Chapter 4

Conclusions

4.1. Concluding Remarks

In the present work, the structural analysis of smart laminated composite plates resting on an elastic foundation is carried out using an interlaminar shear stress continuous non-polynomial ZZ theory namely, the trigonometric zigzag theory (TZZT). A closed form analytical solution based on Navier's method is presented for the first time to carry out the static and dynamic analysis of traditional laminated composites and smart composite plate structures. Further, a generalized C^0 FE formulation is also developed to derive the static and dynamic responses of both traditional laminated composites and smart composite plate structures. A detailed investigation of the static and dynamic responses of traditional laminated composites and smart composite plates are presented in this research by exploiting the various geometrical and material features of the plate structures. The forced vibration responses are presented under the action of various time-dependent loads and various forms of blast loads with parameters like shock pulse length factor, decay parameter and positive phase duration of the pulses. The counteracting electrical loads that diminish the unwanted mechanical vibrations from the system are obtained for various forms of electromechanical loads. The suppressed free and forced vibration responses of smart composite plates are also presented by coupling the actuators and sensors with a negative feedback controller. The main conclusions of the present investigations are stated below:

- The present model considers a trigonometric function namely, the secant function in the kinematic expansion of the 3 D in-plane displacements as a higher order mathematical function of the thickness coordinate. This function is chosen carefully by fulfilling the criteria's of constructing the shear-strain functions (see Sec. 1.4.3.4). The present model is a refinement of the CPT by introducing the non-linearity of the transverse shear stresses/strains across the thickness of the smart composite plates. Also, the slope discontinuities of in-plane displacements are included in the kinematic model with auxiliary variables defined at the interfaces with piecewise linear functions of the thickness coordinate. The present model consists of only five variables defined at the midplane namely, the three constant deformations in x_1, x_2 and x_3-direction and the two rotations about the x_1 and x_2-direction. Thus the computational cost associated with the present model is like that of the FSDT.

- The static and dynamic responses of both traditional laminated composites and smart composite plates obtained using the present model is found to be in good agreement with the elasticity solutions. The results obtained using FEM are in close agreement with the present analytical results.
- The number of primary variables is also less when the refinements are made on the CPT as opposed to the FSDT. When the refinements are made on CPT, then an additional higher-order term in conjunction with an antisymmetric shear strain function is required along with the local ZZ functions. The various possible refinements that can be made on CPT and FSDT are shown below:

Refinement over Poisson-Kirchhoff theory (Model 1)

$$U_1 = \underbrace{u_1 - x_3 \frac{\partial u_3}{\partial x_1}}_{\boxed{\textit{Primary variables of CPT}}} + \underbrace{\sum_{i=1}^{n_u-1}(x_3 - {x_3}^{iu})H(x_3 - {x_3}^{iu})\alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - {x_3}^{jl})H(-z + {x_3}^{jl})\alpha_{1l}^{j}}_{\boxed{\textit{Zigzag terms}}}$$

$$U_3 = u_3 \qquad 4.1$$

The transverse shear strain calculated from Model 1 is written as

$$\gamma_{13} = \sum_{i=1}^{n_u-1}(x_3 - x_3{}^{iu})\delta(x_3 - x_3{}^{iu})\,\alpha_{1u}^{i} + \sum_{i=1}^{n_u-1} H(x_3 - x_3{}^{iu})\,\alpha_{1u}^{i}$$
$$+\ \sum_{j=1}^{n_l-1}(x_3 - x_3{}^{jl})\delta(x_3 - x_3{}^{jl})\,\alpha_{1l}^{j} + \sum_{j=1}^{n_l-1} H(x_3 - x_3{}^{iu})\,\alpha_{1l}^{j} \qquad 4.2$$

It can be observed in Eq. 4.2 that the expression of transverse shear strain does not have any primary variables of the CPT and has only the auxiliary variables defined at the midplane. The auxiliary variables cannot be expressed in terms of the primary variables. Thus the refinement is not useful.

Refinement over Poisson-Kirchhoff theory (Model 2)

$$U_1 = \underbrace{u_1 - x_3\frac{\partial u_3}{\partial x_1}}_{\text{CPT terms}} + \underbrace{f(x_3)\,\beta_1}_{\text{Higher-order term introduced with non-linear function of thickness coordinate}} + \underbrace{\sum_{i=1}^{n_u-1}(x_3 - x_3{}^{iu})H(x_3 - x_3{}^{iu})\alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - x_3{}^{jl})H(-z + x_3{}^{jl})\alpha_{1l}^{j}}_{\text{Zigzag terms}}$$

$$U_3 = u_3 \qquad 4.3$$

The transverse shear strain calculated from Model 2 is expressed as follows

$$\gamma_{13} = \sum_{i=1}^{n_u-1}(x_3 - x_3{}^{iu})\delta(x_3 - x_3{}^{iu})\,\alpha_{1u}^{i} + \sum_{i=1}^{n_u-1} H(x_3 - x_3{}^{iu})\,\alpha_{1u}^{i}$$
$$+\ \sum_{j=1}^{n_l-1}(x_3 - x_3{}^{jl})\delta(x_3 - x_3{}^{jl})\,\alpha_{1l}^{j} + \sum_{j=1}^{n_l-1} H(x_3 - x_3{}^{iu})\,\alpha_{1l}^{j} + \frac{df\,(x_3)}{dx_3}\,\beta_1 \qquad 4.4$$

The last term in Eq. 4.4 is responsible for creating the non-linear profile of the transverse shear strains across the thickness of the plate structures while the other terms are responsible for creating the discontinuous transverse shear strains at the interfaces of the plates.

Refinement over Reissner-Mindlin theory (Model 3)

$$U_1 = \underbrace{u_1 + x_3\theta_1}_{\text{Reissner-Mindlin terms}} + \underbrace{\sum_{i=1}^{n_u-1}(x_3 - x_3{}^{iu})H(x_3 - x_3{}^{iu})\alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - x_3{}^{jl})H(-z + x_3{}^{jl})\alpha_{1l}^{j}}_{\text{Zigzag terms}}$$

$$U_3 = u_3 \qquad 4.5$$

The transverse shear strain calculated from Model 3 is presented below

$$\gamma_{13} = \sum_{i=1}^{n_u-1}(x_3 - {x_3}^{iu})\delta(x_3 - {x_3}^{iu})\, \alpha_{1u}^{i} + \sum_{i=1}^{n_u-1} H(x_3 - {x_3}^{iu})\, \alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - {x_3}^{jl})\delta(x_3 - {x_3}^{jl})\, \alpha_{1l}^{j} + \sum_{j=1}^{n_l-1} H(x_3 - {x_3}^{iu})\, \alpha_{1l}^{j} + (\theta_1 + \frac{\partial u_3}{\partial x_1}) \qquad 4.6$$

It is observed in Eq. 4.6 that the transverse shear strains are constant across the thickness of the smart composite plate. Though the discontinuity of the transverse shear strains can be created at the interfaces, however the parabolic nature of the transverse shear stresses cannot be attained.

Refinement over Reissner-Mindlin theory (Model 4)

$$U_1 = \underbrace{u_1 + x_3\theta_1}_{\textit{Reissner-Mindlin terms}} + \underbrace{f(x_3)\, \theta_1^{*}}_{\textit{Higher-order term introduced with non-linear non-polynomial function of thickness coordinate}} + \underbrace{\sum_{i=1}^{n_u-1}(x_3 - {x_3}^{iu})H(x_3 - {x_3}^{iu})\alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - {x_3}^{jl})H(-z + {x_3}^{jl})\alpha_{1l}^{j}}_{\textit{Zigzag terms}}$$

$$U_3 = u_3 \qquad 4.7$$

The transverse shear strains calculated from Model 4 is shown below

$$\gamma_{13} = \sum_{i=1}^{n_u-1}(x_3 - {x_3}^{iu})\delta(x_3 - {x_3}^{iu})\, \alpha_{1u}^{i} + \sum_{i=1}^{n_u-1} H(x_3 - {x_3}^{iu})\, \alpha_{1u}^{i} + \sum_{j=1}^{n_l-1}(x_3 - {x_3}^{jl})\delta(x_3 - {x_3}^{jl})\, \alpha_{1l}^{j} + \sum_{j=1}^{n_l-1} H(x_3 - {x_3}^{iu})\, \alpha_{1l}^{j} + \underbrace{(\theta_1 + \frac{\partial u_3}{\partial x_1}) + \frac{df\,(x_3)}{dx_3}\, \theta_1^{*}}_{\textit{Shear deformation getting refined `}(\frac{\partial w_0}{\partial x} + \theta_x)\textit{+ additional term'}} \qquad 4.8$$

It is observed in Eq. 4.8 that the non-linear profile of the transverse shear strains along with its discontinuity at the interfaces can be attained simultaneously. Therefore, it is now evident that Model 2 and Model 4 can model the non-linear through-thickness variations of the transverse shear stresses/strains. Additionally, both the models satisfy the piecewise continuity requirements of in-plane displacements, which can further create discontinuous transverse shear strains and continuous transverse shear stresses at the interfaces of the plates. However, we see that the number of primary variables in the refined FSDT (Model 4) is seven while the number of primary variables in refined CPT

(Model 2) is five. Therefore, Model 2 is computationally less expensive than Model 4. A complexity noticed while developing a FE model with Model 2 is that it requires C^1 continuity of the transverse displacement at the element boundaries while Model 4 requires C^0 continuity in the FE formulation.

- The transverse shear stresses are fairly estimated using the constitutive relations (CR) of the material in the case of traditional laminated composites and sandwich plates. However, the estimations are too poor for the smart composite plate structures under the action of electromechanical loads.
- The accuracy of the transverse shear stresses is enhanced using the equilibrium equations (EE) of elasticity. The estimations obtained using EE are found to be in excellent agreement with the 3 D solutions for traditional laminated composites, soft-core sandwich plates and smart composite plates.
- A significant change in the through-thickness variation of transverse shear stress (τ_{13}) is noticed in the smart composite plates under the action of combined electrical and mechanical loading. However, the variation is more or less similar to the traditional laminated composite plate under the action of only mechanical loads. When the combined electromechanical loads are applied, it is observed that the maximum value is attained at the interface of the smart composite plate (PFRC/0/90/0) while the maximum value is attained at the midplane under the action of mechanical load only.
- The through-thickness variations of τ_{23} in smart composite plates (PFRC/0/90/0) are more or less similar to the traditional laminated composites (0/90/0) subjected to both mechanical and electromechanical loadings. The maximum value of τ_{23} is attained at the midplane under the action of both electromechanical loads and purely mechanical loads.

- The actuation in the static responses is more in case of smart composite plates with low span thickness ratio. The actuation in σ_{11} is much more pronounced than σ_{22} in a PFRC/0/90/0 and PFRC/0/90/90/0 plate due to the placement of the piezoelectric fibers in the x_1-direction and also due to the large value of the piezoelectric coefficient 'e_{31}'.
- The in-plane displacements are non-zero at the midplane in PFRC/0/90/0 plate under the action of transverse electromechanical and purely mechanical loads as there is coupling between the membrane and bending stiffness components due to the presence of the PFRC layer.
- The natural frequencies and the higher-modes of vibration are largely affected by the material and the geometrical features of the plates like density, core-thickness, aspect-ratio and modular ratio.
- The foundation stiffness has a significant impact on the static and dynamic responses of traditional laminated composites and smart composite plate structures. The magnitudes deflection and stresses have significantly reduced due to the Winkler and the shear stiffness of the elastic foundations. The fundamental frequencies of the plate structures tend to increase due to the stiffness of the foundations.
- The combined Winkler and shear stiffness in the Pasternak's foundation model has a greater impact on the structural responses of plate structures than the Winkler's foundation model as it accounts for the shear interactions among the points in the elastic soil in addition to the proportional interaction between the pressure and deflection of any point on the surface of the soil.
- In the case of laminated composite plates, it is found that the effect of enforcing the continuity of inter-laminar stresses are not much significant as the difference in the

results of the natural frequencies from the present model and the ESL-based theories are small. However, in the case of soft-core sandwich systems, the differences in the responses are significant. Thus, it is essential to employ an inter-laminar continuous plate theory to model the dynamic responses of soft-core-sandwich plate structures.

- The displacement-time responses obtained in the transient analysis indicate that the dynamic responses are largely affected by the shock pulse length factor, decay parameter, and the positive phase duration of the pulse in the blast loads. The amplitudes of the free vibration responses after a strong blast is dependent on the duration of the blast load. The laminated composite plates with a higher aspect ratio and lower span thickness ratio have more stiffness and experiences less transverse deflection. The non-linear behavior of the transverse shear stresses should be carefully accommodated in the mathematical model as improper modeling of the transverse shear stress behavior will result in the overestimation/underestimation of the stiffness of the plate, resulting in erroneous responses.
- The dynamic responses obtained using the present model and the ESL-based FSDT and HSDTs does not have much difference when the transverse shear modulus ‘G_{13}’ and ‘G_{23}’ are same for all the layers in multi-layered structures. This is because there are no discontinuities in the material properties at the interfaces when ‘G_{13}’ and ‘G_{23}’ are equal. However, the differences in the dynamic responses are evident when unequal shear modulus is considered. The unequal shear modulus creates discontinuities in the material properties at the interfaces and as a result the present ZZ model produces more accurate responses than the ESL-based models.
- The amplitude of the forced-vibration response is maximum in the case of pulse load followed by triangular, exponential and sinusoidal loads. The reason behind this is that the time-dependent response of a structure depends on the magnitude of

the load at a particular time '*t*' along with the initial conditions of displacement, velocity and acceleration at the previous time '*t*-1'. In the case of pulse loads, the amplitude of the load remains constant in time, in triangular and exponential profile the amplitude of the load linearly and non-linearly decreases from the peak value at *t* = 0 and in the case of sinusoidal loads, the amplitude increases from zero load at *t* = 0 to the peak value at some time instant. The initial conditions in the case of pulse, triangular and exponential loads are higher than sinusoidal loads as the displacement, velocity and acceleration will be higher due to higher magnitude of load during the initial time instants. Thus the amplitude of response under the sinusoidal load is the smallest. Among the pulse, triangular and exponential loads, the initial conditions at any time '*t*-1' will be maximum for pulse, followed by triangular and exponential as the amplitude of load in the pulse loading remains constant in time, *i.e*, at the peak value whereas in the case of the triangular and exponential loads, the amplitude decreases linearly and exponentially in time.

- The amplitude of the dynamic responses is also largely affected by the magnitude and polarity of the electrical loadings. The counteracting electrical loads required to remove the mechanical vibrations from the system is dependent on the span-thickness ratio. The amplitude of the electrical loads is found to increase with the increase in the span-thickness ratio.
- The static and dynamic responses of smart composite plates are not significantly different from the responses of the traditional laminated composite plates when the thickness of the piezoelectric layer is very small in comparison to the thickness of the laminated composite plates.
- The negative feedback controller used in the active vibration analysis creates an active damping matrix by which the amplitude of the responses decreases with time.

The magnitude of the free-vibration and forced-vibration responses decreases with the increase in the gain of the controller.

4.2. Contribution of the thesis

New analytical and FE models are derived for the structural responses of traditional laminated composites and smart composite plate structures resting on an elastic foundation. The developed models can produce accurate responses of multilayered laminated composites and smart structures with less computational effort. Both the actuator and the sensor responses of the smart composite plates are derived. The variation of the transverse shear stresses in smart composites under the action of the electromechanical load is significantly different from those of traditional composite structures. Thus the constitutive relations fail to predict the accurate variation of transverse shear even if an interlaminar shear stress continuous plate theory is used to model the responses. Therefore, the accuracy of the results of the transverse shear stresses is enhanced by using an efficient post-processing scheme of integrating the equilibrium equations (EE) of elasticity. The coupled electromechanical actuator responses under the action of both static and dynamic electromechanical loads are investigated in detail to check the actuation in the responses of deflection and stresses. Multilayered composite plates made up of advanced composites are extensively used in subsonic/supersonic flight vehicles and these structures are often subjected to severe dynamic excitations like sonic boom pulses and nuclear explosions, etc. It is highly important to understand the dynamic behavior of composite structures subjected to blast loads. In this research, the forced vibration responses of composite plate structures are obtained for various forms of blast loads to understand the behavior of composites under extreme conditions. In addition, various forms of time-dependent electromechanical loads are considered for evaluating the coupled electro-mechanical dynamic behavior of smart composite plates with various piezoelectric materials like polyvinylidene fluoride (PVDF) and piezoelectric-fiber reinforced composites (PFRC). The PFRCs are recently developed piezoelectric materials in the literature with greater actuating capacity than the PVDF as the piezoelectric coefficient (e_{31}) coupling the in-plane normal stress and the electric field in the thickness direction is higher in PFRCs. The controlling capacity of the piezo patches is verified by deriving the electrical loads required to completely diminish the mechanical vibrations from the system. The Active Control analysis is also presented by deriving the suppressed free-vibration and forced vibration responses of smart composite plates by creating a control strategy using a negative feedback controller. Finally, the complex soil-structure interaction problem of traditional laminated composites and smart composite plate structures supported by the elastic foundation is presented in which both the static and the dynamic behavior of the plates are investigated.

4.3. Scope for the Future Research

In this section, some of the possible areas of research which can be carried out in the future are presented below:

- The present work may be extended to derive the deformation responses of advanced structures like smart FG and smart CNT-reinforced composite plates.
- Shell structures are more economical than those of plate structures due to their shape which helps to transfer the load through axial as well bending stiffness. The present work can be extended for determining the static and dynamic responses of traditional laminated composite and smart composite shells.
- Analytical solutions are considered as the best solutions of any problem and are often useful to test new numerical methods. However, the analytical solutions based on Navier’s scheme are restricted to simply supported boundary conditions.

Therefore, solutions for other boundary conditions are required to be obtained analytically.

- The environmental conditions like variation of temperature and moisture often degrade the strength of structures. The present work can be extended to study the static and dynamic deformation responses of structures subjected to hygro-thermo-mechanical loads.
- In many research works the hygro-thermal loads are considered to be linearly varying through the thickness of the plate structures. However, more realistic approach would be to solve the Fourier's law of heat conduction and the Fick's law of moisture concentration for deriving the exact variation of the hygro-thermal load through the thickness.
- The present mathematical model can be refined by including the effects of transverse normal stresses/strains. Also, the constant axial stretching mode in the present model can be improved by considering nonlinear stretching modes.
- The present formulation is developed in the framework of linear elasticity. The incorporation of material and geometrical non-linearity in the formulation is essential to study the non-linear effects on the deformation responses of traditional composites and smart composite plate structures.