Chapter 7

Summary and Future scope

7.1 Overall Summary

The work in the thesis is focused on the study of analytical and numerical solution of the specific problems, formulated mathematically as IVPs/BVPs, associated with quasilinear hyperbolic system of partial differential equations. It also concerns with the problems associated with the classical and non-classical solutions of the Riemann Problem of one-dimensional homogeneous and non-homogeneous quasilinear hyperbolic system. The major contribution of this thesis includes:

- To solve the problem for non-homogeneous hyperbolic system which is modified into homogeneous hyperbolic system of conservation laws to study the solution of Riemann Problem with constant initial data by introducing new variable for the velocity.

- To study the rarefaction wave solution of the Riemann Problem for non-homogeneous hyperbolic system with non-constant Riemann initial data through Differential constraint method.

- To study the phenomena of concentration and cavitation in the solution of the Riemann Problem for quasi-linear hyperbolic system describing the one-dimensional unsteady flow of an ideal polytropic dusty gas.

- To study delta shock wave for a hyperbolic system of conservation laws whose governing system occurs in gasdynamics.

- Effects of small solid dust particles on the evolutionary behavior of non-linear wave for several geometry of flows in reacting gas.

The first chapter is introductory and gives a general idea of when and how a discontinuity appears. Certain terminologies commonly used in the current work have been defined. The mathematical theory and their fundamental properties have also been briefly discussed. The physical properties of hyperbolic systems, equation of state, dusty gas, reacting gas and methods which are used throughout the thesis are briefly reviewed.

The second chapter concerns with the study of the exact solution of the generalized Riemann Problem for the 2×2 hyperbolic $p-$system with linear damping by using Differential constraint method. This method is used to derive the consistency conditions and constraint equations for the considered non-homogeneous hyperbolic system and obtained the exact solution of the system of governing equations. Further, the generalized Riemann Problem of non-homogeneous hyperbolic $p-$system which involves non-constant discontinuous initial data is solved.

In the third chapter, the structure of the Riemann solutions for one-dimensional compressible hyperbolic system of PDEs with logarithmic equation of state, so called Logotropic model, in the presence of a Coulomb-type friction is analyzed. The system considered in this chapter is hyperbolic in nature and the characteristic fields associated with the characteristics are genuinely non-linear. The classical wave solutions of the Riemann Problem for the Logotropic model are structured explicitly for all cases. It is shown that the Riemann solutions for the Logotropic model with a Coulomb-type friction term composed of the rarefaction wave and shock wave. Also, it is found that the Coulomb-type friction term, appearing in the governing equations, influences the Riemann solution for the system.

In the fourth chapter, the solution of the Riemann Problem for hyperbolic system of PDEs with modified Chaplygin gas (MCG) equation of state in the presence of constant external force is studied. The analysis leads to the fact that in some special circumstances delta shock appears in the solution of the Riemann Problem. Further, the Rankine-Hugoniot relations for delta shock wave which are utilized to determine the strength, position and propagation speed of the delta shocks have been derived. Delta shock wave solution to the Riemann Problem for the non-homogeneous hyperbolic system with modified Chaplygin gas equation of state is obtained. It is found that the external force term, appearing in the governing equations, influences the Riemann solution for the system.

In the fifth chapter, the concentration and cavitation phenomenon in the solution of the Riemann Problem to the pressure-less isentropic Euler equations for the dusty gas model is investigated by using two parameter flux approximation. The similar solution of the Riemann Problem for dusty gas model is obtained. The formation of delta - shock and vacuum state in the flow field is discussed. Also, it is shown that the solution, containing two shock waves, of the Riemann Problem to the isentropic Euler equations for dusty gas converges to the delta - shock wave solution of the transport equations and the solution, containing two rarefaction waves, of the Riemann Problem converges to the vacuum state solution of the transport equations.

In the sixth chapter, the evolutionary process of shock wave along the characteristic path under the effect of dust particles in a polytropic reacting gasdynamics is investigated. Using the characteristics of the governing quasilinear hyperbolic system as a reference coordinate system, we transform the governing equations and obtain the solution of it. It is shown that a linear solution in the characteristic plane can exhibit a non-linear behaviour in the physical plane. It is shown how the reacting gas parameter influences the evolutionary process of the compressive and expansive waves, respectively, in reacting gas flows. Also, it is shown how the presence of dust in reacting gas affects the growth and decay of the compressive and expansive waves. The transport equation leading to the evolution of shock wave is determined which provides the relations for the shock formation. The comparative study of the effect of reacting gas parameter and dust particles on the flow patterns and distortion of shock wave for planar, cylindrically symmetric and spherically symmetric flows is also performed.

7.2 Future Scope

There are some interesting and challenging research Problems which can be studied in future. This section provides future work to consolidate the study presented in this thesis. Our study is restricted to a one-dimensional system of non-linear partial differential equations in gas dynamics. However, this analysis can be extended for two or higher dimensional non-linear partial differential equations in gas dynamics. The key areas that can be focused for future research are identified here. We can highlight some of the proposed extensions of the work made in the thesis as follows:

- In this thesis, we focused only on the one-dimensional Riemann Problems. One may study two-dimensional Riemann Problems admitting delta shock waves and nonlinear wave interactions for hyperbolic system of conservation laws which involves much more analysis and challenging task.

- We studied the structure of the Riemann solution for non-homogeneous hyperbolic system. It would be more interesting to further investigate the formation of delta shock and vacuum state solutions to the corresponding problem. So this will be a very potential work in the field of hyperbolic system of conservation laws.

- We studied the classical elementary waves (shock waves, rarefaction waves and contact discontinuities) and delta shock wave for hyperbolic system. One can extend these ideas to discuss the interactions of delta shock wave with weak discontinuities which may lead to develop interesting concept, which is an open problem.

- The closed form solution of the Generalized Riemann Problem for the higher dimensional Euler's equation for various gas dynamic regimes by using the Differential Constraint Method. On the behalf of applications of Differential Constraint Method, one can use this method to determine the exact solution to the Generalized Riemann Problem for two-phase flow in gasdynamics.

- Numerical study of the Riemann Problem in higher dimensional Euler's equation for various gas dynamic regimes using different types of Riemann solvers.

- To analyze the evolutionary behavior of non-linear wave propagating in higher-dimensional flow in different gaseous media by using the method of asymptotic analysis, wavefront analysis.

- To derive the high frequency small amplitude asymptotic solution of the two-dimensional quasilinear hyperbolic system of partial differential equations characterizing compressible, unsteady flow with generalized geometry in reacting gas using the theory of weakly non-linear geometrical acoustics .

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