Chapter 6

The propagation of shock wave in planar and non-planar polytropic reacting gas with dust particles *

"Without mathematics, there's nothing you can do.
Everything around you is mathematics.
Everything around you is numbers."

– Shakuntala Devi.

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6.1 Introduction

In the nonlinear system, the wave is considered to be moving surface along which the flow variables and their derivatives occupy certain kind of discontinuity that are carried along by the surface. The occurrence of these type of discontinuities represented by mathematical system of quasilinear PDEs are natural phenomenon in several physical situations like collision of galaxies, space science, supernova explosion, space re-entry vehicles, photo ionized and other astrophysical situations. The study of shock waves are of great importance from the point of view of both fundamental research and practical applications. Since, the shock waves arise due to the deposition of large amount of energy in very small region over short intervals as in case of spark discharges in air, explosions. The shock waves in fluids received considerable attention because they are a particular class of discontinuous wave processes, which are studied by some analytical and numerical methods. The process of shock formation and decay in the characteristic plane is one of the most critical aspects of non-linear wave theory in gasdynamics. The problem of the study of flow patterns of nonlinear waves in different material mediums has gained significant attention over a recent couple of decades due to their advantage in numerous fields such as space science, nuclear science, and engineering sciences. The shock waves are discontinuities that occur in the derivative of the solution along the characteristics however, the solution itself remains continuous. The study of shock wave in two phase flow have also significant role due to its applications to coal mines blast, volcanic and cosmic explosions, underground, metalized propellant, nozzle flow, lunar ash flow, supersonic flight in polluted air and many engineering science problems (See [2], [113], [114],[115],[116],[117],[118]). Furthermore, in several astrophysical events, the mixture of gases play a decisive role. This mixture consists of an ideal gas containing dust particles, so called dusty gas. In dusty gas, the volume of the small dust particles does not occupy more than five percent of the mixture's total volume. Many researchers worked in gas dynamics, where they examined the particle's effect on the propagating waves in various medium of flow. Miura [2, 116] have examined the wave propagation and its behaviour through dusty gas layer. Also, he described the condition for the separation of pure gas from a mixture of gas with dust particles. Higashino [115] have investigated the problem of blast waves in a dusty medium and discussed that as to how the decay of blast waves is affected in dusty gas. In recent, Pai [3], Steiner and Hirschler [118], Vishwakarma and Nath [127], Chadha et al. [128], Nath [129], Chaturvedi et al. [112, 130] and Srivastava et al. [131] have studied the solution of the shock wave propagation in dusty gas by several approaches. Mehla et al. [132] have further discussed the wave propagation in relaxing gases with solid particles. The dusty gas flow has drawn the focus of the many authors due to its advantages in industrial and environmental fields. Several researchers have generalized the theory of nonlinear waves in various material media, such as ideal gas dynamics, magnetogasdynamics, and established the conditions for shock formation and its distortion in the medium. Ram [133], Shankar [134], Keller [135] and Chaturvedi et al. [136] studied the several effects of the propagation of shock wave and obtained the conditions for examining the evolutionary behaviour of the acceleration wave.

The study of the mixture of two species of gases has a large number of diverse phenomenon due to which, many authors has inspired to investigate the theoretical and numerical problems of the formulation of an adequate mathematical model. Teng et al. [137] proposed the first rational model of mixture of unburnt and burnt gases for the Riemann problem and later on, Torrisi [138, 139], Goppi et al. [140], Barenblatt et al. [141], Harle et al. [142], Logan and Bdzil [143], Godlewski [18] have studied the theoretical and numerical problem related to the propagation of nonlinear waves in reacting gases. Since past two decades, the problem related to the propagation of shock waves and its interaction in reacting gases has received attention of many researchers like Lung et al. [144], Medvedev [145], Singh et al. [146, 147], Shah and Singh [148].

In the present chapter, we have studied the system of quasilinear PDEs describing the unsteady planar and non-planar flow of inviscid polytropic reacting gas with dust particles. The governing equations consist of two species (the burnt gas and the unburnt gas) in which the burnt gas has mass fraction z and unburnt gas has mass fraction $(1 - z)$. Both gases are ideal gases. Also, we consider that the dust particles presented in reacting gas are solid, uniform in size and occupy less that 5 percent of total volume. In this study, we investigate the evolutionary process of shock wave and derive the amplitude of the shock wave propagating along characteristic. Further, the effect of reacting gas parameter and the dust particles on the evolution of shock wave in planar, cylindrically symmetric and spherically symmetric flows is shown.

The complete structure of this chapter is summarized into some sections: in the second section, we derive the governing equation for reacting polytropic gas flow with dust particles and determine the characteristic curves that represent the propagation of the waves. In the next section, we introduce characteristic variables and change the fundamental equations in terms of these new variables. In the fourth section, we derive a differential equation and its solution that investigates the process of shock formation. The behaviour of the solution obtained in the fourth section is discussed in the fifth section. Also, the fifth section refers to the interpretation of various parameter effects on the shock formation process and its deformation. The last section contains the conclusions of this study.

6.2 Governing equations and its characteristics

In this study, the system of governing equations describing the planar, cylindrically symmetric and spherically symmetric flows of polytropic reacting gas with dust particles is considered with the following assumptions: The fluid consists a single irreversible exothermic reaction between two species of ideal gas, the burnt gas and the unburnt gas, with constant specific heat. The dust particles present in the reacting gas are solid, spherical and uniform in size. These small solid dust particles do not occupy more than 5 percent of the volume. Also, the mass transfer between two phases, the effect of boundary layer on duct walls and heat transfer are ignored in this study. The EoS for such system of equations can be written in the form [18, 148]

$$e = e(\rho, p, z), \tag{6.1}$$

where e, ρ, p and z are internal energy, density, pressure and the mass fraction of burnt gas. So that $(1 - z)$ is mass fraction of unburnt gas. The internal energy for both gases (burnt and unburnt gases) are defined as

$$e = ze_b(\rho, p, z) + (1 - z)e_u(\rho, p, z), \tag{6.2}$$

where $e_b(\rho, p, z) = \frac{(1-V)p}{\rho(\Gamma-1)} + Q_b$ and $e_u(\rho, p, z) = \frac{(1-V)p}{\rho(\Gamma-1)} + Q_u$, here Q_b and Q_u are the energies of formation of burnt and unburnt gases, respectively. The parameter Γ is called Grüneisen coefficient, defined by

$$\Gamma = \frac{\gamma(1 + \lambda\beta)}{(1 + \lambda\beta\gamma)}, \lambda = \frac{k_p}{(1 - k_p)}$$

where $\gamma = c_p/c_v$, $\beta = c_{sp}/c_p$ and $k_p = m_{sp}/m_g$, called the mass fraction of the solid particles, where m_{sp} is the mass of the solid particles and and m_g denotes the total mass of the mixture. The parameter c_{sp} is the specific heat of dust particles, c_p represents the specific heat of the gas at constant pressure and c_v is the specific heat of the gas at constant volume, . Here, the parameters Z and k_p are related by $Z = \theta\rho$, $\theta = k_p/\rho_{sp}$, where ρ_{sp} is the specific density of the dust particles.
The governing equations describing the one-dimensional unsteady polytropic reacting gas with dust particles for planar ($m = 0$), cylindrically symmetric ($m = 1$) and spherically symmetric ($m = 2$) flows may be written as [148]

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} + \frac{m\rho u}{x} = 0,\\
&\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} = 0,\\
&\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} - \rho C^2\left(\frac{\partial u}{\partial x} + \frac{mu}{x}\right) = \frac{(\Gamma - 1)\rho}{(1-\theta\rho)}\left(Q_u - Q_b\right) r(\rho, p, z),\\
&\frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} = r(\rho, p, z),
\end{aligned} \tag{6.3}$$

where ρ denotes the density, p is the pressure, t is the time and u represents the velocity in $x-$direction. C is the speed of sound in reacting gas with dust particles defined as

$$C^2 = \frac{\Gamma p}{(1-Z)\rho}, \tag{6.4}$$

where $Z = V_{sp}/V_g$ denotes the volume fraction of the dust particles. The volume of the dust particles is denoted by V_{sp} and V_g denotes the entire volume of the mixture. When $\theta = 0$, then Γ becomes γ, $C^2 = \frac{\gamma p}{\rho}$ i.e. the flow becomes ideal reacting gas flow.

Equation (6.3) can be written in the matrix form as

$$U_t + AU_x + B = 0, \tag{6.5}$$

here U, B and A are the column vectors of order 4×1 and matrix of order 4×4, respectively, given below

$$U = \begin{pmatrix} \rho \\ u \\ p \\ z \end{pmatrix}, \quad B = \begin{pmatrix} \frac{m\rho u}{x} \\ 0 \\ \frac{m\rho C^2 u}{x} - \frac{(\Gamma-1)(Q_u - Q_b)\rho r}{(1-Z)} \\ -r \end{pmatrix}, \text{and} \quad A = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 1/\rho & 0 \\ 0 & \rho C^2 & u & 0 \\ 0 & 0 & 0 & u \end{pmatrix}, \tag{6.6}$$

where $U(x,t)$ is a continuous function which satisfy equation (6.3) everywhere in the characteristic plane except at the characteristic curve $S(t)$. But its first derivatives U_t and U_x may suffer finite jump across the characteristic curve $S(t)$. This type of discontinuity is called weak discontinuity. Now, along the characteristic curve $S(t)$, we have (see [133, 149])

$$\frac{\partial}{\partial t}[U] = [U_t] + \frac{dS(t)}{dt}[U_x], \tag{6.7}$$

where $\frac{\partial}{\partial t}$ denotes the time-partial derivative across the characteristic curve. Since U is a continuous function, therefore $[U] = 0$ and $[B] = 0$. Taking jump in (6.3) and using equation (6.7) with the conditions $[U] = 0$ and $[B] = 0$, we obtain

$$\left(A - \frac{dS}{dt}I\right)[U_x] = 0, \tag{6.8}$$

where I denotes the identity matrix of order 4×4. The Eq. (6.8) shows that if there occur finite discontinuities of acceleration along the characteristic curve $S(t)$, the characteristic speed of propagation $\frac{dS}{dt}$ is an eigenvalue of A. It follows immediately that there are four families of characteristic curves corresponding to the system (6.5), two of which are given by

$$\frac{dx}{dt} = u \pm C, \tag{6.9}$$

where the characteristic curve $u+C$ represents the wave propagation in the positive x-direction, and $u-C$ represents the wave propagation in negative x-direction. The other two of the characteristic curves

$$\frac{dx}{dt} = u, \tag{6.10}$$

represent the particle path.

6.3 Shock waves in characteristic plane

We define two characteristic variables ζ and ψ, where ζ is the particle tag and ψ is the wave tag such that ζ is constant along the particle path $\frac{dx}{dt} = u$, and the wave tag ψ is constant along the characteristic $\frac{dx}{dt} = u+C$. Therefore, if the characteristic wavefront moves through a particle at time t^*, then the particle tag ζ can be marked as $\zeta = t^*$, and if the piston generates outgoing wave at time t', then we can label the wave tag ψ as $\psi = t'$.

Hence, corresponding to each pair (ψ, ζ), we can find a pair (x, t) such that $x = x(\psi, \zeta)$, $t = t(\psi, \zeta)$. Now, the characteristic variables ψ and ζ satisfy the following conditions

$$x_\psi = u t_\psi, \qquad x_\zeta = (u + C) t_\zeta. \tag{6.11}$$

Under the above transformation (6.11), U_t and U_x transformed into the following form

$$U_t = \frac{U_\zeta x_\psi - U_\psi x_\zeta}{J}, \qquad U_x = \frac{U_\psi t_\zeta - U_\zeta t_\psi}{J}, \tag{6.12}$$

where $J = \dfrac{\partial(x,t)}{\partial(\psi,\zeta)} = -Ct_\psi t_\zeta$, is the Jacobian of transformation.
By using equation (6.12), system (6.3) can be written as

$$C\rho_\psi t_\zeta - \rho\left(u_\psi t_\zeta - u_\zeta t_\psi - \frac{muCt_\psi t_\zeta}{x}\right) = 0, \tag{6.13}$$

$$C\rho u_\psi t_\zeta - p_\psi t_\zeta + p_\zeta t_\psi = 0, \tag{6.14}$$

$$Cp_\psi t_\zeta - \rho C^2\left(u_\psi t_\zeta - u_\zeta t_\psi - \frac{muCt_\psi t_\zeta}{x}\right) = -\frac{(\Gamma-1)\rho}{(1-\theta\rho)}\left(Q_u - Q_b\right) r(\rho,p,z)Ct_\psi t_\zeta, \tag{6.15}$$

$$z_\psi = -t_\psi r(\rho,p,z). \tag{6.16}$$

Using eqs. (6.14 − 6.16) in (6.13), we obtain

$$p_\zeta + \rho C u_\zeta + \frac{m\rho C^2 u t_\zeta}{x} = -\frac{(\Gamma-1)\rho}{(1-\theta\rho)}\left(Q_u - Q_b\right) r(\rho,p,z)t_\zeta. \tag{6.17}$$

The sub-scripts ψ and ζ represents the partial derivative with respect to ψ and ζ, respectively.
The interface conditions at the wave front at $\psi = 0$ are

$$[p] = 0, \;\; [\rho] = 0, \;\; [u] = 0, \;\; [z] = 0, \;\; t = \zeta. \tag{6.18}$$

Since the gas flow ahead of the wavefront is uniform and at rest, we observe that

(6.18) demands that

$$\rho_\zeta = 0,\ p_\zeta = 0,\ u_\zeta = 0,\ z_\zeta = 0 \text{ and } t_\zeta = 1 \quad \text{at } \psi = 0. \tag{6.19}$$

On insertion of (6.18) and (6.19) in (6.16), (6.14) and (6.11), and evaluating at $\psi = 0$, yields

$$\rho_\psi = \left(\frac{\rho_0}{C_0}\right) u_\psi, \tag{6.20}$$

$$p_\psi = \rho_0 C_0 u_\psi, \tag{6.21}$$

$$x_\psi = 0, \quad x_\zeta = C_0. \tag{6.22}$$

Here the flow variables evaluated at the front of the shock are signified by the subscript '0'. Using (6.19) in (6.12), we get

$$\left[\frac{\partial u}{\partial x}\right] = Y = -\frac{u_\psi}{C_0 t_\psi}, \quad \text{at } \psi = 0, \tag{6.23}$$

where Y denotes the amplitude of the shock wave at $\psi = 0$.

6.4 Amplitude of the disturbance

In this section, the basic equations are transformed from physical plane to characteristic plane. The transformation of variables from (x, t) to (ζ, ψ) has been defined by Eq. (6.11) and Eq. (6.12), and then a transport equation governing the evolution of the amplitude of the disturbance has been obtained and given by Eq. (6.26). When a small perturbation is given to the disturbance at rest, a wave starts moving along the characteristics across which the flow variables velocity, density and pressure are uniform, but their derivatives suffer finite jump discontinuities. These discontinuities are called acceleration wave. Now, we shall derive the transport equation governing the evolution of weak discontinuities which move along the initial wavefront. We determine the relations for the dependence of u_ψ and t_ψ on time, and with the help of these relations, we find the solution to the problem. By taking derivative of equation (6.11) and (6.17) with respect to ψ and combining with ζ derivative of (6.21) , at $\psi = 0$, we get

$$\frac{t_{\psi\zeta}}{t_\psi} = \frac{\Gamma + 1}{2(1 + Z_0)} Y, \tag{6.24}$$

$$\frac{u_{\psi\zeta}}{t_\psi} = \left(\frac{\Gamma - 1}{2(1 - Z_0)^2 C_0} (Q_u - Q_b) \left(\frac{1 - \Gamma}{2} r_0 + (r_{\rho_0} + C_0^2 r_{p0}) \rho_0 (1 - Z_0) \right) + \frac{m C_0^2}{2\zeta} \right) Y, \tag{6.25}$$

where $Z_0 = \theta \rho_0$.

On taking derivative of equation (6.23) with respect to ζ and using equations (6.24) and (6.25), we get

$$\begin{aligned} \frac{dY}{d\zeta} + &\left(\frac{\Gamma - 1}{2(1 - Z_0)^2 C_0^2} (Q_u - Q_b) \left(\frac{1 - \Gamma}{2} r_0 + (r_{\rho_0} + C_0^2 r_{p0}) \rho_0 (1 - Z_0) \right) + \frac{m C_0}{2\zeta} \right) Y \\ &+ \left(\frac{\Gamma + 1}{2(1 + Z_0)} \right) Y^2 = 0, \end{aligned} \tag{6.26}$$

at $\quad \psi = 0$.

Now, we introduce some non-dimensional quantities given as

$$\eta = \frac{Y}{Y^*}, \quad \alpha = \frac{\zeta - \zeta^*}{2\zeta^*} \ \text{and} \ \delta = Y^* \zeta^*, \tag{6.27}$$

where δ, η and α are the initial disturbance, wave amplitude and time respectively. The superscript '*'has been used to indicate the value of the parameters at $t = t^*$. In view of (6.27), equation (6.26) can be reduced in the following dimensionless form

$$\frac{d\eta}{d\alpha} + \left(\frac{\Theta}{(1 - Z_0)} + \frac{m}{2\alpha + 1} \right) \eta + \frac{\delta(\Gamma + 1)}{(1 - Z_0)} \eta^2 = 0, \quad \text{at} \ \psi = 0. \tag{6.28}$$

where $\Theta = \frac{(\Gamma-1)(Q_u-Q_b)\rho_0}{C_0^2}\left(\frac{(1-\Gamma)r_0}{(1-Z_0)\rho_0} + (r_{\rho_0} + C_0^2 r_{p_0})\right)\zeta^*$.

Equation (6.28) is a Bernoulli type differential equation with η as a dependent variable and α is an independent variable. The solution of (6.28) is given as

$$\eta = \left\{(2\alpha+1)^{\frac{m}{2}} e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha}\left(1 + \left(\frac{(\Gamma+1)\delta}{(1-Z_0)} I(\alpha)\right)\right)\right\}^{-1}, \qquad (6.29)$$

where $I(\alpha) = \int_0^\alpha \frac{-e^{\left(\frac{\Theta}{(1-Z_0)}\right)s}}{(2s+1)^{\frac{m}{2}}} ds$.

From equations (6.23) and (6.29), it is clear that, for the shock formation we must have $t_\psi = 0$, i.e.

$$1 + \left(\frac{(\Gamma+1)\delta}{(1-Z_0)} I(\alpha)\right) = 0. \qquad (6.30)$$

Equation (6.30) indicates that the compressive waves ($\delta < 0$) terminate in to the shock wave.

6.5 Results and discussion

Now, we investigate the behavior of the solution obtained in the previous section for both planar ($m = 0$) and cylindrically symmetric ($m = 1$) case and discuss the possibilities of formation and distortion of the shock. Consider the following cases to analyze the evolution of shock waves under the effect of various parameters.

Case I. Planar flow ($m = 0$):

By substituting $m = 0$ in (6.29), we get

$$\eta = \left\{e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha} + \frac{\delta(\Gamma+1)}{\Theta}\left(e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha} - 1\right)\right\}^{-1}. \qquad (6.31)$$

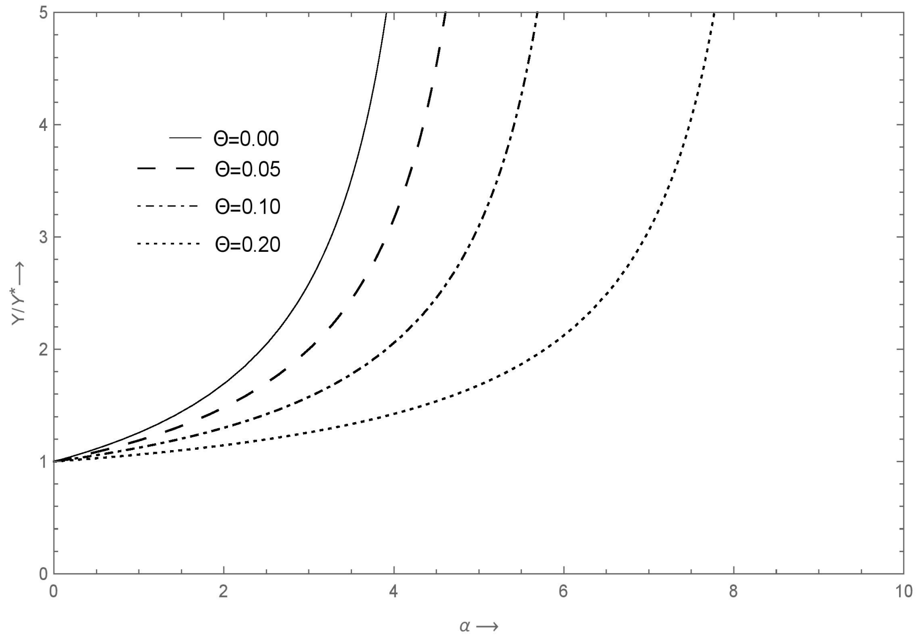

FIGURE 6.1: Variation in compressive wave for planar case with $Z_0 = 0.001, \gamma = 1.4, \delta = -0.1, \beta = 0.5$ and $k_p = 0.2$..

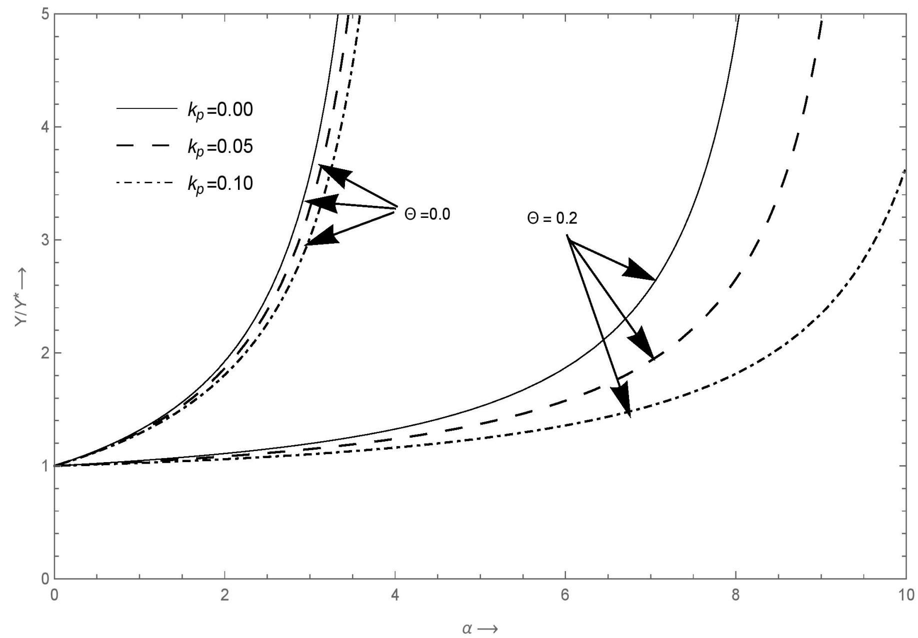

FIGURE 6.2: Variation in compressive wave in reacting and non-reacting gas for planar case with $Z_0 = 0.001, \gamma = 1.4, \delta = -0.1, \beta = 0.5$.

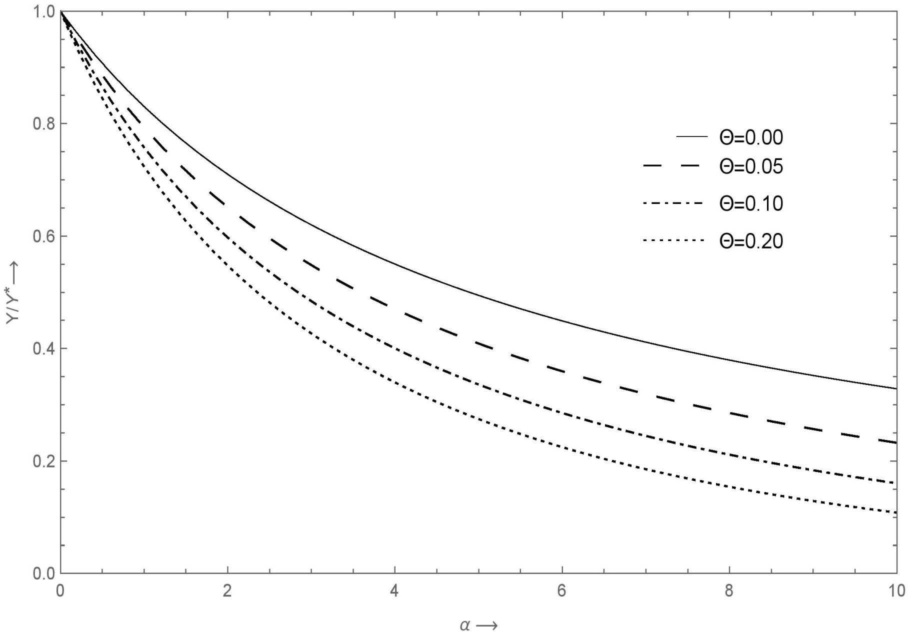

FIGURE 6.3: Variation in expansive wave for planar case with $Z_0 = 0.001, \gamma = 1.4, \delta = 0.1, \beta = 0.5$ and $k_p = 0.2$...

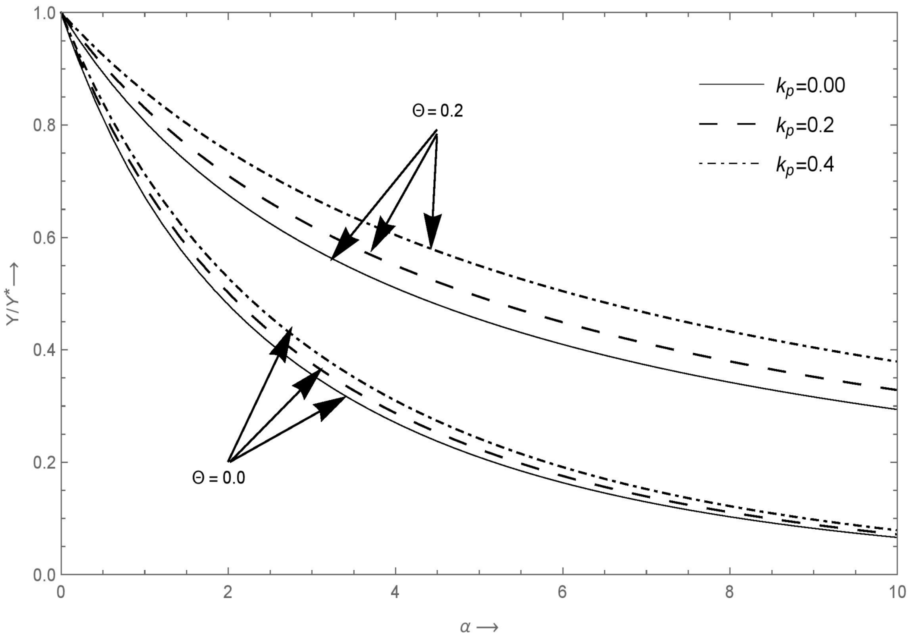

FIGURE 6.4: Variation in expansive wave in reacting and non-reacting gas for planar case with $Z_0 = 0.001, \gamma = 1.4, \delta = 0.1, \beta = 0.5$.

Using equation (6.27), yields

$$Y = \frac{Y^*}{e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha} + \frac{\delta(\Gamma+1)}{\Theta}\left(e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha} - 1\right)}. \tag{6.32}$$

From equation (6.32), it can be seen that the amplitude of nonlinear waves (known as expansive wave for ($\delta > 0$)) decays and eventually vanishes. Also, the curves representing (6.32) shows that the decay time for the expansive waves is increased due to an added effect of mass fraction of the dust particles presented in reacting gas. Here, $\delta > 0$ represents the expansive waves and $\delta < 0$ represents the compressive waves.

The curves presented in Figure 6.1 shows the growth process for $\delta < 0$ in planar ideal reacting gas flow with dust particles. It is also obtained that the steepening of the propagating waves for $\delta < 0$ is slowed down in the presence of reacting gas parameter and an increase in the value of reacting gas parameter causes to delay in shock formation, where $\Theta = 0$ corresponds to the non- reacting gas. Also, Figure 6.2 depicts that if we increase the value of mass fraction k_p, then it causes to slow down the growth of propagating waves for $\delta < 0$ in both medium planar ideal reacting gas and planar ideal non-reacting gas. In Figure 6.2, the behaviour of solution curves is presented for reacting gas and non-reacting gas for planar case with different values of k_p. Thus, the growth rate of a compressive wave in a dusty gas is lower than that of a dust-free gas ($k_p = 0$). Similarly, the variation in propagating waves for $\delta > 0$ in planar flow of reacting and non-reacting gases with dust particles is shown by Figure 6.3 and Figure 6.4. From Figure 6.3, it is clear that an increase in the value of reacting gas parameter Θ causes to increase the decay rate of expansive wave in the presence of dust particles. Also, it is observed that the decay rate of expansive wave is very slow in reacting gas as compared to non-reacting gas which is shown in

Figure 6.4.

Case II. Cylindrically symmetric flow $(m = 1)$**:**

For cylindrically symmetric flow, the solution of (6.28) is given as

$$Y = \frac{Y^*}{\sqrt{(2\alpha+1)}e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha}\left(1+\left(\frac{(\Gamma+1)\delta}{(1-Z_0)}I(\alpha)\right)\right)}, \tag{6.33}$$

where $I(\alpha) = \int_0^\alpha \frac{-e^{\left(\frac{\Theta}{(1-Z_0)}\right)s}}{\sqrt{(2\alpha+1)}}ds$.

In cylindrically symmetric flow case, it is obtained that the propagating wave for $\delta < 0$ and $\delta > 0$ have similar phenomenon to the planar case in reacting gas flow with dust particles which can be seen by the solution curves depicted in Figs. 6.5, 6.6. The effect of reacting gas parameter in dusty and dust free gases on the compressive wave and expansive wave is shown in Figure 6.5 and Figure 6.6, respectively. It is obtained that the presence of dust particles in reacting gas causes to delay the shock formation for cylindrically symmetric flow (see Figure 6.5). Also, the presence of

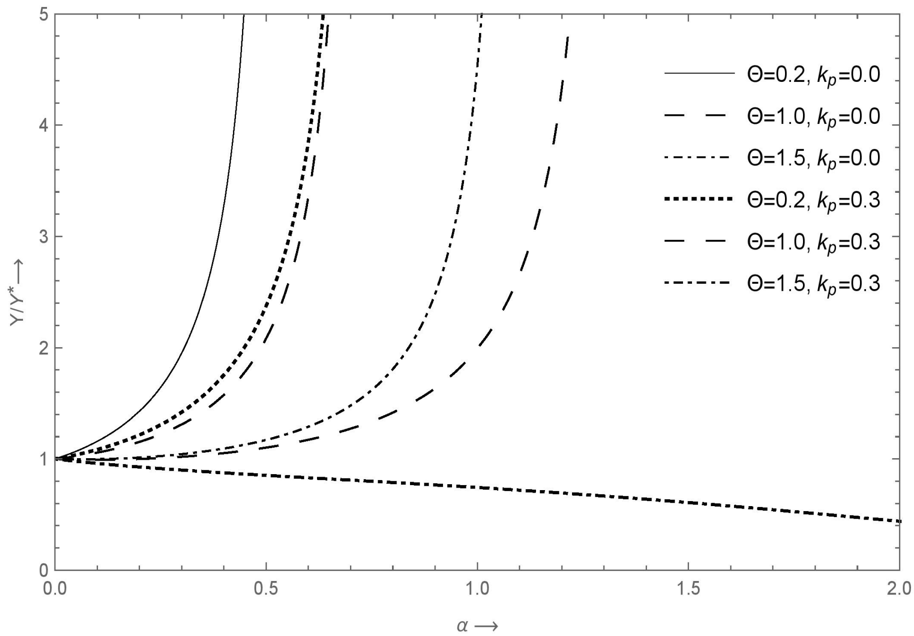

FIGURE 6.5: Variation in compressive wave in reacting gas for cylindrically symmetric case with with $Z_0 = 0.001, \gamma = 1.4, \delta = -1, \beta = 0.5$.

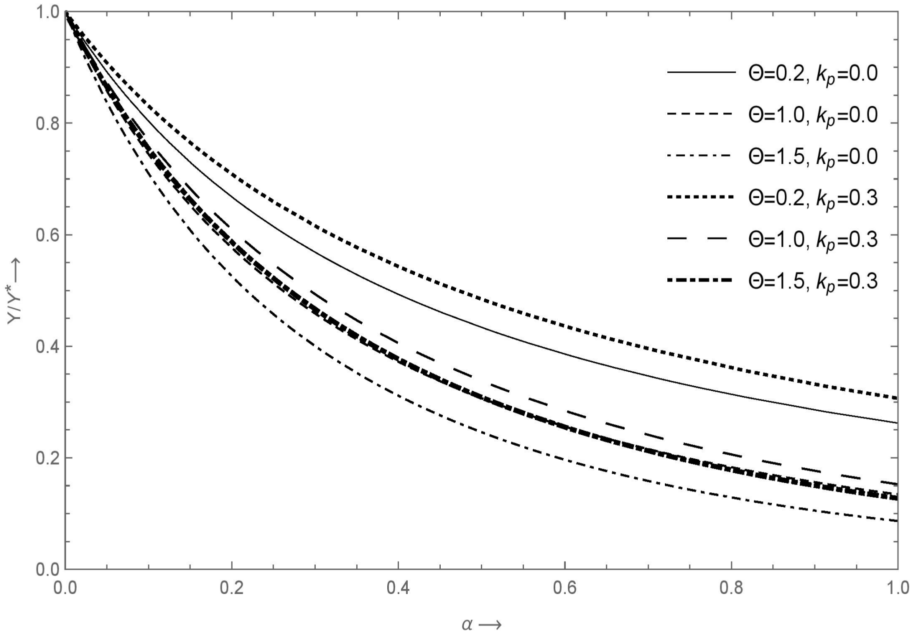

FIGURE 6.6: Variation in expansive wave in reacting gas for cylindrically symmetric case with $Z_0 = 0.001, \gamma = 1.4, \delta = 0.1, \beta = 0.5$.

the dust particles in reacting gas accelerates the decay rate of expansive wave for cylindrically symmetric flow (see Figure 6.6). We observed that the reacting gas parameter Θ together with the mass fraction k_p enhances the flattening of expansive waves and reduces the time for shock formation in reacting gas flow.

Case III. Spherically symmetric flow $(m = 2)$**:**

For spherically symmetric flow, the solution of (6.28) is given as

$$Y = \frac{Y^*}{(2\alpha+1)e^{\left(\frac{\Theta}{(1-Z_0)}\right)\alpha}\left(1+\left(\frac{(\Gamma+1)\delta}{(1-Z_0)}I(\alpha)\right)\right)}, \tag{6.34}$$

where $I(\alpha) = \int_0^\alpha \frac{-e^{\left(\frac{\Theta}{(1-Z_0)}\right)s}}{(2\alpha+1)}ds$.

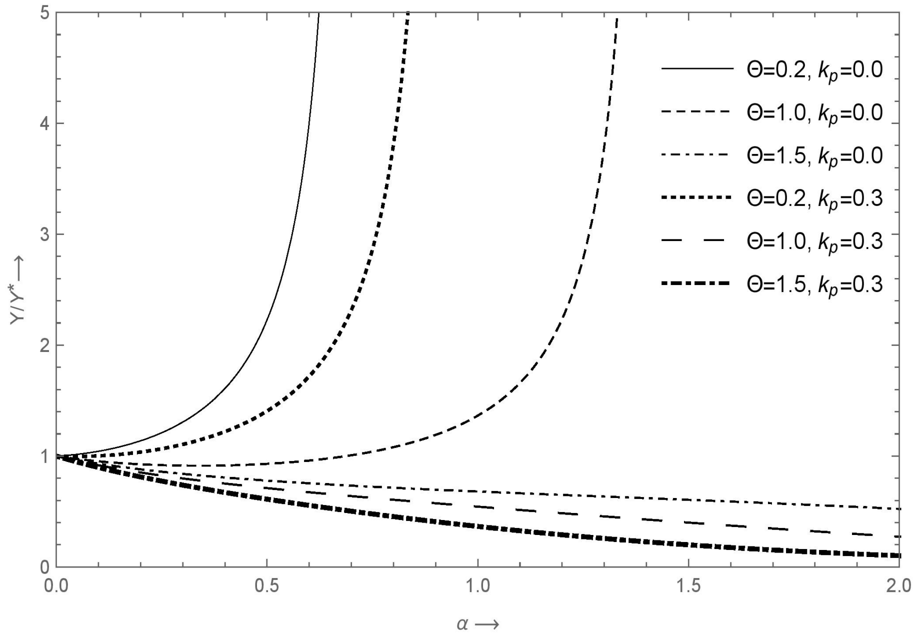

FIGURE 6.7: Variation in compressive wave in reacting gas for spherically symmetric case with $Z_0 = 0.001, \gamma = 1.4, \delta = -1, \beta = 0.5$.

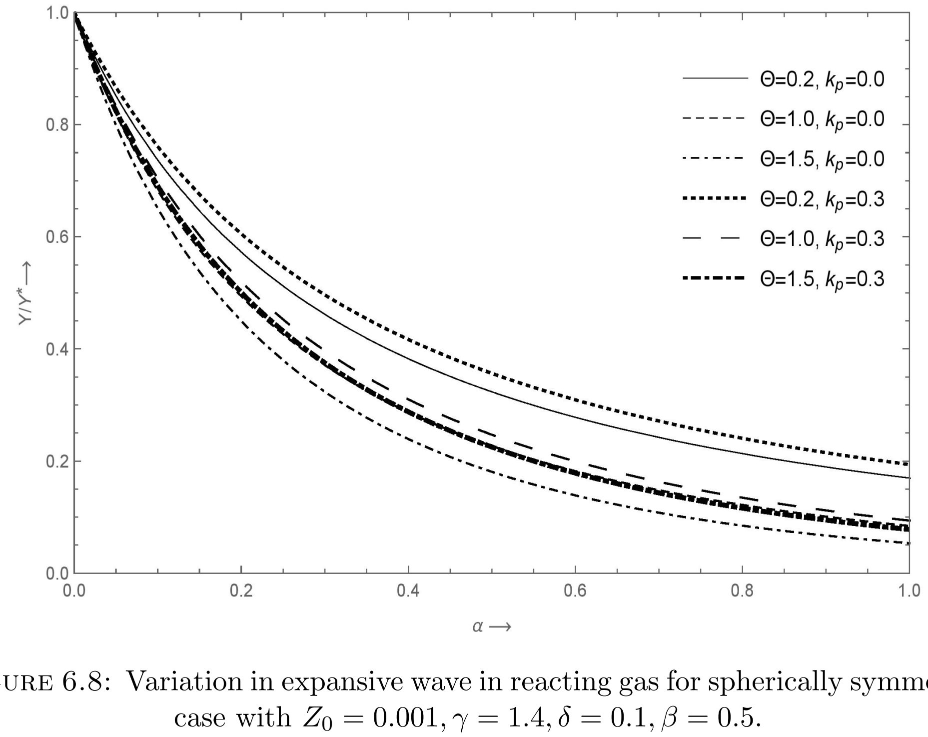

FIGURE 6.8: Variation in expansive wave in reacting gas for spherically symmetric case with $Z_0 = 0.001, \gamma = 1.4, \delta = 0.1, \beta = 0.5$.

It is also obtained that the propagating wave for $\delta < 0$ and $\delta > 0$ in spherically symmetric flow case have similar phenomenon to the planar and cylindrically symmetric flow case in reacting gas flow with dust particles which can be seen by the solution curves presented by Figure 6.7 and Figure 6.8.

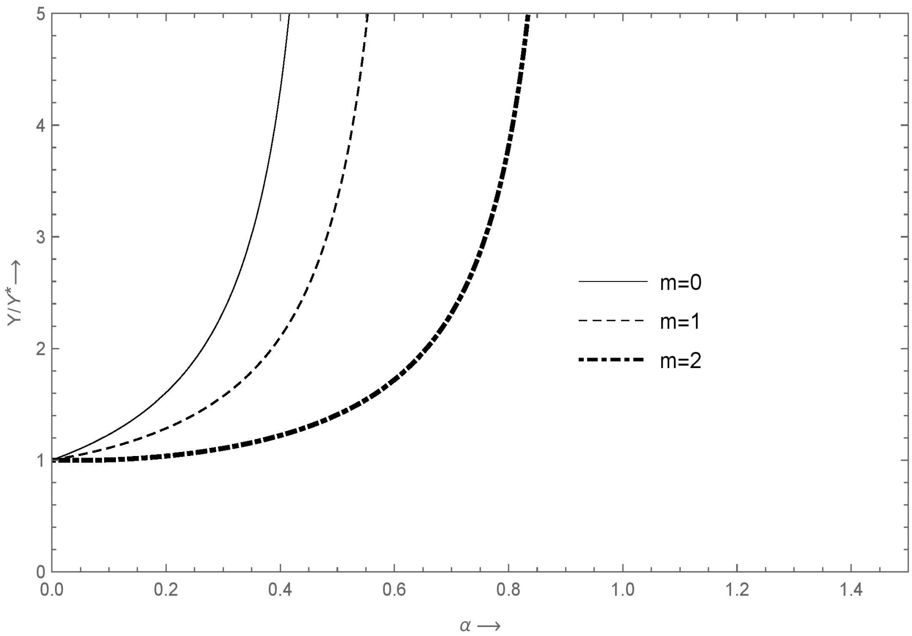

FIGURE 6.9: Compressive wave in reacting gas with dust particles with $k_p = 0.2, \gamma = 1.4, \delta = -1, \beta = 0.5$ and $\Theta = 0.2$.

A comparative analysis for expansive and compressive waves in planar, cylindrically symmetric and spherically symmetric flows of dusty reacting gas is shown by Figure 6.9 and Figure 6.10, respectively. Figure 6.9 shows that the compressive waves grow later in the case of spherically symmetric flow as compared to the planar and cylindrically symmetric flows. Also, we observe that in the case of spherically symmetric flow, the shock formation is delayed in comparison to the case of planar and cylindrically symmetric flows, i.e. the compressive waves terminate into the shock wave earlier in case of planar flow as compared to the cylindrically and spherically symmetric flows of reacting gas with dust particles (see Figure 6.9). From Figure 6.10, it is clear that the expansive waves decay earlier in the case of non-planar flow (cylindrically and spherically symmetric flows) as compared to the planar flow in reacting gas with dust particles.

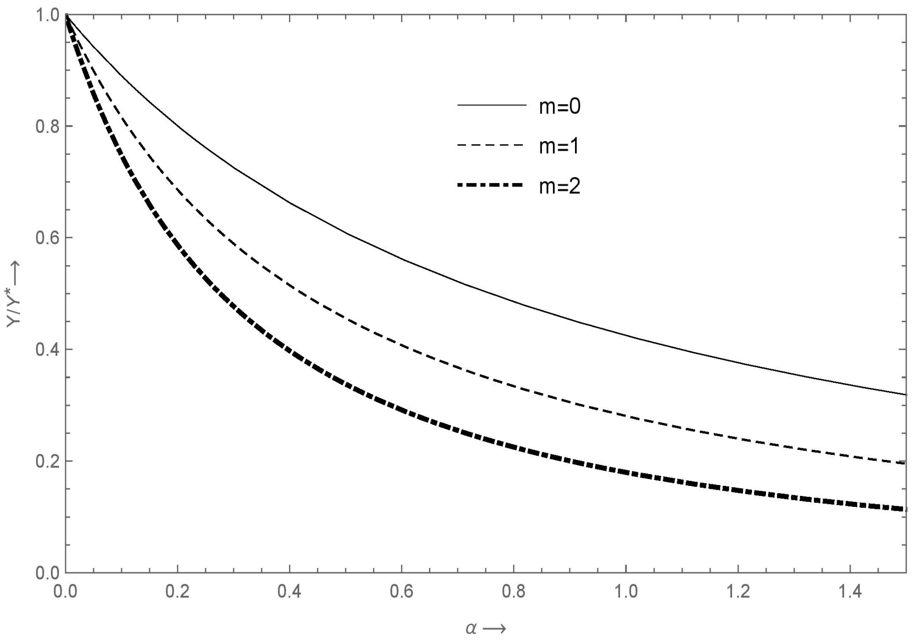

FIGURE 6.10: Expansive wave in reacting gas with dust particles with $k_p = 0.2, \gamma = 1.4, \delta = 0.5, \beta = 0.5$ and $\Theta = 0.2$.

6.6 Conclusions

In this study, we investigated the evolutionary process of the wave propagating in one-dimensional inviscid polytropic reacting gas with small solid dust particles for planar, cylindrically symmetric and spherically symmetric cases, and obtained the condition of shock formation. We found that the compressive disturbances terminate into the shock. It is also analyzed that how the shock formation process is influenced by the presence of dust particles in reacting gas. The influence of reacting gas on the evolution of the shock wave is also discussed. Throughout this study, we observed that the steepening of compressive wave and decay rate of expansive wave decreases in reacting gas with dust particles as compared to ideal gas with dust particles.

For more clarification of the effect of reacting gas on flow pattern and distortion of waves for planar and non-planar cases, a comparative study has been presented. It is observed that in planar flow, the expansive waves decay later as compared to the non-planar cases (cylindrically symmetric case and spherically symmetric case). It is obtained that the compressive waves end up with the shock earlier in planar flow as compared to the non-planar cases (cylindrically symmetric case and spherically symmetric case). The entire study concluded that the decay rate of expansive waves decreases in reacting gas, and growth rate of compressive wave also decreases, i.e. the time for the shock formation increases. The results obtained in this work is in agreement with a recent work reported in Ref [130].

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