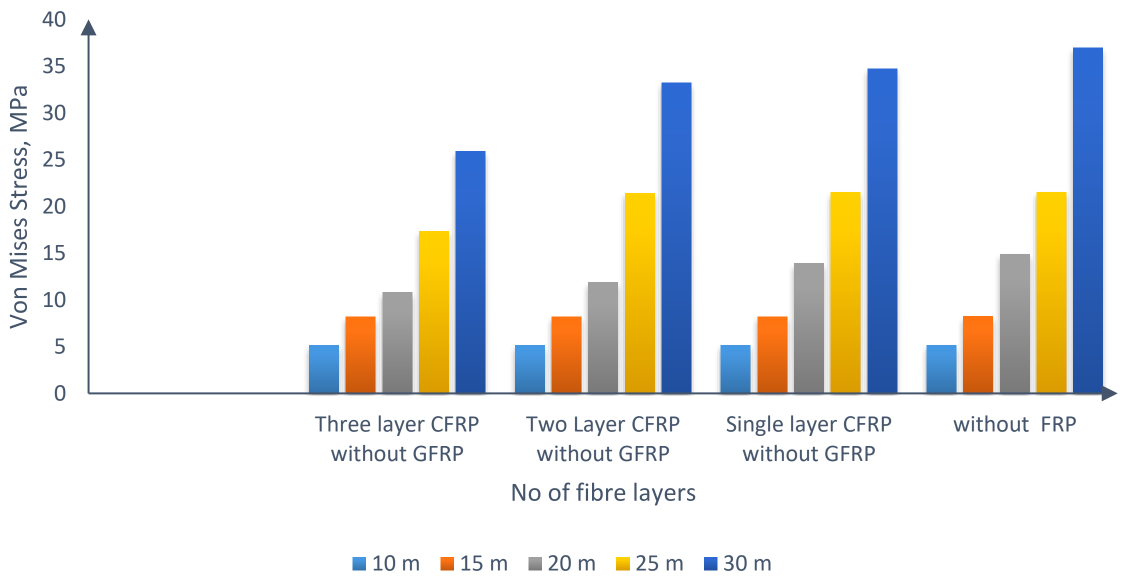



**Figure 3. 58** Maximum Von-Mises stress vs. no of layer of in carbon laminate girder

### 3.6.6 The strain energy of prismatic girder

Figure 3.59 explains the effect of larger variation of fibre on the different sizes of girder. The single-layer composite fibre produces the 26514 Joule strain energy for 30-meter girder. After doubling the composite layer strain energy comes to 21166 Joule and in the case of triple layer fibre the strain energy was found 10981 Joule. In the 25m and 20 meter long spans the strain energy reported 10255 joule and 4190.7 joules when doubling carbon and glass fibre wrapping. Figure 3.60 explains the case where only carbon fibre is used for retrofitting the girder. In this analysis the strain energy in the unstrengthened beam was found 26560 Joule for the longest girder and decreasing gingerly when the layer of fibres was single, doubled and tripled. In the case of single-layer carbon wrapping the strain energy in 30 meter girder was found 25530 joule and in other two girders it was 12382 joule and 4560 joule for 25 meter and 20 meter respectively. If we see the difference in terms of length of girder, then the strain energies' difference is more than three time of each other. A similar pattern was noticed in the double and tripled layer laminated girder of all three sizes.

Similarly, in the shorter span, the reduction in strain energy was not as much as seen in the composite retrofitted girder. In the 25 meter un-retrofitted girder the strain energy was found 5164.2 Joule and decreasing very minutely when the number of layers was increased to double and triple. A similar pattern was continued in the 20m long girder.

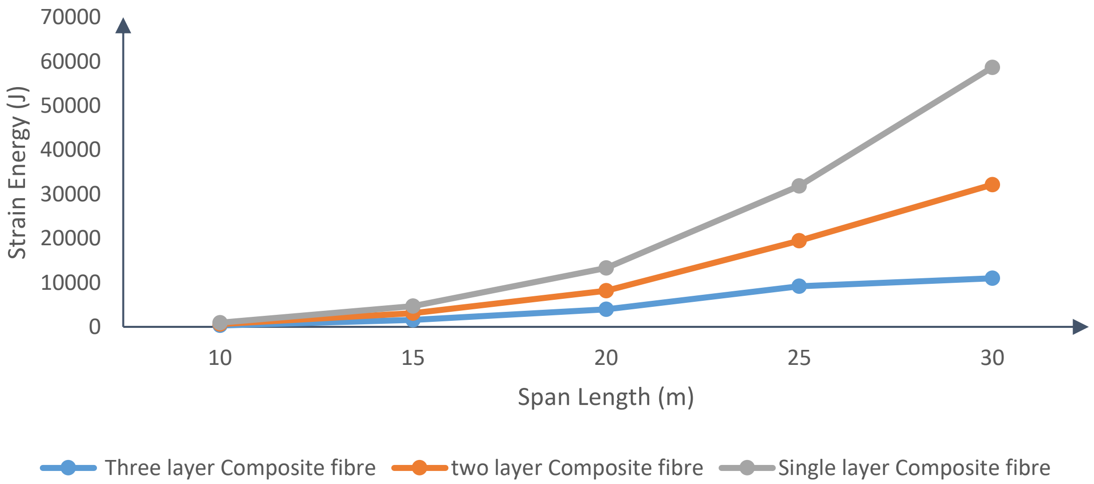



**Figure 3. 59** Strain energy vs span length in composite laminate girder

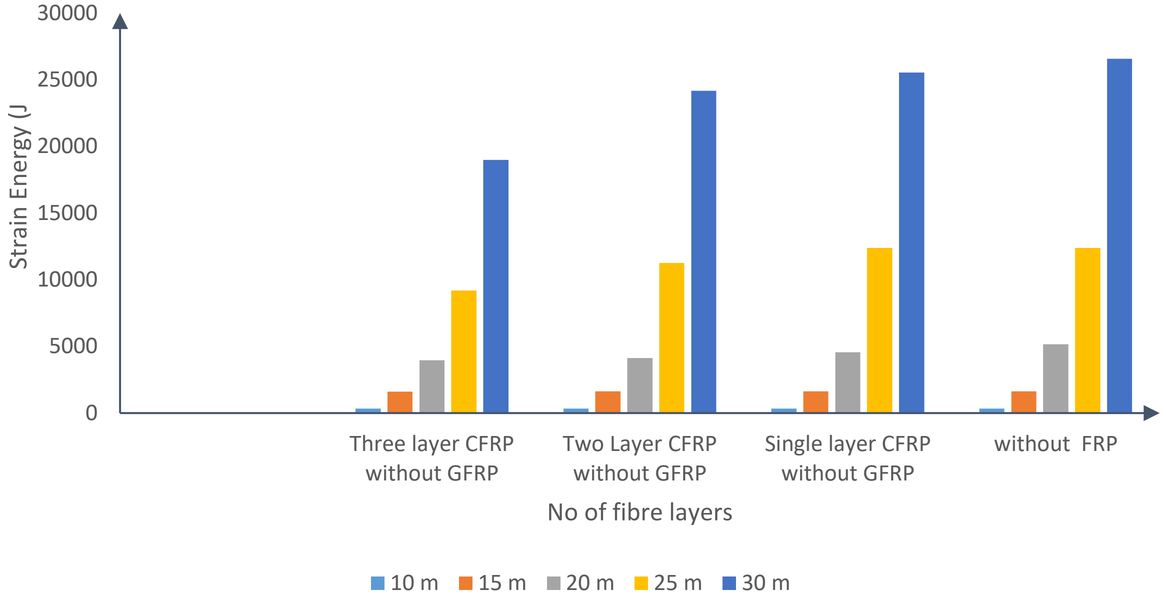



**Figure 3. 60** Strain energy vs. no of the layer of carbon laminate girder

### 3.6.7 Frequency of prismatic girder with laminated carbon fibre for different modes

The frequency of girder for the first five modes have been recorded and represented below:

i. **Frequency of prismatic girder with laminated carbon fibre for mode 1**

To find out the natural frequency of girder 5 mode shape have analysed use and for each mode, the cases of composite fibres and without glass fibre have been discussed. In Figure 3.61 for mode 1 the frequency of single layer composite fibre of largest girder i.e. 30 meter was found 2.50 Hz and of 25m girder it is found 5.72 Hz and for 20 m girder it was found 10.139 Hz.

To find out the vibration (mode shape, natural frequency), the modal analysis of any structure is very useful. It can represent any structure's motion under dynamic loading because of lateral force originated by electrostatic impellers. In Figure 3.61 and Figure 3.62 the frequency of different lengths of girder was examined in three cases without any fibre, with composite fibre and with carbon fibre. The result are compared with each span length and also with the different layers of fibres. The frequency for longest girder with three layer laminated fibre is noted as 5.53 Hz while for 25 m and 20m it is 7.81 Hz and 12.48 Hz in triple layer coating of fibre. Figure 3. 63 Consists the frequency of girder of different length and wrapped with carbon fibre only. The frequency of the un-laminated largest girder was found 4.5 Hz while for 25m and 20m girders it was 6.72 Hz and 11.14 Hz respectively. With the triple layer of carbon fibre in the longest girder of span 30m the frequency was found 5.53 Hz and for 25m and 20 meter girders the frequency was reported as 7.81 Hz and 13.48 Hz.

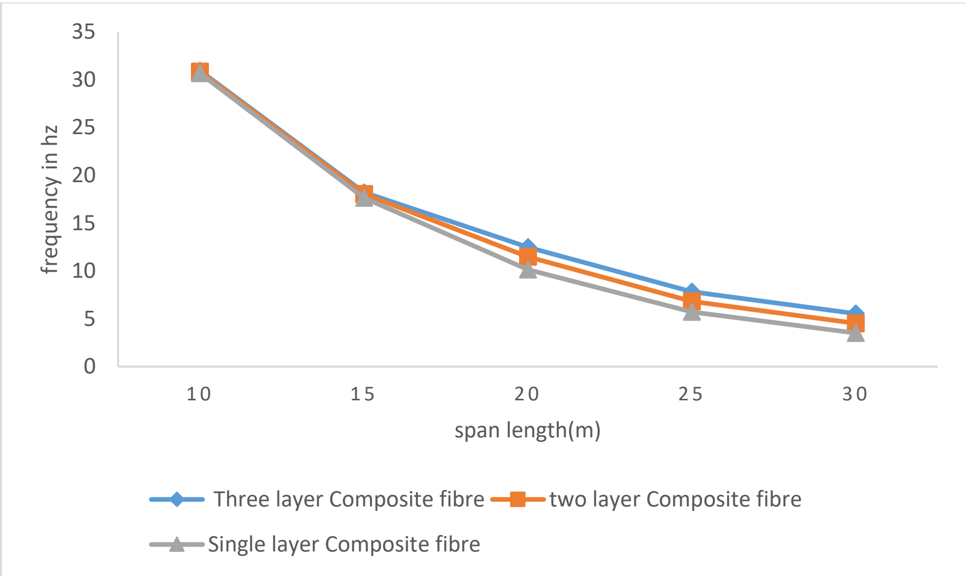



**Figure 3. 61** Frequency in mode 1 vs span length in composite laminate girder

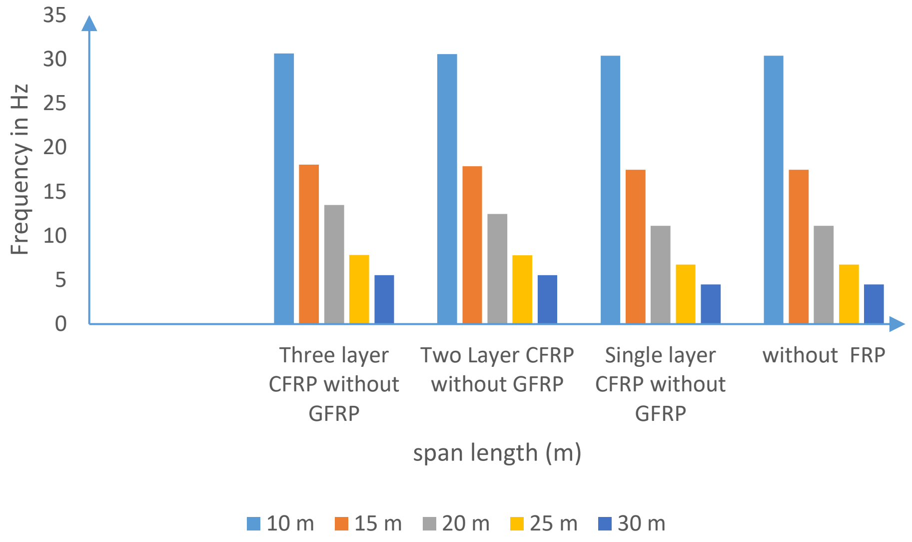



**Figure 3. 62** Frequency of girder in mode 1 with carbon laminated fibre

ii. **Frequency of prismatic girder with carbon laminated fibre for mode 2**

Figure 3. 63 describes the vibrations of girder in mode 2. for single layer composite laminated girder of length 30m the frequency was found 5.69 Hz, In girder length 25m and 20 meter it was found as 6.78 Hz and 11.44 Hz. Similar pattern was observed in double and triple layer of composite fibre laminated girder. For triple layer the frequency of 30 metre long girder increased by 10% and no such improvement was observed in the girder below 20 m span. When the girder was laminated only using carbon fibre the frequency in mode has been displayed in Figure 3.64. In this figure the frequency of triple layer carbon fibre laminated girder of length 30 meter was found 7.07 HZ. While other girders of length 25 meter and 20 meter was noted 8.81 Hz and 11.43 Hz. the frequency in doubly laminated girder of 30 Meter, 25 meter and 20 meter was relieved as 6.71 Hz, 8.49 Hz and 11.01 Hz. when the strengthening of girder reduces to the single layer carbon fibre then the frequency in mode 2 was 10.83 Hz in 30 meter long girder, 7.78 Hz in 25 meter long girder and 10.94 Hz in 20 meter sample girder. In the case where all girders are un-laminated the frequency was found as 5.69 Hz for 30m length, 7.78 Hz for 25 meter and 9.46 Hz for 20 meter girder.

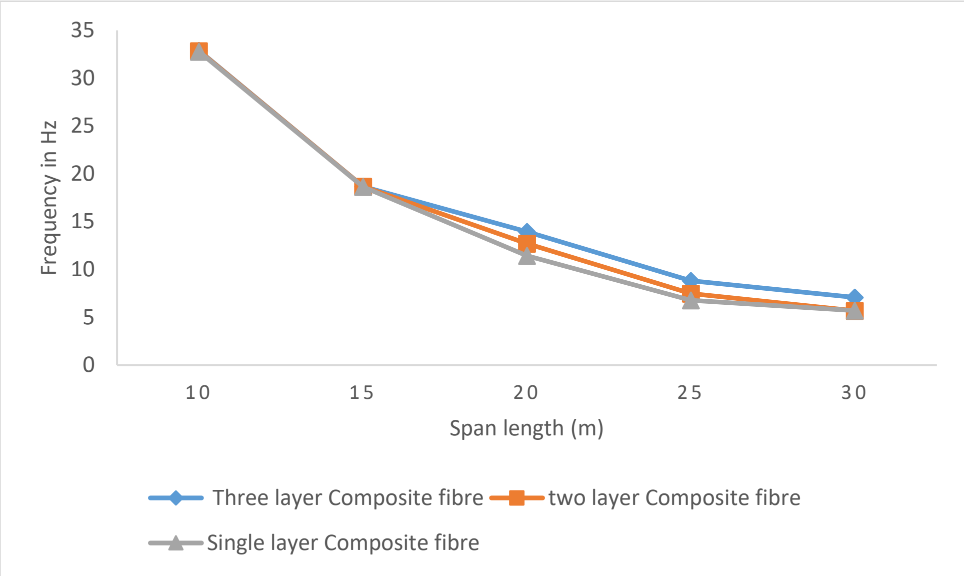



**Figure 3. 63** Frequency in mode 2 vs span length in composite laminate girder

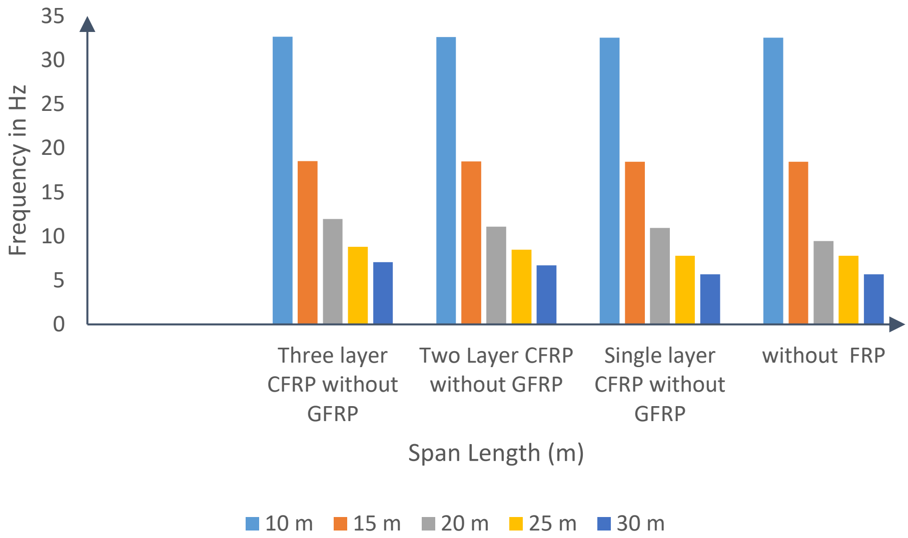



**Figure 3. 64** Frequency of girder in mode 2 with carbon laminated fibre

iii. **Frequency of prismatic girder with carbon laminated fibre for mode 3**

Figure 3.65 and 4.66, describes the frequency of all the three different length girder for mode shape 3 which was retrofitted in two different types of fibre combination i.e. first with composite fibre of carbon and glass, and second only strengthen by Carbon fibres. In Figure 3.66 it can be seen that the Single layer Composite fibre when applied in longest span then the frequency obtained Comes to 10.47 Hz and when it was applied to 25 meter and 20 meter long girder the frequency was reported 14.89 Hz and 23.29 Hz. It can also concluded that the frequency difference in the shortest and longest sample girder was just more than double. The frequency for double and triple layer composite fibre of span 30 meter was 10.52 Hz and 12.54 Hz and for 25 meter and 20 meter span it was confirmed to 14.95 Hz, 25.39 Hz and 16.98 Hz, 26.43 Hz respectively.

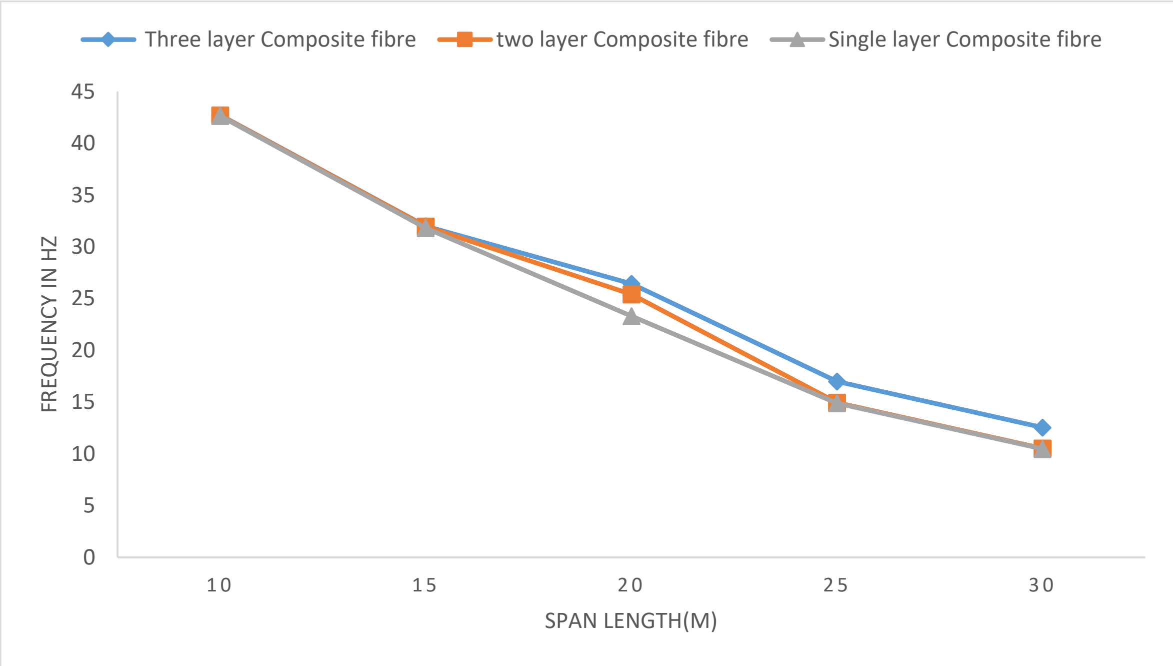



**Figure 3. 65** Frequency in mode 3 vs span length in composite laminate girder

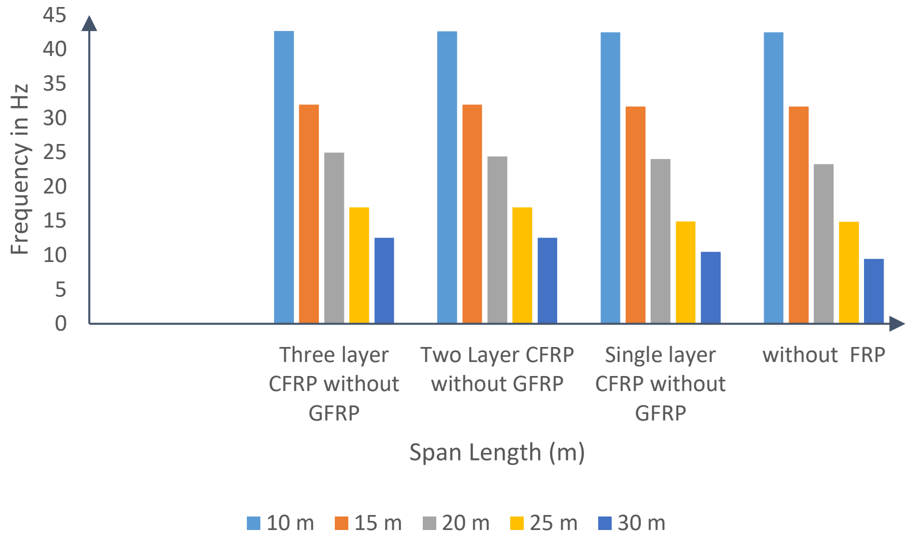



**Figure 3. 66** Frequency in mode 3 vs. span length in carbon laminate girder

### iv. Frequency of prismatic girder with carbon laminated fibre for mode 4

Analysis of modal frequency depicts about the dynamic properties of tested structure. The primary objective of this analysis was to find out the mode shape and frequency of girders. Figure 3.67 and Figure 3.68 Contains frequency in mode 4 with and without glass fibre while carbon fibre was always in flexure. In Figure 3.67 the single layer Composite fibre wrapped girder showed more than 30 percent of frequency in shortest girder than the longest girder of case. This trend was continued in double and triple layered girder too. In Figure 3.68 the frequency of single layered carbon strengthened girder of span length 30 meter was noted as 12.22 Hz while girder of length 25 meter and 20 meter was found a 14.38 Hz and 24.89 Hz respectively. When the Carbon fibre layered was increased in all the girder the frequency obtained in 30 meter, 25 meter and 20m girder was 14.28 Hz, 18.42 Hz, and 25.13 Hz respectively. Similarly the frequency in all the three carbon swaddled girders in descending order was found at 14.31 Hz, 18.44 Hz and 25.95 Hz.

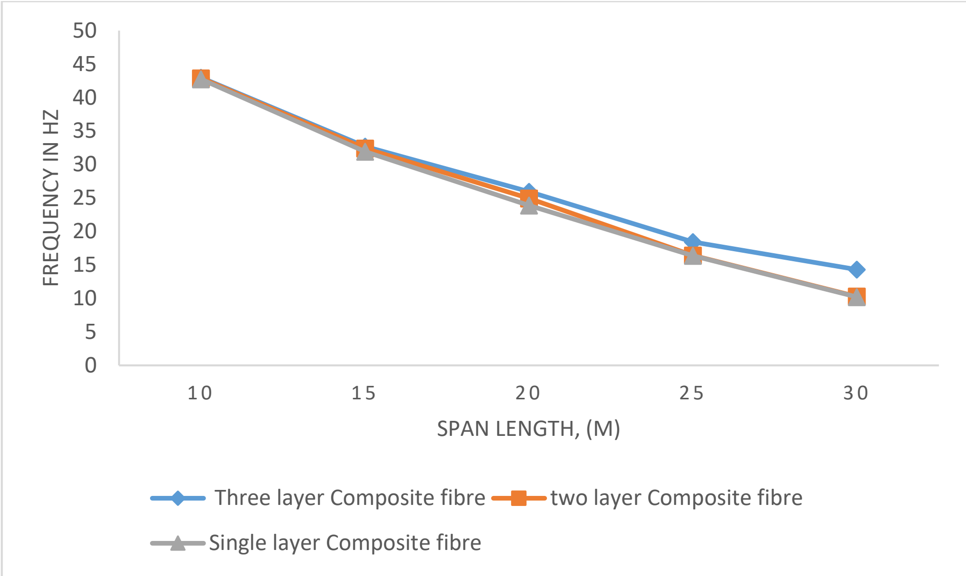



**Figure 3. 67** Frequency in mode 4 vs span length in composite laminate girder

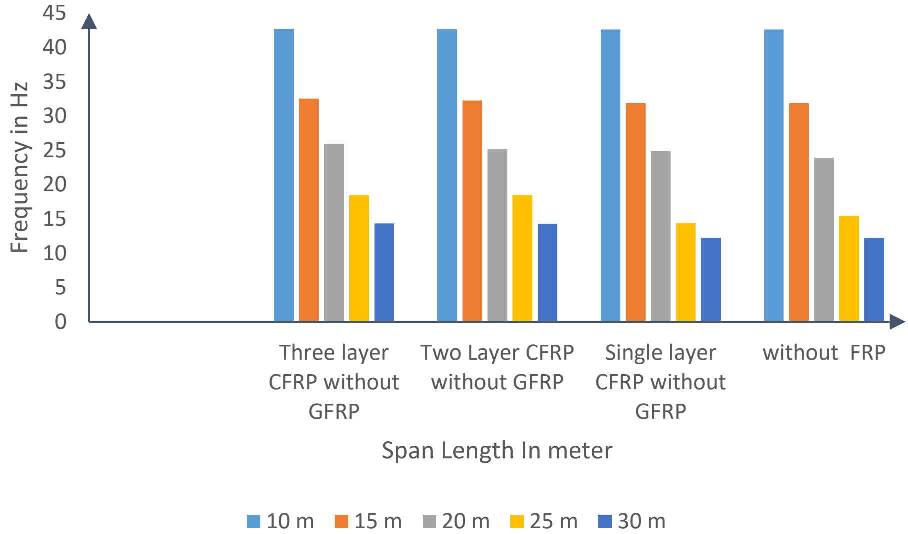



**Figure 3. 68** Frequency in mode 4 vs span length in carbon laminate girder

v. **Frequency of prismatic girder with carbon laminated fibre for mode 5**

Figure 3.69 and Figure 3.70 represents the frequency of laminated girder in mode shape 5. In Figure 3.70 the single layer composite fibre on the girder of length 30 meter showed the frequency of 12.418 Hz and the girder of 25m and 20m showed the 17.16 Hz and 24.97 Hz respectively. The double layered composite fibre of the longest girder showed the frequency of

12.91 Hz and other two girder in descending order showed 17.63 and 25.83 Hz respectively. Similar stance was observed in triple Layered girders of all the three spans. In Figure 3.70 the mode shape 5 and frequency for three different spay and three type of Carbon fibre arrangement were compared. In the first combination the single layer carbon fibre on the girder of three different span i.e. 30 meter 25 meter and 20 meter was applied and frequency was measured for mode shape 5. The frequency for largest and second largest girder is 12.22 Hz and 14.38 Hz. The frequencies in the 20 meter girder after lamination of single layer is 24.89 Hz. For the double layer the frequency in the 30 meter girder are noted 15.92 Hz and the girder with 25 meter span and 20 meter span reported their frequency 18.42 Hz and 25.13 Hz respectively. Similar Easel was noticed for next added layer of FRP. For three layer FRP retrofitted girder the frequency of 30 meter girder was 16.316 Hz, for 25 meter it was 20.82 Hz and for 20 meter long girder's frequency was 26.26 Hz.

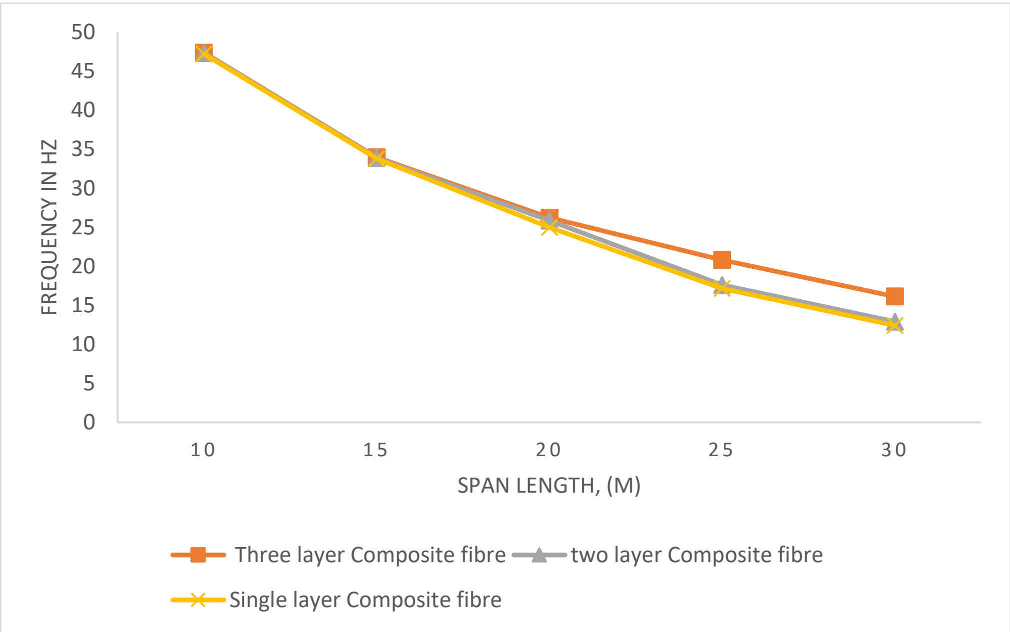



**Figure 3. 69** Frequency in mode 5 vs span length in composite laminate girder

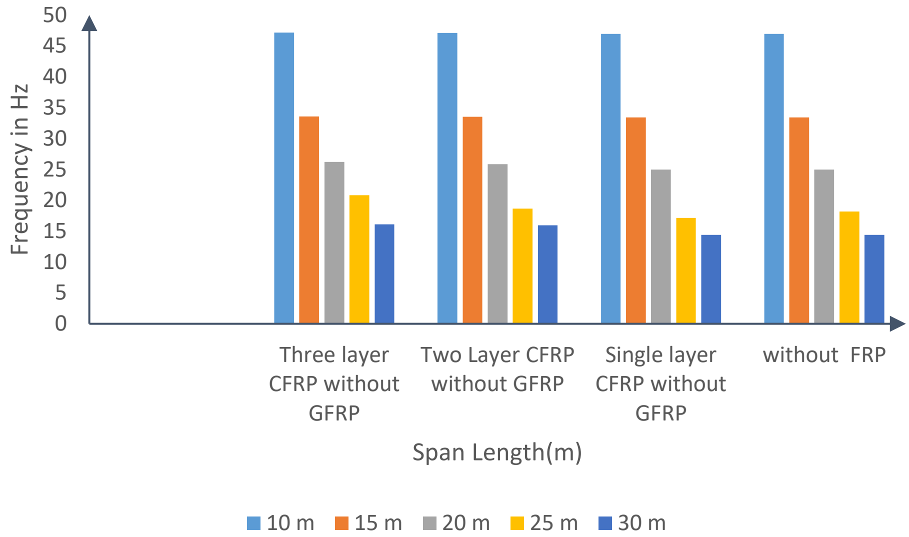


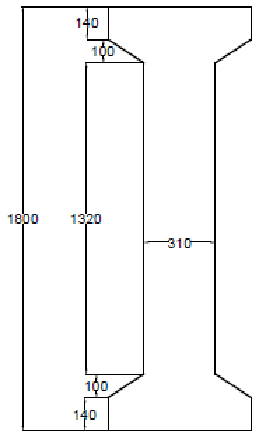

**Figure 3. 70** Frequency in mode 5 vs span length in carbon laminate girder

## 3.7 Analysis of I-girder

In the case where girders need to carry the heavy loads, preferably I-girders come in the top priorities due to several advantages. The girders have higher stiffness, wide flange which enables it to carry the more loads. The high precision value have been found in compared to other type of girder. In the rail bridges it is widely used because of lower cost than steel, lesser residual stress. It saves around 30 percentage of total construction cost than the steel girder.

### 3.6.1 Dimension of I- girder

Here Six I girder with constant width to depth ratio and varying in the length. The length of girder is chosen as for very short to long span. The girder length were chosen as 5 m, 10m, 15m, 20m, 25m and 30 meter have been analysed with carbon and glass fibre, also only with carbon fibre for all the lengths of girder and result were compared with each lamination cases and lengths using finite element modelling. The section of the girder are as shown in Figure 3.71




(All dimension are in mm)

**Figure 3. 71** Section diagram of I-girder

### 3.6.2 Loading on I- girder

According to the axle plan, the loading pattern needs in the ANSYS to define the WAGON 7 rail locomotive (Figure 3.72) of the Indian Railway. The axle details of WAGON 7 is presented in Table 3.3.

Table 3. 3 Loading details of Indian locomotive WAGON 7

| S.N. | Parameter | Values |
|---|---|---|
| 1 | Weight of WAGON | 123 t |
| 2 | Total No of Axle | 6 nos |
| 3 | Axle Load | 20.5 t |
| 4 | Max Design Speed | 110 Km/h |
| 6 | Lateral Force/Axle | 4 t |
| 7 | Dynamic Augment | ≤ 50 % |
| 8 | Un sprung Mass/Axle | 4.3 t |

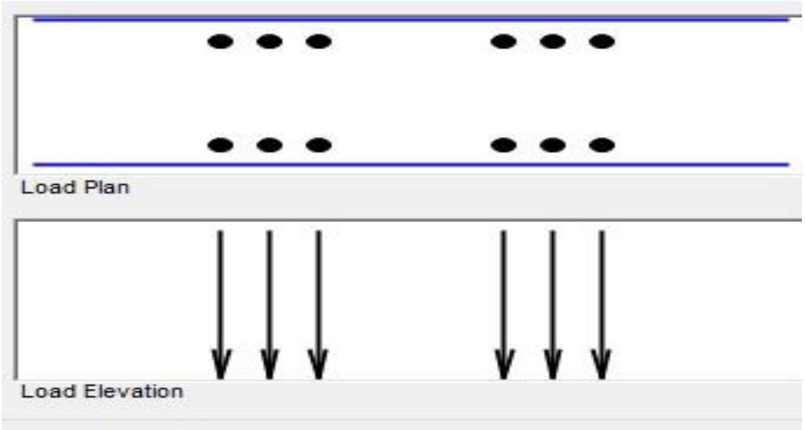



**Figure 3. 72** loading plan of I-girder

## 3.8 Result and discussion

All the five girders of dimensions 5m, 10m, 15m, 20m, 25m, and 30 m have been analysed for deformation, stress, strain and natural frequency for the first five mode shapes. The detailed analysis of result is presented in the form of various suitable graphs and figures.

### 3.8.1 Maximum deformation of laminated I- girder

The deformation due to live load has been displayed after finite element analysis of girder for different IRC loads. Few apparent changes have observed when the layers of composite fibre were increased. In Figure 3.73 it is clearly visible that for the longest span of bridge i.e. 30 m, the deformation was vastly reduced when layer of composite fibres CFRP and GFRP were tripled. In the case 1 when the length of the girder was 20 meter and the single layer of composite fibre were glued then the deformation in the girder was noticed 4.99 mm and when this layer increased by one more layer then the deformation was found 3.55 mm and in the case of triple layer it decreased and reached to 2.93 mm. In the case 2 when the length of girder was 25 meter, the deformation in the single layer fibre 11.11 mm, in the double layered fibre it was 9 mm and in the triple layered fibre it came to 7.92 mm.

In the case 3 when the length of the girder was 30 meter, then in the single layer composite fibre the deformation was 21.79 mm, as soon as the layer of fibres were doubled the deformation was decreased and reached up to 18.49 mm and in the next case when the layers of fibres were tripled the deformation was reduced up to 10.25 mm.

The girder was again modelled without glass fibres (Figure 3.74) with the increasing number of layer of carbon fibres for all the three girder of 20mm, 25 mm and 30mm. The span of 10m, 15, and 20 m also does not impact the results after increasing the layer of fibre. In the case of 20 meter the deformation of the girder when there was no fibre, was found 5.02 mm. with the single layer of carbon fibre it was 4.70 mm, as doubled the layers for this girder the deformation came to 4.45 mm and in the three time CFRP wrapped girder the deformation was found 4.03 mm. For the second girder whose length was 25 meter the deformation was 11.17 in no fibre strengthening case.  The deformation was reduced and reached to 11.12 mm when single layer CFRP was applied similarly it further sink to 11.01 mm and 10.92 mm for two and three layer lamination of girder. The third girder with span length 30 meter was examined by wrapping with  one, two and three layer of carbon fibre and the deformation was noticed 21.84 mm, 15.53 mm and 17.4 mm respectively.

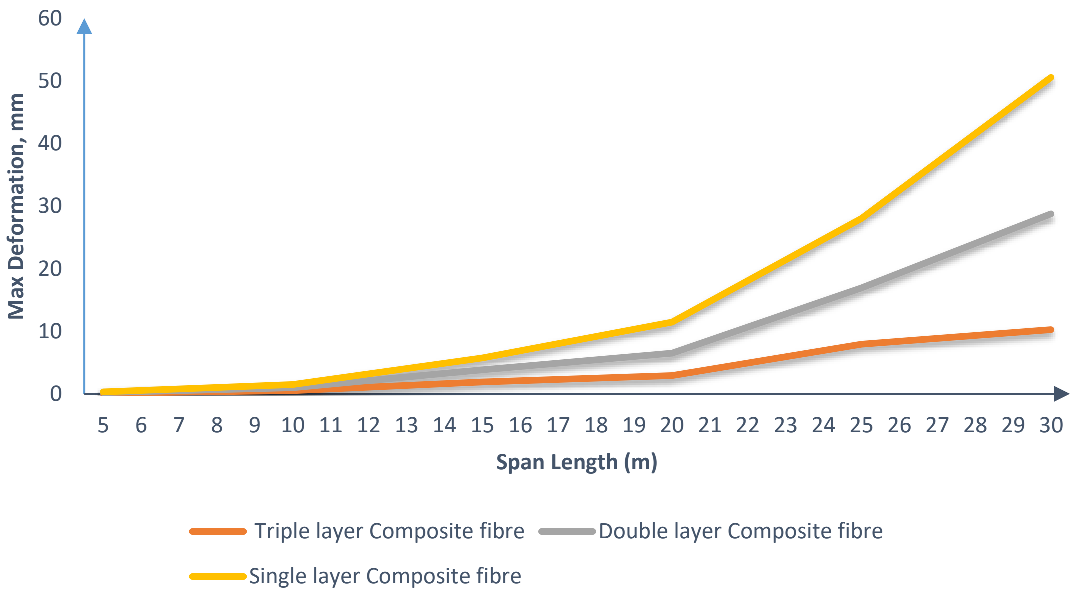



**Figure 3. 73** Maximum deformation vs span length in composite laminate girder

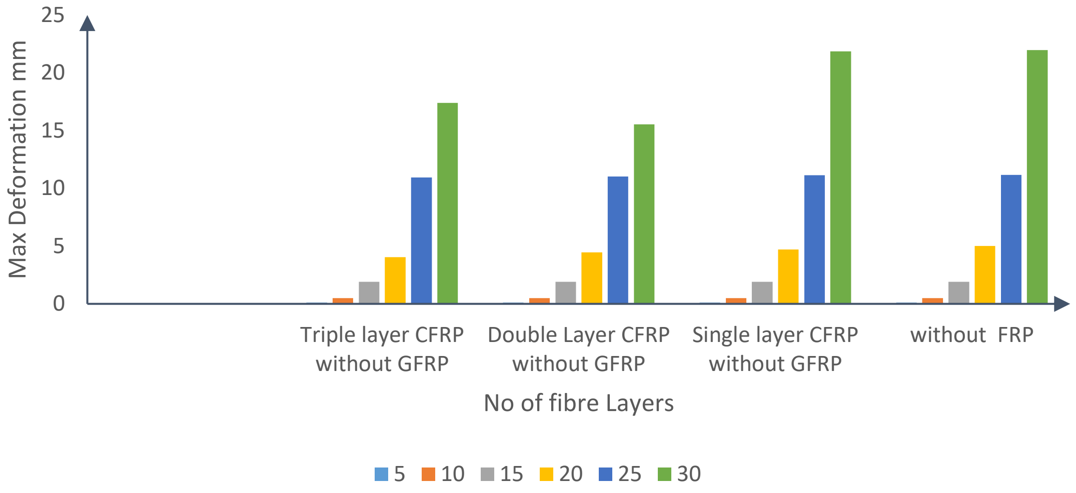



**Figure 3. 74** Maximum deformation vs. no of layer of carbon laminate girder

### 3.8.2 Max Von Mises strain of laminated I- girder

Figure 3.75 and Figure 3.76 confirmed that every laminated bridge girder experiences the yielding before going in failure mode as their Von Mises strain is greater than their simple deformation in each case. In Figure 3.75 the single, double and triple layer composite fibre stratum on all the three girder of sample. In the single layer and longest girder the strain was found 0.9 mm while it got reduced by 43 percent in the case of triple layer composite fibre on the same length of girder. The Von misses strain on the single layer laminated girder of 25 meter and 20 meter length is 0.648 mm and 0.53 mm. When the composite fibre layer tripled the strain found as 0.54 mm, 0.48 mm and 0.39 mm for 30m, 25m, and 20m girder.

Figure 3.76 consist the von mises strain of girders which was laminated only using the carbon fibre only. The single layered girder of 30 meter length showed von misses a strain of 0.95 mm while the other two girder of comparatively smaller length showed a difference of 0.63 mm and 0.49 mm for 25 meter and 20 meter respectively. The similar trend observed in the double and triple layered girder of all length.

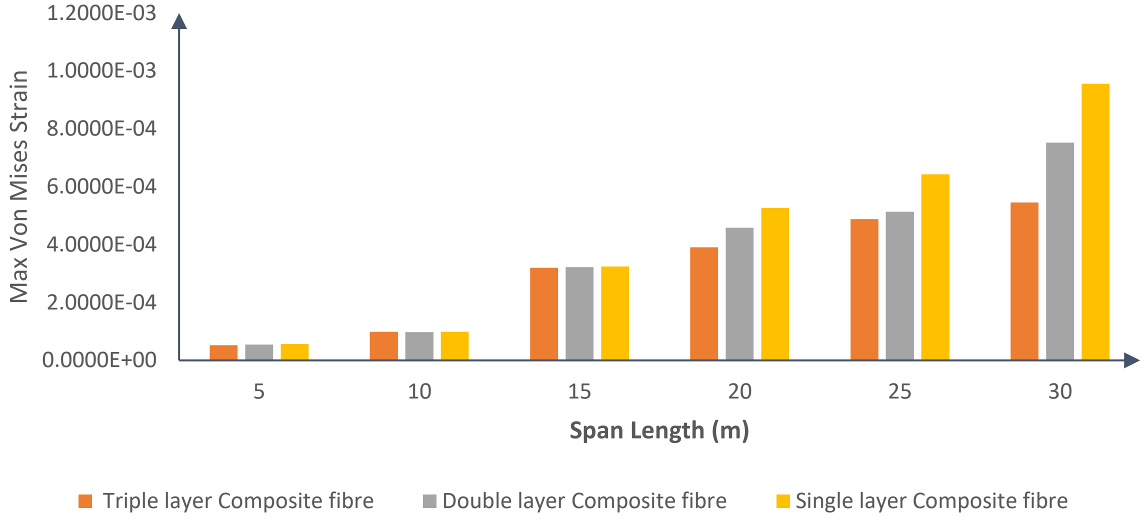



**Figure 3. 75** Maximum Von Mises strain vs span length in composite laminate girder

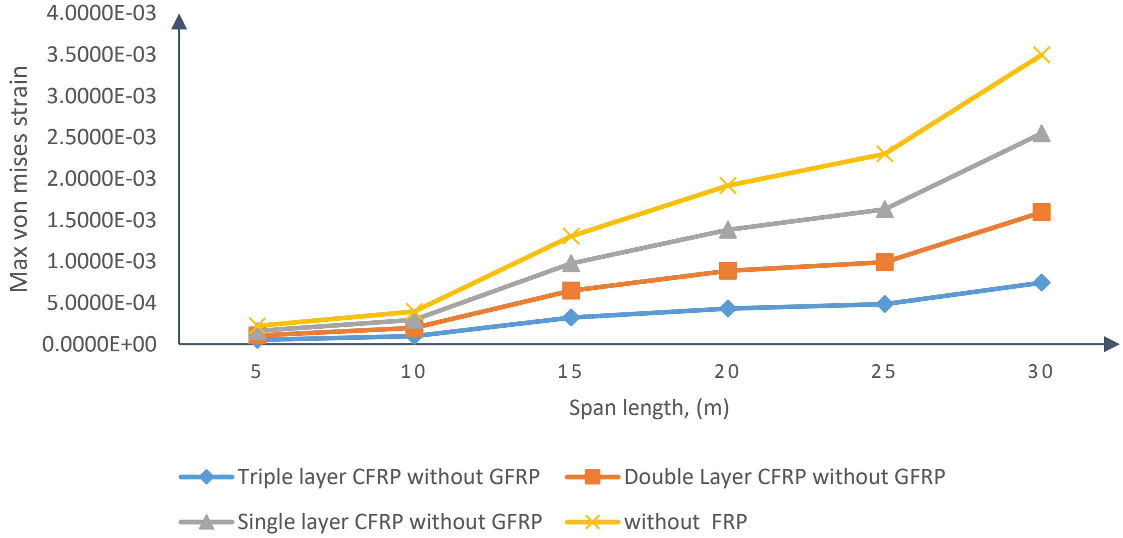



**Figure 3. 76** Maximum Von Mises strain vs no of layer of carbon laminate I girder

### 3.8.3 The maximum compressive stress of laminated I- girder

Figure 3.77 and 3.78 result show that all the five girder were retrofitted by both carbon and glass fibre the maximum compressive have stress 10.55 MPa for 30 meter span in single layer fibre and reduced this by approximately 62 percent when the layers were tripled in the same beam. The girder of 25-meter length was 7.39 MPa for the longest span and reduced to 4.29 MPa when increased the layer of girder to three. For the 20 meter span and three layer lamination the stress was 4.94 MPa and decreased to 3.90 MPa when it was wrapped by triple layer. In the other two beam of 5m and 10 m, the reduction was prolonged, so it should be avoided for economical purpose.

In the case of carbon laminated girder of length 30 meter, the compressive stress was found 10.56 MPa for single layer and reduced it to 8.39 MPa for triple layer lamination. When the girder became slightly smaller of length 25 m, the compressive stress for a single layer was found 7.4 MPa and reduced to 6.9 MPa for a single layer laminated girder. For the 20 meter long girder, the compressive stress was 4.9 MPa in single laminated cases and 3.5 for triple layer lamination.

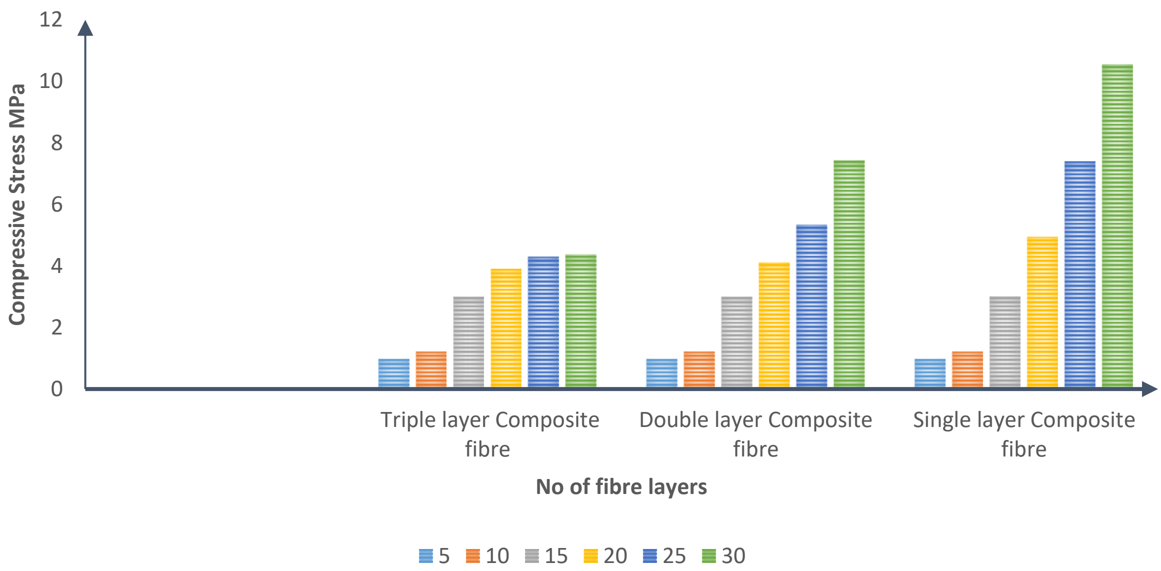



**Figure 3. 77** Maximum compressive stress vs span length in composite laminate girder

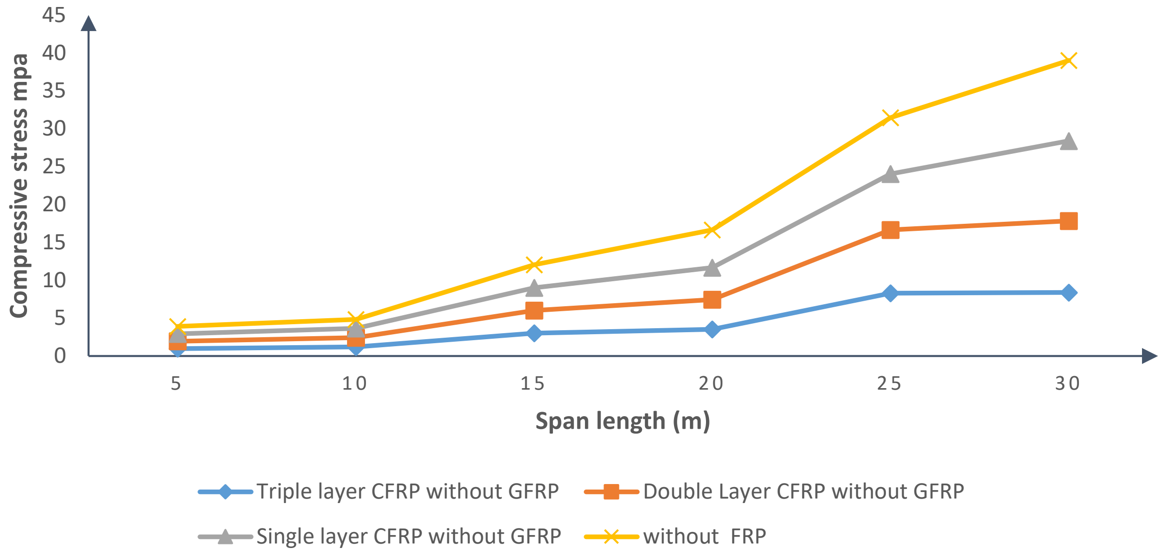



**Figure 3. 78** Maximum compressive stress vs no of layer of carbon laminate I girder

### 3.8.4 The maximum tensile stress of laminated I- girder

It is well known that the concrete is weak in tension, especially simple reinforced girder of all the structures. So it should ensure that if the load is traveling in the tension zone due to any reason, then the tension zone's strengthening is constitutive. Figure 3.79 assured that the effect of multiple layer FRP increased the strength of concrete considerably in the tension zone. The maximum tensile stress in singly, doubly, and triple layer wrapped girder of three different length is being analysed in this figure. The first case of the girder length 30 meter was singly wrapped with stress of 8.9 MPa, 6.4 MPa for the 25 meter long girder and 4.21 MPa for 20 meter girder. As the strengthening of all girder were completed and tested again the maximum reduction was observed in the longest girder about 80 percent. In double coated lamination the stress in longest girder of 30 meter length it is 5.8 MPa, in 25 meter long girder it is 5.3 MPa and in shortest girder it is 3.7 MPa. In the three layered laminated girder case the maximum tensile stress is 4.8 MPa, In 25 m span it is 3.2 MPa and in the 20 meter span it is 2.9 MPa

Figure 3.80 proved that without glass fibres there will be no such increment in the tensile strength of concrete. The reduction in the tensile stress in all the girder was not as found in composite fibres.

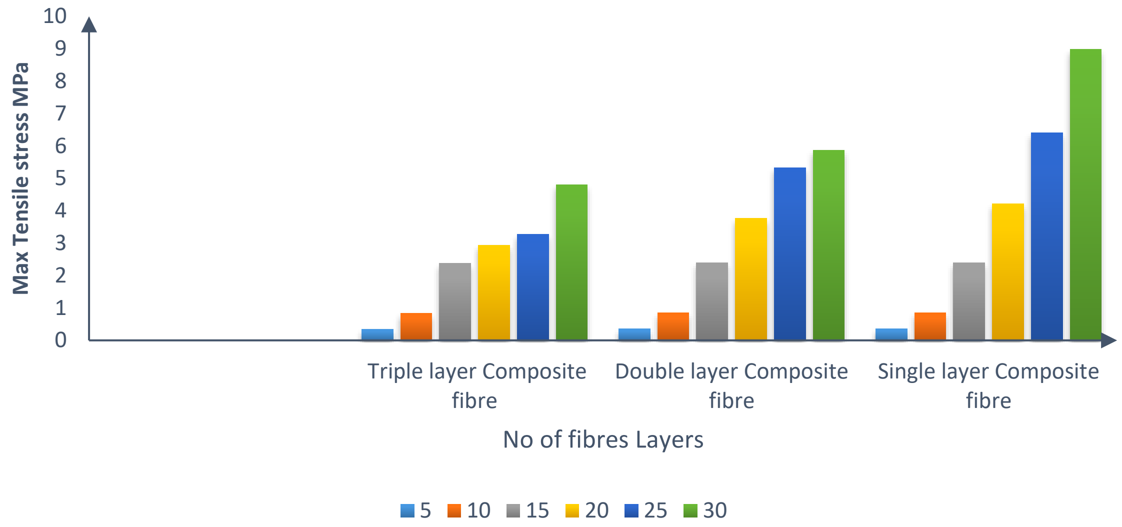



**Figure 3. 79** Maximum tensile stress vs span length in composite laminate I girder

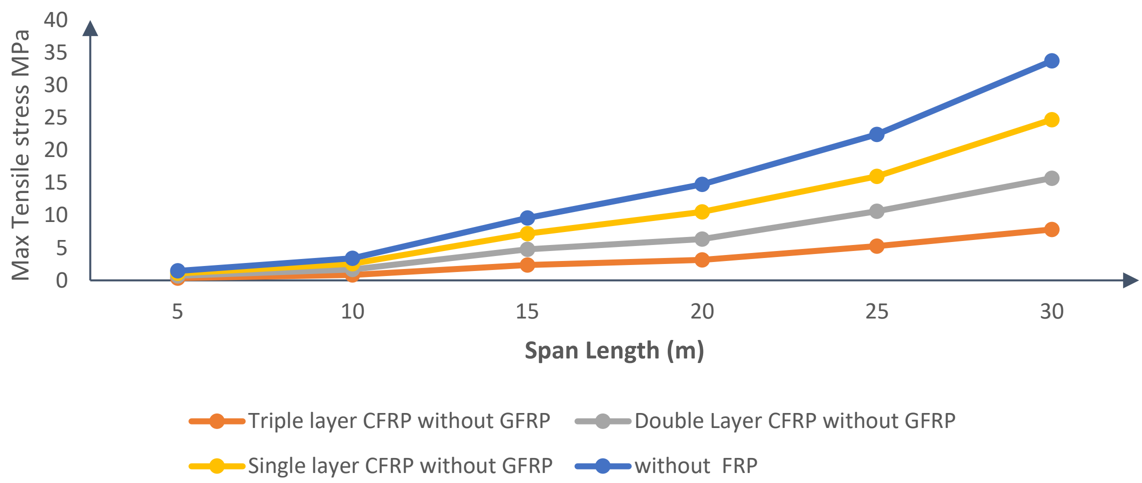



**Figure 3. 80** Maximum tensile stress vs. no of layer of carbon laminate I girder

### 3.8.5 Maximum Von-Mises stress of laminated I- girder

Figure 3.81 the von-Mises stress of any structure describes its ability to yield before discomfiture. Figure 3.81 consists of the behavior of composite laminated girders. The stress in single layered laminated girder the von Mises stress was found 26.14 MPa for the longest span which was 30m. Respectively for double and triple layer it was 20.61 Mpa and 14.18 MPa. For shorter span length i.e. 25m and 20m the von Mises stress was reported 11.95 MPa and 12.63 MPa when girder was double coated by composite fibre and 9.5 MPa and 11.4 MPa for triple coated composite layer.

In the Case where glass fibre was not attached with girder and only strengthened by one, two and three layers of carbon fibre are being discussed in Figure 3.82. In the case of triple-layered wrapping of fibre the von mises stress on the girder length 30 m, 25 m and 20 m was 20.13 MPa, 12.42 MPa and 11.46 MPa, respectively. If the strengthening was done using double-layered carbon fibre then the stress was observed 22.62 MPa, 13.81 MPa and 12.65 MPa for descending length of last three girders. When the layers of carbon downed to a single layer, the stress in the longest girder was 26.2 MPa while in the girders of 25 meter and 20 meter, the stress was 16.16 MPa and 13.8, respectively.

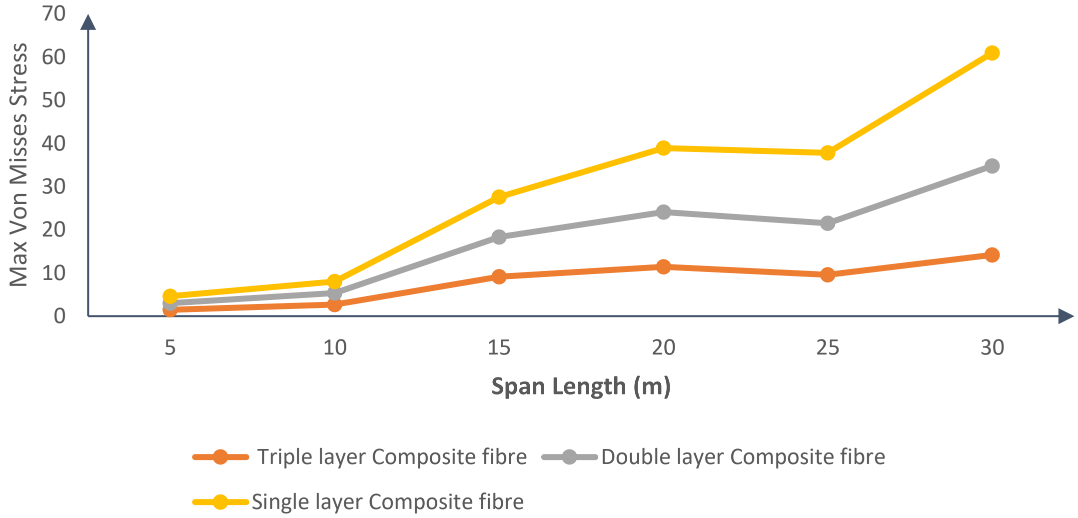



**Figure 3. 81** Maximum Von-Mises stress vs span length in composite laminate I girder

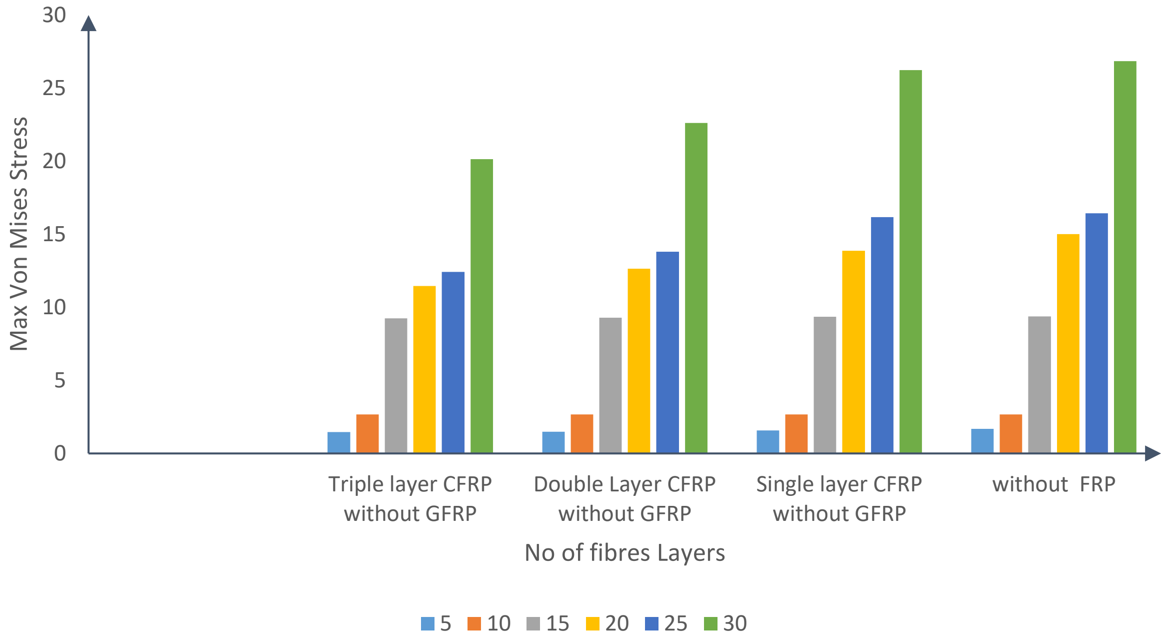



**Figure 3. 82** Maximum Von-Mises stress vs no of layer of in carbon laminate I girder

### 3.8.6 Strain Energy of laminated I- girder

Figure 3.83 explains the effect of lager variation of fibre on the different size of girder. The single layer composite fibre produces the 43249 Joule strain energy for 30 meter girder. After doubling the composite layer strain energy comes to 32488 Joule and in the case of triple layer fibre the strain energy was found 22106 Joule. In the 25m and 20 meter long spans the strain energy reported 11028 joule and 9789 joule when doubling the both carbon and glass fibre wrapping. Figure 3.83 explains about the case where only carbon fibre used for retrofitting the girder. In this analysis the strain energy in unstrengthen beam was found 43617 Joule for longest girder and decreasing gingerly when the layer of fibres were single, doubled and tripled. In the case of single layer carbon wrapping the strain energy in 30 meter girder was found 43381 joule and in other two girders it was 18385 joule and 6490 joule for 25 meter and 20 meter respectively. If we see the difference in the length of girder, then the difference in the strain energies is more than three times each other. A similar pattern was noticed in the double and tripled layer laminated girder of all three sizes. Similarly, in the shorter span, the reduction in strain energy was not as much as seen in the composite retrofitted girder. In the 25 meter un-retrofitted girder the strain energy was found 18532 Joule and decreasing very minutely when the number of layers were increased to double and triple. The similar pattern was continued in the 20m long girder. (Figure 3.84)

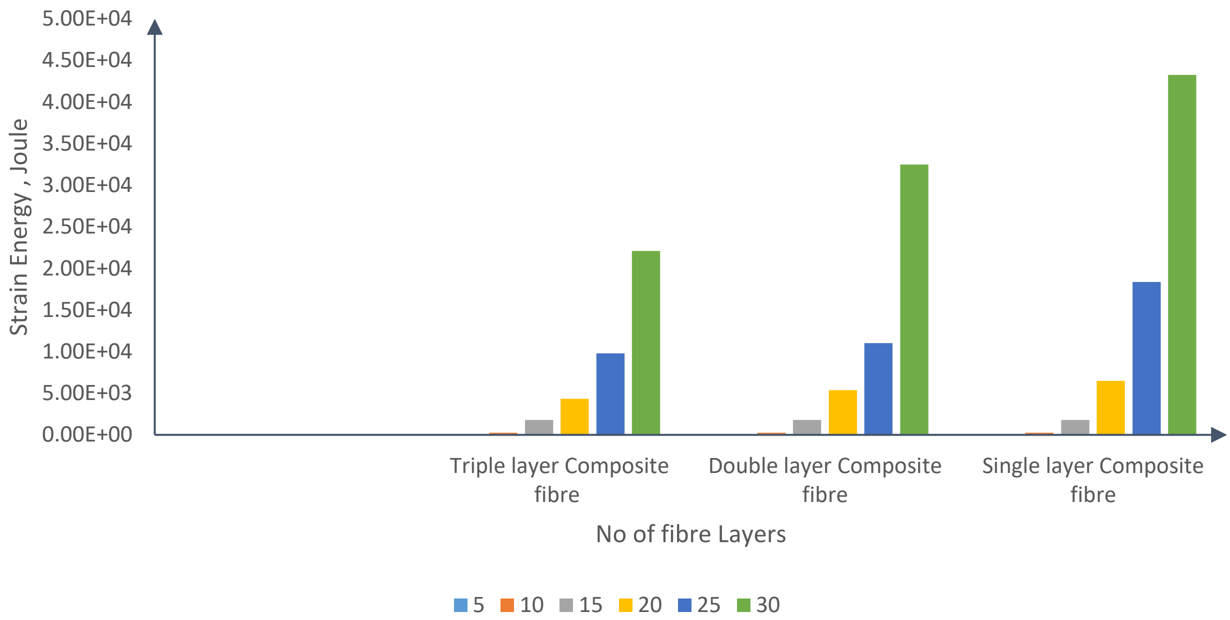



**Figure 3. 83** Strain energy vs span length in composite laminate I girder

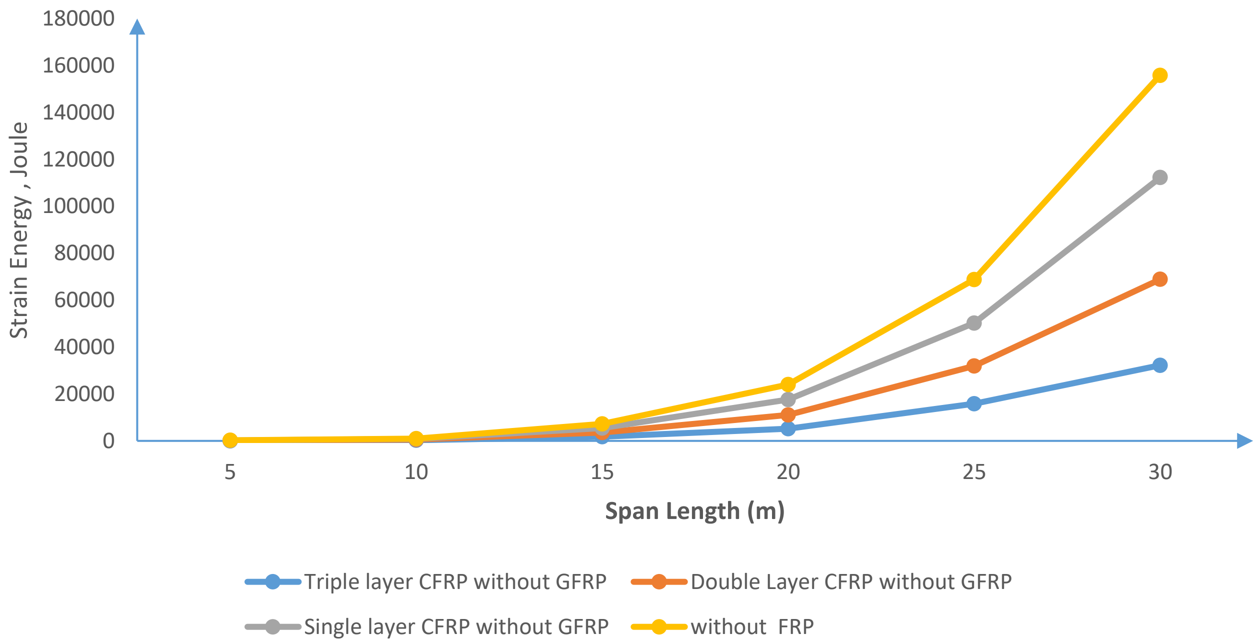



**Figure 3. 84** Strain energy vs no of layer of carbon laminate I girder

### 3.8.7 Frequency of laminated I -girder for different modes

The natural frequency of girders for different modes have been discussed here

**i. Frequency of laminated I -girder for mode 1**

Figure 3. 85 describes the vibrations of girder in mode 2. for single-layer composite laminated girder of length 30 m the frequency was found 4.27 Hz, In girder length 25m and 20 meter it was found as 7.87 Hz and 9.77 Hz. A similar pattern was observed in a double and triple layer of composite fibre laminated girder. For the triple layer the frequency of 30 meter long girder increased by 15% and no such drastic improvement was observed in the girder below 20 m span. When the girder was laminated only using carbon fibre the frequency in mode has been displayed in Figure 3.86. In this figure the frequency of triple layer carbon fibre laminated girder of length 30 meter was found 4.43 HZ. While other girders of length 25 meter and 20 meter was noted 8.02 Hz and 10.67 Hz. The frequency in doubly laminated girder of 30 Meter, 25 meter and 20 meter was found as 6.77 Hz, 7.62 Hz and 9.93 Hz. when the wrapping of girder reduces to the single layer carbon fibre then the frequency in mode 2 was 4.73 Hz in 30 meter long girder, 5.57 Hz in 25 meter long girder and 8.87 Hz in 20 meter sample girder. If all girders are un-laminated the frequency was found as 4.69 Hz for 30m length, 5.78 Hz for 25 meter and 8.46 Hz for 20 meter girder.

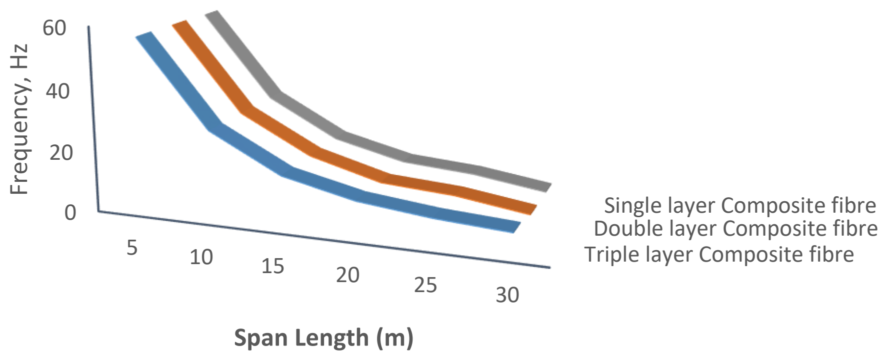



**Figure 3. 85** Frequency in mode 1 vs span length in composite laminate I girder

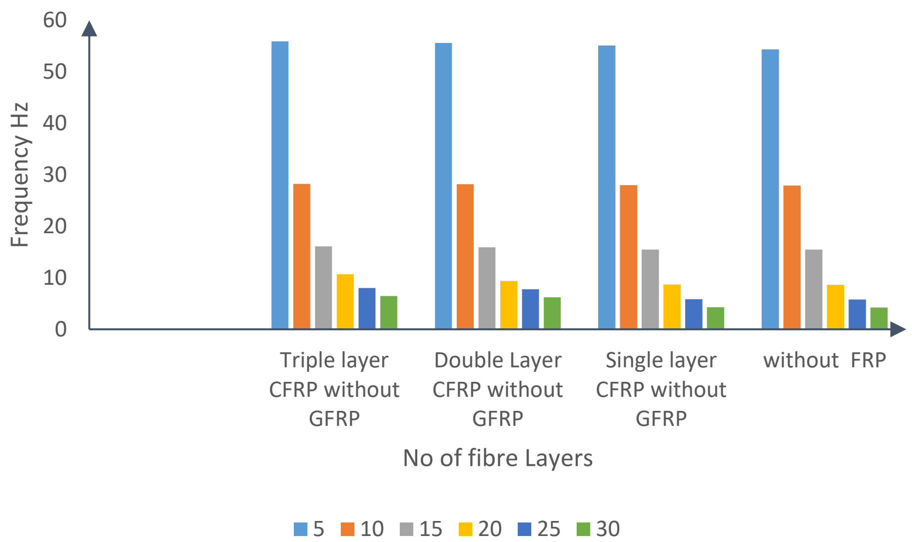



**Figure 3. 86** Frequency of girder in Mode 1 with carbon laminated I girder

### ii. Frequency of laminated I -girder for mode 2

Figure 3. 87 describes the vibrations of girder in mode 2. for single layer composite laminated girder of length 30m the frequency was found 4.73 Hz, In girder length 25m and 20 meter it was found as 8.57 Hz and 13.87 Hz. Similar pattern was observed in double and triple layer of composite fibre laminated girder. The frequency of 30 metre long girder increased by up to

12.8% for triple layer, and no such drastic improvement was observed in the girder below 20 m span. When the girder was laminated only using carbon fibre the frequency in mode has been displayed in Figure 3.87. In this figure the frequency of triple layer carbon fibre laminated girder of length 30 meter was found 7.79 HZ. While other girders of length 25 meter and 20 meter was noted 10.67 Hz and 15.97 Hz. the frequency in doubly laminated girder of 30 Meter, 25 meter and 20 meter was relieved as 4.77 Hz, 7.62 Hz and 12.93 Hz. when the strengthening of girder reduces to the single layer carbon fibre then the frequency in mode 2 was 3.73 Hz in 30 meter long girder, 5.57 Hz in 25 meter long girder and 10.87 Hz in 20 meter sample girder. In the case where all girders are un-laminated the frequency was found as 5.69 Hz for 30m length, 8.78 Hz for 25 meter and 14.46 Hz for 20 meter girder.

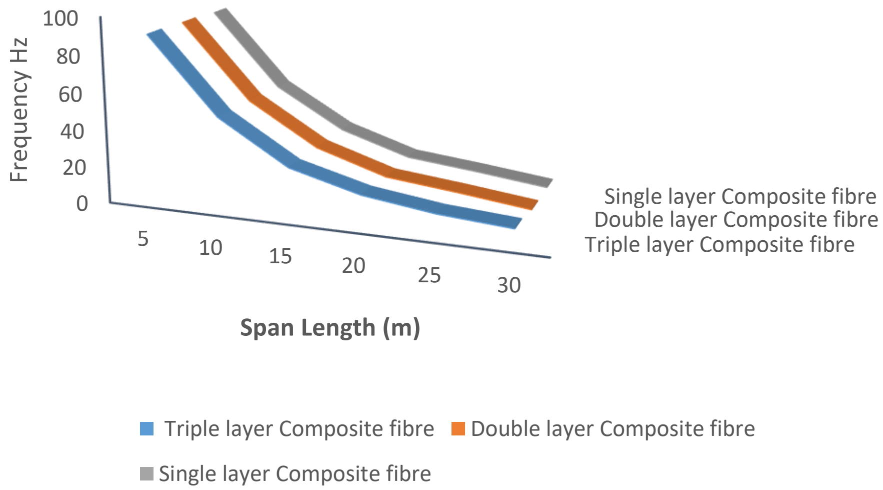



**Figure 3. 87** Frequency in mode 2 vs span length in composite laminate girder

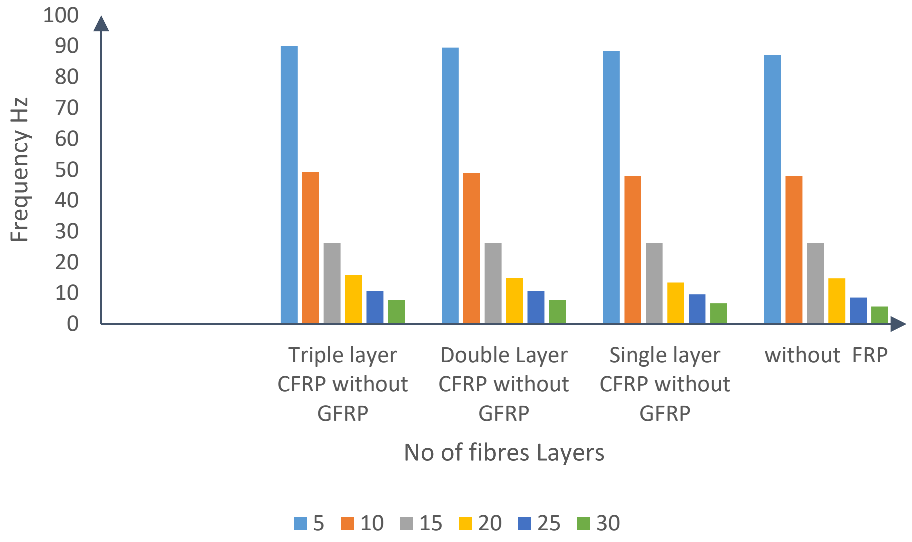



**Figure 3. 88** Frequency of girder in Mode 2 with carbon laminated fibre

### iii. Frequency of laminated I -Girder for mode 3

Figure 3.89 and 3.90, describes the frequency of all the three different length girder for mode shape 3 which was retrofitted in two different types of fibre combination i.e. first with composite fibre of carbon and glass, and second only strengthen by Carbon fibres. In Figure 3.89 it can be seen that the Single layer Composite fibre when applied in the longest span then the frequency obtained Comes to 11.49 Hz and applied to 25 meter and 20 meter long girder the frequency was reported 16.25 Hz and 24.77 Hz. It can also concluded that the frequency difference in shortest and longest sample girder was increased by upto 20 percent. The frequency for double and triple layer composite fibre of span 30 meter was 15.36 Hz and 15.637 Hz and for 25 meter and 20 meter span it was confirmed to 19.79 Hz, 20.07 Hz and 25.56 Hz, 26.36 Hz respectively.

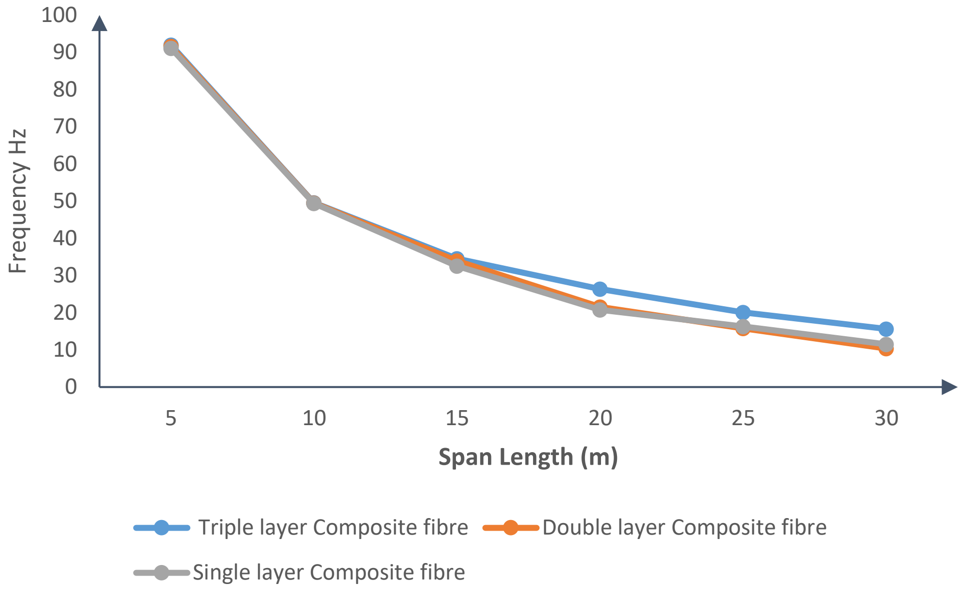



**Figure 3. 89** Frequency in mode 3 vs span length in composite laminate girder

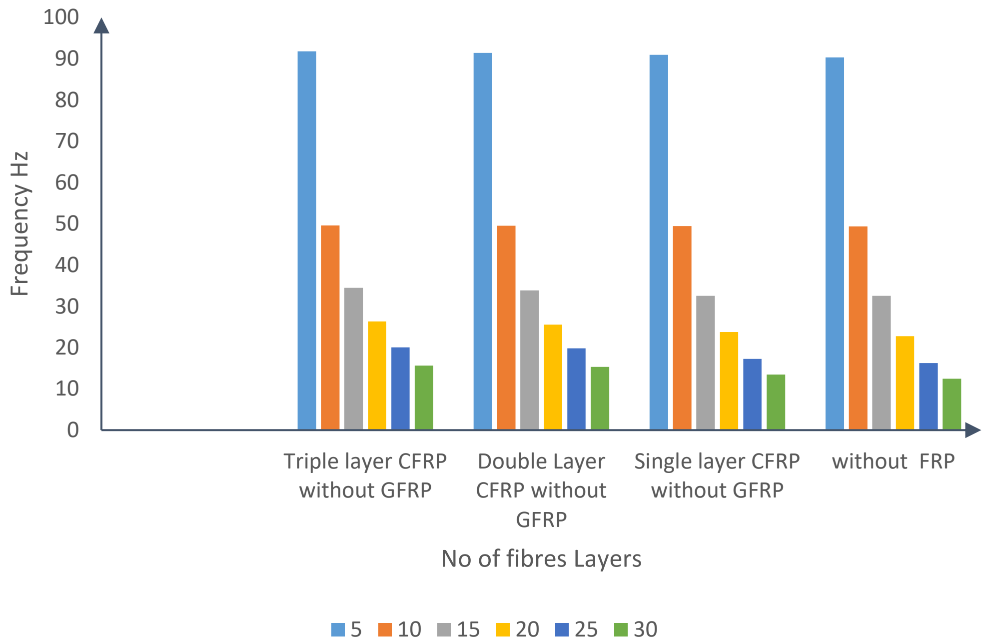



**Figure 3. 90** Frequency in mode 3 vs span length in carbon laminate girder

### iv. Frequency of laminated I -girder for mode 4

Analysis of modal frequency depict about the dynamic properties of tested structure. The primary objective of this analysis was to find out the mode shape and frequency of girders.

Figure 3.91 and Figure 3.92 Contain frequency in mode 4 with and without glass fibre while carbon fibre was always flexure. In Figure 3.91 the single layer Composite fibre wrapped girder showed approx. 15 percent increment in frequency in shortest girder than the longest girder of case. This trend was continued in double and triple layered girder too. Figure 3.91 the frequency of single layered carbon strengthened girder of span length 30 meter was 14.63 Hz while girder of length 25 meter and 20 meter was found 18.16 Hz and 26.91 Hz respectively. When the Carbon fibre layered was increased in all the girder the frequency obtained in 30 meter, 25 meter and 20m girder was 17.32 Hz, 20.58 Hz, and 27.35 Hz, respectively. Similarly, the frequency in all the three carbon swaddled girders in descending order was found at 17.91 Hz, 21.60 Hz and 27.56 Hz.

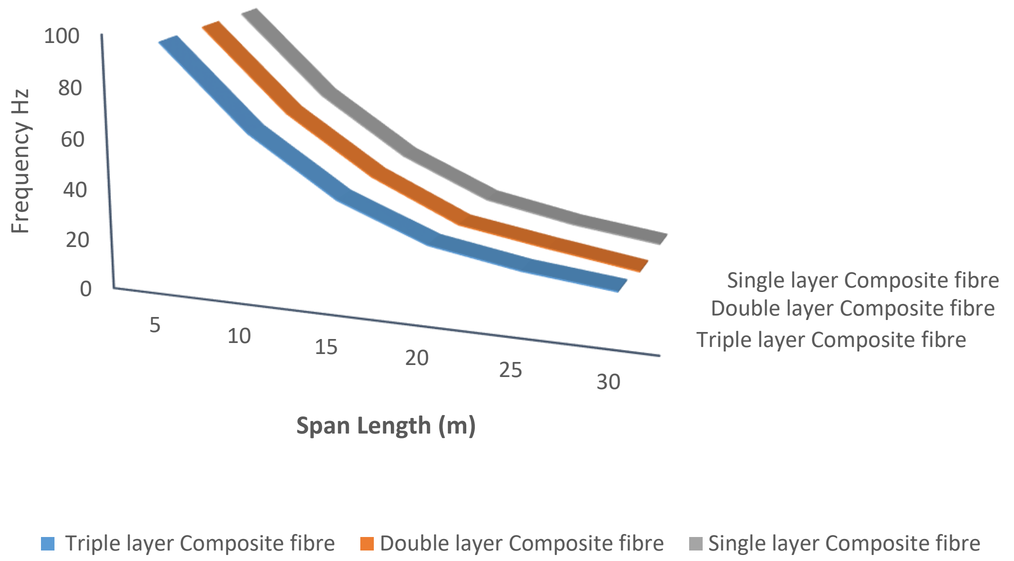



**Figure 3. 91** Frequency in mode 4 vs span length in composite laminate girder

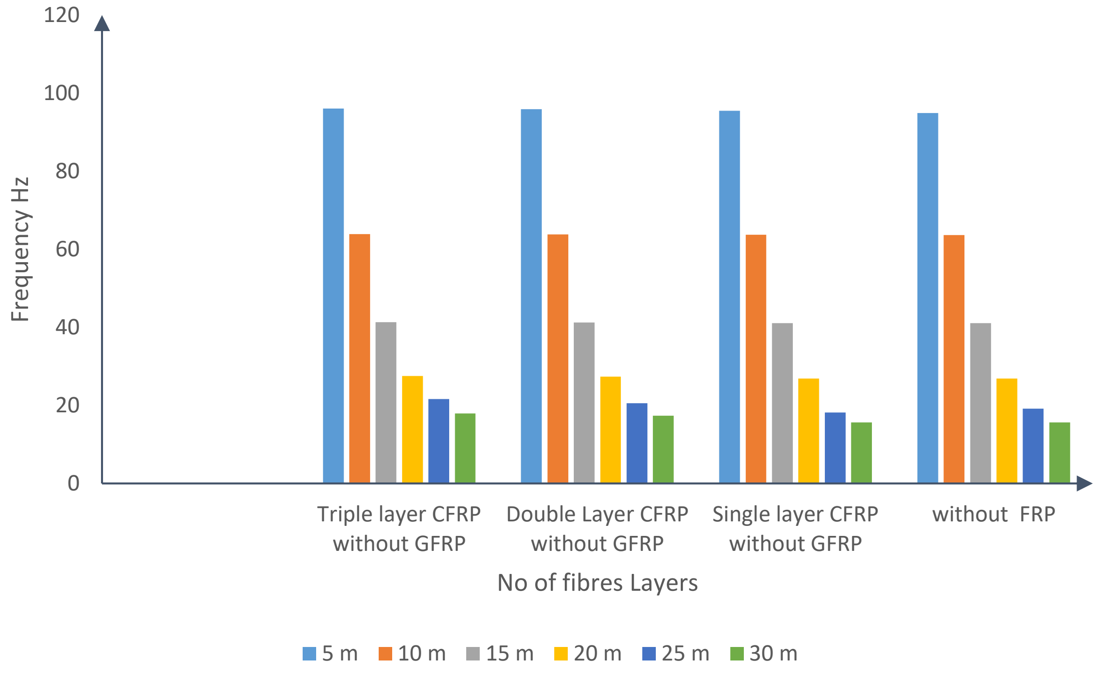



**Figure 3. 92** Frequency in mode 4 vs span length in carbon laminate girder

### v. Frequency of laminated I -girder for mode 5

Figure 3.93 and Figure 3.94 represents the frequency of laminated girder in mode shape 5. In Figure 3.93 the single layer composite fibre on the girder of length 30 meter showed the frequency of 16.83 Hz and the girder of 25m and 20m showed the 21.4 Hz and 30.39 Hz respectively. The double-layered composite fibre of longest girder showed the frequency of 17.89 Hz and other two girder in descending order showed 13.55 and 30.46 Hz respectively. Similar stance was observed in triple Layered girders of all the three spans. In Figure 3.93 the mode shape 5 and frequency for three different spay and three type of Carbon fibre arrangement were compared. In the first combination the single layer carbon fibre on the girder of three different span i.e. 30 meter 25 meter and 20 meter was applied and frequency was measured for mode shape 5. The frequency for largest and second largest girder was 16.82 Hz and 22.46 Hz. The frequencies in the 20 meter long after lamination of single layer was found the highest and it was 28.89 Hz. For the double layer, the 30 meter girder frequency was noted 17.88 Hz and the girder with 25 meter span and 20 meter span reported their frequency 23.54 Hz and 32.46 Hz respectively. Similar Easel was noticed for next added layer of FRP. For three layer FRP retrofitted girder the frequency of 30 meter girder was 18.48 Hz, for 25 meter it was 23.601 Hz and for 20 meter long girder's frequency was 32.50 Hz.

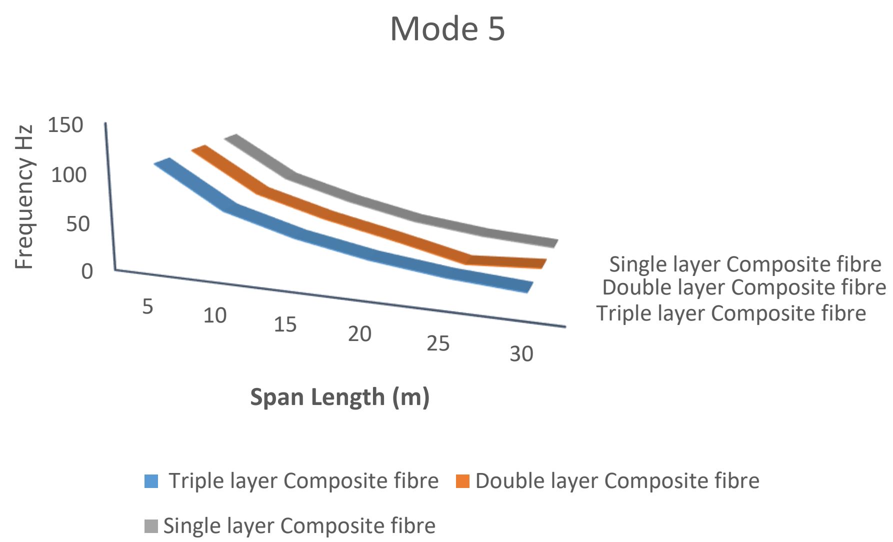



**Figure 3. 93** Frequency in mode 5 vs span length in composite laminate girder

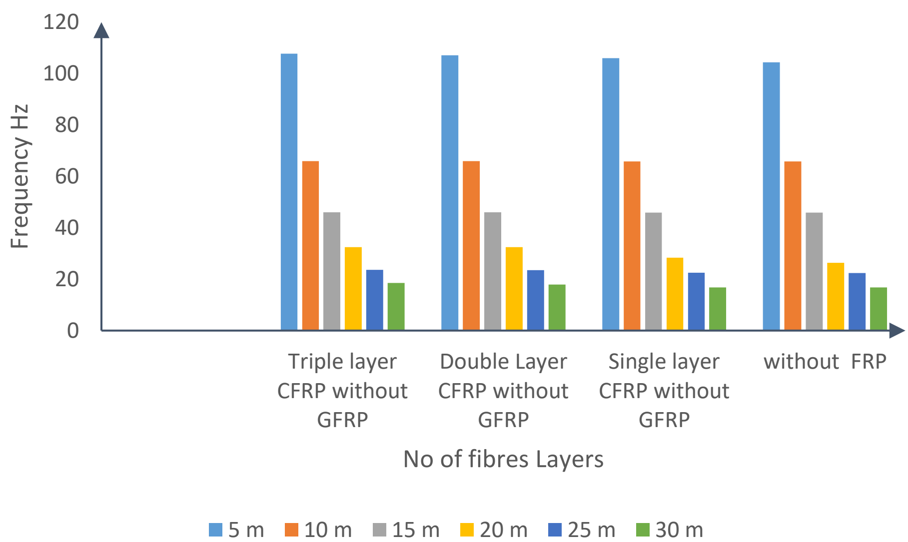



**Figure 3. 94** Frequency in mode 5 vs span length in carbon laminate girder

### 3.9 Regression analysis

The process mentioned above generates an extensive array of data, and this data is then used in Microsoft Excel to develop relations between the reduction in deflections, stresses, strain energies, increase in strength, change in natural frequencies etc. and the configuration of beam with the extent of retrofitting. For this purpose a multivariate polynomial regression analysis is done with polynomials up to fourth-degree in case of frequency analysis and up to third-degree in case of static analysis.

The data obtained from the simulation of different cases is used to obtain the relation between various parameters for additional layers of retrofitting. The developed equations are listed in this section

#### 3.9.1 Parametric relations of the beam without FRP

i. The non-dimensional deflection

$$\frac{\delta}{l} = -1.445 + 0.169w - 0.213P + 0.0165P^3 - 1.893m + 7.009m^3 - 0.0206m^3 \qquad ; r^2 = 0.999$$

$$\sigma = -124.074 + 7.585w - 10.955P + 4.07P^2 + 362.99m - 390.031m^2 + 231.046m^3 \qquad ; r^2 = 0.989$$

ii. Strain energy

$$E = -72.744 + 1.012w + 0.742w^2 - 9.855P + 0.723P^2 - 88.448m + 341.64m^2 - 22.579m^3 \qquad ; r^2 = 0.999$$

iii. Natural frequency

$$\nu = -11.822 + 4.659n^2 - 0.135n^3 + 0.613P + 96.103m - 163.203m^2 + 70.774m^3 \quad ; r^2 = 0.941$$

Where,

$\frac{\delta}{l}$ = Non-dimensional deflection due to static loading i.e. deflection to span ratio

$\sigma$ = Tensile stress in bars (MPa), $E$ = Total strain energy (J), $n$ = Number of mode of vibration

$\nu$ = Natural frequency (Hz), $w$ = uniformly distributed loading intensity (kN/m)

$P$ = Percentage of steel (%),$m = \frac{b}{d}$ = Aspect ratio, i.e. width to effective depth ratio

$\frac{\Delta}{l}$ = Non-dimensional deflection due to natural vibration i.e. deflection to span ratio

### 3.9.2 Parametric relations of the beam with single layer FRP

i. Reduction in non-dimensional deflection (%)

$$= -0.196 + 8.729P - 4.013P^2 + 0.512P^3 - 0.6785m + 2.889m^2 \quad ; r^2 = 0.502$$

ii. Reduction in tensile stress in bar (%)

$$= 32.521 + 43.097P - 20.465P^2 + 3.227P^3 + 178.266m - 379.194m^2 + 204.44m^3 \quad ; r^2 = 0.713$$

iii. Reduction in total strain energy (%)

$$= -0.326 + 15.025P - 6.922P^2 + 0.884P^3 - 11.265m + 4.637m^2 \quad ; r^2 = 0.497$$

### 3.9.3 Parametric relations of the beam with double layer FRP

i. Reduction in non-dimensional deflection (%)

$$= 1.165 + 3.272P - 2.197P^2 + 0.292P^3 - 21.726m + 108.409m^2 - 45.153m^3$$

$$; r^2 = 0.999$$

ii. Reduction in tensile stress in bar (%)

$$= -17.498 + 31.032P - 15.952P^2 + 2.584P^3 + 77.42m - 103.447m^2 + 66.916m^3$$

$$; r^2 = 0.896$$

iii. Reduction in total strain energy (%)

$$= -2.923 + 3.913P - 2.847P^2 + 0.378P^3 - 4.375m + 149.458m^2 - 74.583m^3$$

$$; r^2 = 0.998$$

### 3.9.4 Parametric relations of beam with triple layer FRP

i. Reduction in non-dimensional deflection (%)

$$= -1.281 + 0.114w - 0.1086P + 0.0107P^2 + 0.398m + 4.15m^2 - 2.119m^3$$

$$; r^2 = 0.999$$

ii. Reduction in tensile stress in bar (%)

$$= -12.005 + 25.417P - 13.717P^2 + 2.252P^3 + 45.342m + 10.769m^2$$

$$; r^2 = 0.934$$

iii. Reduction in total strain energy (%)

$$= -16.036 - 1.044P^2 + 0.1346P^3 + 82.711m + 81.052m^2 - 62.537m^3$$

$$; r^2 = 0.997$$

## 3.10 Summary

After the analysis methodology was verified, different cases of beams are generated approximately 90 in numbers. These beams are very similar to the benchmark beams except for variation in the following criteria: Aspect ratio, Percentage of steel uniformly distributed loading intensity, Number of layers of GFRP used CFRP used.

From the study it can be concluded that the effect of CFRP and GFRP retrofitting on prismatic girder resulting the reduction in deflection and stress and the increase in strength. However the same cannot be said about the increase in natural frequency of vibrations with such certainty. As suspected, the triple layer setup is found to be the most effective with up to 70% reduction in deflection, 78% reduction in tensile stress and 91% reduction in case of total strain energy. In the case of frequency, most of the cases show an increase in frequency even up to 75% with an increase in FRP layers, but there are also non-negligible numbers of discrepancies showing a decrease of up to 2.3%. Here, a definite trend is not established. Still, since the increase is comparatively substantial than the decrease, it can be said that there is overall an increase of natural frequencies across all modes with an increase in the number of layers of FRP. A regression analysis based on the above data has been numerated below to correlate the data and to found the proportional design for efficient performance.

The effect of retrofitting I girder with all three combinations of fibre used it can be concluded that the deformation got reduced up to 59 % in the length of girder less than 20 meter and upto 47 % in more than 20 meter length. While yielding before failure is greatly increased in triple layer fibre and length more than 20 meter because the von misses strain reduce by 59 percent in this case. The increment in frequency was more than 100 percent in the case of 30 meter girder and triple layer while it is noted about up to 29 % increment in the case of girder less than 15 meter and triple-layer girder. The strain energy in the 25 meter span is reduced by 85 percent when the triple-layer is applied on the girder while this increment is very negligible in the case of girder less than 10 meters. So it is clear here that the effect of increasing fibre layers is more effective in longer girders than the smaller one.

The case of deflection of the natural mode of vibration is uncertain, and trends for this case are not well established. The deflections for different modes don’t appear to follow any particular trend as studied in different parametric studies. However the most consistent results for this case are observed in variations with respect to the percentage of steel which clearly shows that the deflection decreases with increase in the number of FRP layers.

The regression analysis resulted in very accurate relations for static deflection, stress, strain energy, and natural frequency for different layers. The relative percentage decrease in all these quantities except frequency are also quantified as equations quite accurately but for the case of frequency the relations developed were not accurate and hence discarded.

It can also be concluded that effect of increasing FRP layer in the girder length equal to or less than 15 meter does not have any significant change on the concerned quantities of deflection, stress etc, but for noticeable effects on the girder equal to or more than 20 meter when layer of FRP is been increasing are found suitable. From the strength and economic point of view double layer glass fibre wrapping in shear zone and double-layer carbon fibre in flexure zone in this case and for this length of all girder wherever it is.