DECLARATION BY THE CANDIDATE

I, **Aniket Chanda**, certify that the work embodied in this thesis is my own bonafide work and carried out by me under the supervision of **Dr. Rosalin Sahoo** from July 2018 to May 2021 at the Department of Civil Engineering, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, etc., reported in journals, books, magazines, reports dissertations, theses, etc., or available at websites and have not included them in this thesis and have not cited as my own work.

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Date: 12/05/2021 **Aniket Chanda**

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CERTIFICATE BY THE SUPERVISOR

It is certified that the above statement made by the student is correct to the best of my knowledge.

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Acknowledgements

I would like to express my deepest gratitude to my Ph.D. supervisor **Dr. Rosalin Sahoo,** Assistant Professor in the Department of Civil Engineering at Indian Institute of Technology BHU (Varanasi) for her invaluable guidance, encouragement, motivation, and support throughout this research work. Despite her busy schedule, she has always managed to devote her time to discussions, regularly monitoring my thesis progress, and in preparation of the manuscripts for publications. Her enthusiasm, sublime work ethic, and optimistic attitude towards research and life have inspired me a lot during my Ph.D. tenure.

I am also extremely thankful to my doctoral committee members, **Dr. Pabitra Ranjan Maiti**, Associate Professor in the Department of Civil Engineering, and **Dr. Nilanjan Mallik**, Associate Professor in the Department of Mechanical Engineering for their valuable suggestions, feedback, and critical assessment at various stages of my work. Their suggestions have helped me a lot to expand my research area and also to give a better shape to my thesis. My sincere gratitude also goes to the Head of the Department of Civil Engineering, Professor **Dr. Prabhat Kumar Singh Dikshit** for providing all the valuable resources that were required for the successful completion of my research work. It is truly an honor for me to thank all the faculty members of the **Department of Civil Engineering** at the Indian Institute of Technology BHU (Varanasi) for giving me an excellent opportunity to pursue my Doctoral degree.

As a special mention, I would also like to thank and express my gratitude towards Professor **Dr. Tarun Kant**, Institute Chair Professor of Civil Engineering at Indian Institute of Technology, Bombay, my M.Tech supervisor **Dr. Devesh Punera**, Assistant Professor in the School of Infrastructure at Indian Institute of Technology, Bhubaneswar and **Dr. Ashutosh Kainthola**, Assistant Professor at Department of Geology, Banaras Hindu University (BHU) for their immense help in carrying out my research and for keeping me motivated during my Ph.D. I am fortunate enough to meet them in person which also helped me to advance my research further.

I would also like to extend my heartfelt thanks to all my professors from my graduation and post-graduation years who had helped to pave the way for Doctoral research. I am also thankful to the **Ministry of Human Resource and Development (MHRD)**, Government of India for providing me financial support during my Ph.D. tenure at IIT BHU (Varanasi).

I feel deep appreciation and love for my father, **Shri. Chandra Sekhar Chanda**, and my mother, **Smt. Gopa Chanda**, who showed me how to recognize the values of life. Last but not the least, I would like to thank my seniors, friends, and juniors who supported me unconditionally during my doctoral research.

Abstract

The present study deals with the development of new analytical and finite element models for the modeling and analysis of traditional multi-layered laminated composites and smart composite plate structures supported by elastic foundations. Layers of piezoelectric materials are stacked on the laminated composite plates along the thickness direction for sensing and actuation of the responses of the smart structures. The plate deformations are modeled with an inter-laminar transverse shear stress continuous plate theory. The plate theory consists of a global non-polynomial shear deformation field and a local segmented zigzag field with changes in slopes of displacement components at each interface of the plates. The non-polynomial shear deformation field is a refinement of the classical plate theory that incorporates a trigonometric shear strain function to accurately model the non-linear variation of the transverse shear strains across the thickness of the plates. The zigzag field consists of piecewise linear mathematical functions that are defined for each layer. Additionally, some auxiliary variables are also defined at the interfaces between adjacent layers of different material properties to incorporate the slope discontinuities of the displacement components. The number of field variables of the present plate model is equal to that of the global non-polynomial shear deformation theory as the auxiliary variables can be expressed in terms of the field variables with the help of inter-laminar continuity conditions of transverse shear stresses at all the interfaces of the plates. The deformation of the elastic foundations is modeled with Pasternak's foundation model, which takes into account the proportional interaction between the load intensity and deflection of any point on the surface of the elastic foundation and also accommodates the continuity of the adjacent displacements by considering shear interactions among the points on the elastic foundation. The governing equations of motion and the boundary conditions of the problem are derived using Hamilton's principle and variational calculus. Two solution methods namely, generalized closed-form analytical solutions of Navier-type and numerical solutions in the framework of finite element method are adopted for solving the governing set of equations. The solution involves a spatial solution and a temporal solution of the primary variables. The spatial solutions of the primary variables in the analytical and finite element method reduce the governing partial differential equations to ordinary differential equations in time. The temporal solutions of the ordinary differential equations are further obtained with Newmark's constant average acceleration method. A detailed investigation of the static and dynamic responses of both traditional laminated composite plates and smart composite plates is carried out by considering different geometrical and material features of the plate structure. The uncontrolled and controlled static responses in the form of deflection and stresses are derived under the action of purely mechanical and electromechanical loads. The free vibration response in the form of natural frequencies and forced vibration response in the form of displacement-time responses under the action of several forms of time-dependent mechanical and electromechanical excitations are obtained. The vibration control of the smart composite plates is also carried out by coupling the laminated composite plate and the piezoelectric materials with a feedback controller.

Additionally, the effects of the elastic foundations on the static and dynamic responses of traditional laminated composites and smart composites are also investigated in detail. It is concluded from the results that both analytical and finite element solutions are capable of accurately predicting the responses of laminated composite plates and smart composite supported on elastic foundations.

***Keywords*:** Trigonometric zigzag theory; Analytical; Finite Element method; Laminated composites; Sandwich structure; Smart Composites; Elastic foundations; Static; Free vibration; Transient analysis; Navier's solution; Newmark's time integration.

Contents

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List of Symbols

x_1, x_2, x_3	Cartesian coordinate system
U_1, U_2, U_3	3D displacements in the global x_1, x_2 and x_3- direction
u_1, u_2, u_3	Mid-plane displacement components in the x_1, x_2 and x_3-direction
β_1, β_2	Rotation of the transverse normal to the mid-plane about x_2 and x_1-direction
$\alpha_{1u}^{i}, \alpha_{1l}^{j}, \alpha_{2u}^{i}, \alpha_{2l}^{j}$	Auxiliary variables defined at the interfaces of the multi-layered plate structures in the x_1 and x_2-direction
$\{\sigma\}$	Stress vector in the global coordinate system
$\{\varepsilon\}$	Strain vector in the global coordinate system
$[\bar{Q}]$	Transformed reduced stiffness matrix
$\{\bar{\sigma}\}$	Stress vector in the material coordinate axis
$\{\bar{\varepsilon}\}$	Strain vector in the material coordinate axis
$[Q]$	Reduced stiffness matrix
E_{11}, E_{22}	Young's Modulus in the longitudinal and transverse direction to the fiber direction
G_{12}	In-plane shear modulus
G_{13}, G_{23}	Transverse shear modulus
$\{\bar{E}\}$	Electric field vector in the material coordinate axis
$[\epsilon]$	Electric permittivity matrix in the material coordinate axis
$\{\bar{D}\}$	Electric displacement vector in the material coordinate axis
$\{E\}$	Electric field vector in the global coordinate axis
$[\bar{\epsilon}]$	Electric permittivity matrix in the global coordinate axis
$\{D\}$	Electric displacement vector in the global coordinate axis
k_w	Winkler stiffness of the foundation
k_s	Shear stiffness of the foundation

p_1, p_2, q_1, q_2	Mathematical functions of the thickness coordinate
U	Strain energy of the smart laminated composite plate
U_F	Strain energy of the elastic foundation
W	Work potential of the applied loads
K	Kinetic energy of the smart laminated composite plate
N_{11}, N_{12}, N_{22}	In-plane stress resultants
M_{11}, M_{12}, M_{22}	Moment stress resultants
N_{11}^*, N_{12}^*	Higher-Order In-plane stress resultants
M_{22}^*, M_{12}^*	Higher-Order moment stress resultants
Q_1, Q_2	Transverse shear stress resultants
T_1^*, T_2	Higher-Order transverse shear stress resultants
h	Overall thickness of the laminated composite plate
t_p	Thickness of the piezoelectric actuator and sensor
q	Mechanical pressure
$\bar{I}_0$, $\bar{I}_1$, $\bar{I}_2$, $\bar{I}_3$, $\bar{I}_4$, $\bar{I}_5$, $\bar{I}_6$, $\bar{I}_7$, $\bar{I}_8$	Inertia components
$[A]$, $[B]$, $[C]$, $[D]$, $[G]$, $[H]$, $[I]$, $[L]$, $[M]$, $[P]$ $[AA]$, $[EE]$, $[FF]$; $[SS]$, $[TT]$, $[UU]$	Rigidity submatrices relating the stress-resultants and derivatives of the primary variables
$\{A\}$, $\{C\}$, $\{E\}$, $\{G\}$, $\{L\}$, $\{N\}$, $\{P\}$	Forces generated due to piezoelectric coefficients
V	Electric voltage
NL	Number of layers in the smart laminated plate
Φ	Electric potential
$[\bar{K}]$	Stiffness matrix of the smart composite plate
$\{\Delta\}$	Displacement vector
$\{\bar{F}_M\}$	Mechanical force vector
$\{\bar{F}_E\}$	Electrical force vector

ω	Natural frequency of the plate
$[\bar{M}]$	Mass matrix
$[\overline{N_\iota}]$	Shape function matrix
ξ, η	Natural coordinate system used in finite element
$\{d^e\}$	Nodal coordinates
$\rho^{(s)}$, $\rho^{(k)}$, $\rho^{(a)}$	Density of piezoelectric sensor, laminated composite plate and actuator, respectively
$[H]$	Matrix relating the strains and derivatives of the primary variables
$[B]$	Matrix relating the derivatives of the primary variables and the nodal coordinates
$\{V(t)^e\}$	Elemental electric force vector
$\{\dot{d}^e\}$	Elemental velocity force vector
P_e	Penalty function
$[K^{(s)}]$	Elemental stiffness matrix of sensor
$[K]$	Elemental stiffness matrix of laminated plate
$[K^{(a)}]$	Elemental stiffness matrix of actuator
$[K_{ds}]$	Piezoelectric coupling matrix of the sensor
$[K_{da}]$	Piezoelectric coupling matrix of the actuator
$[K_{pe}]$	Elemental penalty stiffness matrix
$[K^{(F)}]$	Elemental stiffness matrix of the foundation
$\{F_M\}$	Elemental force vector
$[M]$	Elemental mass matrix
L	Lagrangian
$\{\bar{\mathbf{F}}_M\}$	Global mechanical force vector
$[\bar{\mathbf{M}}]$	Global mass matrix
$[\bar{\mathbf{K}}^{(s)}]$	Global stiffness matrix of the sensor
$[\bar{\mathbf{K}}]$	Global stiffness matrix of the laminated composite plate

$[\bar{\mathbf{K}}^{(a)}]$	Global stiffness matrix of the actuator
$[\bar{\mathbf{K}}^{(F)}]$	Global stiffness matrix of the foundation
$[\bar{\mathbf{K}}^{(pe)}]$	Global stiffness matrix containing the penalty terms
G_c	Constant gain of the amplifier
G	Gain of the amplifier
$[C_R]$	Rayleigh damping matrix
$[\boldsymbol{C_{cnt}}]$	Active damping matrix

Abbreviations

3 D	Three Dimensional
2 D	Two Dimensional
CLPT	Classical Laminated Plate Theory
CPT	Classical Plate Theory
FSDT	First Order Shear Deformation Theory
HSDT	Higher-Order Shear Deformation Theory
FRP	Fiber Reinforced Polymers
FGM	Functionally Graded Material
PZT	Lead Zirconate Titanate
PVDF	Polyvinylidene Fluoride
PFRC	Piezoelectric Fiber-Reinforced Composites
ESL	Equivalent Single Layer
LW	Layerwise
ZZ	Zigzag
PHSDT	Polynomial Higher-Order Shear Deformation Theory
NPHSDT	Non-Polynomial Higher-Order Shear Deformation Theory
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
FEM	Finite Element Method
SBFEM	Scaled Boundary Finite Element Method
IGA	Isogeometric Analysis
XFEM	Extended Finite Element Method
FDM	Finite Difference Method
DSC	Discrete Singular Convolution
DQM	Differential Quadrature Method

CUF	Carrera Unified Formulation
EE	Equilibrium Equations
EKM	Extended Kantorovich method
SCF	Shear-Correction Factor
RHZZT	Refined Higher-Order Zigzag Theory
RFSDT	Refined First Order Shear Deformation Theory
FN	Fundamental Nucleus
BEM	Boundary Element Method
TZZT	Trigonometric Zigzag Theory
AVC	Active Vibration Control
UDL	Uniformly Distributed Load
SSL	Sinusoidal Load
MM	Material Model
ND	Non-Dimensional Parameter