CHAPTER 7

SUMMARY OF THE THESIS AND FUTURE SCOPE

7.1 Summary

The present thesis investigates the thermoelastic vibrations of micro and nanomechanical systems under the purview of the classical and non-classical continuum theories within the frame of some recently developed generalized thermoelasticity theories. Several experimental observations indicate the small-scale influence in the vibrating systems at micron and submicron scales. However, the classical (local) continuum theory is incapable of accurately predicting the mechanical behaviour of such systems. Therefore, to overcome this shortcoming/limitations in the classical theory, the modified couple stress theory (MCST) and the nonlocal elasticity theory as non-classical continuum theory have been used in the current work.

On the other hand, the classical coupled thermoelasticity theory could demonstrate the interrelation between thermal and elastic fields to understand the influence of one on the other. However, this theory predicts that the thermal wave propagates with an infinite speed. This behaviour gives satisfactory explanations for the thermoelastic processes involving small heat flux or large time intervals, but it still offers a poor or inadequate interpretation of thermoelastic responses to the case of fast transient loading or high heat flux. Fast transient processes have applications in structural design, nuclear, chemical, and acoustic engineering, among many others. Taking these factors into account, the generalized thermoelasticity theories such as Lord-Shulman (LS) theory, Green-Naghdi (GN-III) theory, dual-phase-lag (DPL) theory, three-phase-lag (TPL) theory, thermoelasticity theory with a single delay term proposed by Quintanilla, and recently developed Moore-Gibson-Thompson (MGT) theory have been used in this work. The current thesis considers Euler-Bernoulli beam theory, Timoshenko beam theory, and Kirchhoff plate theory for the study of thermoelastic damping (TED) and thermoelastic vibrations in micro and nanomechanical resonators. Resonators as MEMS/NEMS systems and their applications including the energy dissipation mechanisms in such systems is described in the first section (Chapter-1). The present research considers to examine the fluctuation of TED in beams and plates in depth using non-classical continuum theories in the contexts of various generalized thermoelasticity theories. Furthermore, the responses of deflection, temperature, and thermal moment of beams with respect to time are also demonstrated. The key aspects of the present work are summarized as follows:

Firstly, TED in micro and nanobeam resonators is investigated in the context of thermoelastic heat conduction model with a single delay term (Quintanilla, 2011). The Chapter 2 is devoted for this and two different problems have been studied which are presented in two subchapters. The first problem presented in **Subchapter 2.1** attempts to investigate TED in microbeam resonators following frequency approach method. The variation of TED versus normalized frequency and thickness of silicon microbeam resonator for different aspect ratios have been studied. We compare the results of present model with the corresponding results of thermoelasticity theories of type GN-III, TPL, and LS models. It is observed that TED under present model is lower than under TPL and LS models, while shows a similar result with GN-III model. The second problem as given in **Subchapter 2.2** demonstrates TED in nanobeam resonators. The problem is solved here by employing entropy generation approach method. With the help of numerical results, the influence of TED versus the normalized frequency and beam thickness is presented. Further, the effects of the time delay parameter and the material constant (k^*) on TED have been discussed in detail. The results of the present model are compared to those obtained for GN-III model. The results show that the prediction of the quality factor by the present model is higher as compared to the prediction by GN-III model. Further, the higher value of the quality factor of the present model can be observed only at the nanoscale, while the quality factor remains approximately the same as GN-III model at the microscale as reported in subchapter 2.1. It is also demonstrated that the high quality factor can also be obtained for higher values of the material constant.

Next part of the work is presented in **Chapter 3** which is concerned with the study of TED in microbeam resonators under the classical and non-classical continuum theories within the framework of recently proposed Moore-Gibson-Thompson (MGT) thermoelasticity theory. In this chapter, the expressions for deflection, temperature, and thermal moment are obtained by using the finite Fourier sine transform and Laplace transform methods. Here three different problems have been considered which are presented in three subchapters. **Subchapter 3.1** presents the analysis of TED in microbeam resonators under the classical continuum theory and MGT thermoelastic equation. The responses of deflection and thermal moment of beam are validated by comparing the results obtained under MGT model with the corresponding results under LS and GN-III models. The results show that the vibration amplitudes of deflection and thermal moment attenuate at faster rate for small sized-beam, leading to more energy dissipation. Hence, it is clear that TED has a size effect in the microbeam resonators. Further, with the increase in beam thickness, the thermal moment curve shows a clear jump in amplitude and after sometimes it attains quasi-steady vibration mode quickly. Moreover, the jump in amplitude of thermal moment increases significantly with the increase in thickness of the beam. There is significant influence of phase-lag parameters on the vibration of the thermal moment. The amplitudes of deflection under all three models are approximately the same, while the thermal moment curve shows a disagreement. In **subchapter 3.2**, the responses of deflection and thermal moment with respect to time in microbeam resonators are analyzed. In order to capture the small-scale effect on thermoelastic vibration of microbeams, MCST is taken into account. The obtained results under MCST are compared to the existing results for the classical continuum theory. Further, the results are compared between MGT, LS, and GN-III models. The present results illustrate that when MCST is used instead of classical theory, the amplitude of thermal moment decreases with time. Moreover, the vibration response of the microbeam lasts longer small value of the material length-scale parameter. The response of the thermal moment in the MGT and LS models is nearly same, but differs from the GN-III model as presented in subchapter 3.1. **Subchapter 3.3** revealed the study of the dynamic and mechanical behaviours of nanobeams using nonlocal elasticity theory and MGT thermoelastic equation while taking into account of surface energy effects. In the context of nonlocal elasticity and surface elasticity theories, the governing equation of motion for an Euler-Bernoulli nanobeam that is simply supported at both ends, and has a uniform load on the top surface is first derived. Thereafter, the solution for deflection and temperature distribution along the thickness direction of the nanobeam is obtained by using the finite Fourier sine transform and Laplace transform techniques. The influences of nonlocal parameter, phase-lag time, residual surface tension, surface elastic modulus, thickness, and length of nanobeam on deflection and temperature over time are systematically analyzed in depth. It is observed that the amplitudes of deflection and temperature over time are lower under the nonlocal elasticity theory than under the classical theory, which causes the energy to decrease with time. The amplitudes of deflection and temperature decrease significantly over time with the increase in the values of small-scale parameter. Also, the influences of surface residual tension and surface elastic modulus on deflection and temperature responses are insignificant for short time intervals during vibration but it is considerable for longer time intervals. Furthermore, the surface residual tension creates significant effect on the vibration responses of deflection and temperature than surface elastic modulus. Moreover, the variation in the value of phase-lag time has no influence on the vibration response of the nanobeam's deflection with respect to time, whereas a considerable effect has been observed on the temperature response over time.

The third part of the work as described in **Chapter 4** is devoted to the analysis of TED in nanobeam resonators using MCST and Eringen's nonlocal elasticity theory, thus so-called modified nonlocal couple stress (MNCS) theory within the context of MGT theory. The closed-form expression of size-dependent TED is obtained by following frequency approach method. With the help of numerical results, the influences of nonlocal parameter and material length-scale parameter on TED are analyzed in a detailed manner. In addition, the effects of phase-lag time on TED associated with the MGT model is demonstrated. Furthermore, the obtained results under MNCS theory are compared with classical, MCST, and nonlocal elasticity theories. The present analysis shows that MNCS theory estimates less values of TED as compared to MCST, nonlocal, and classical theories at nanoscale. When the nonlocal effect is taken into account, it results lower values of TED at submicron scale. The amount of TED predicted by MCST increases at submicron scale, whereas it decreases at micron scale. The MGT thermoelastic model shows higher TED values than the GN-III model and converges with the LS model. A significant effect of phase-lag time on TED has been observed. Also, TED may decrease for smaller phase-lag time. Moreover, for the smaller phase-lag time, the modeling of nanobeam resonator with combined effects of MNCS and MGT can result better performance.

The next part of the work presented in **Chapter 5** deals with the study of the dynamic behaviour of Timoshenko microbeams by combining the effects of MCST and DPL thermoelastic heat conduction model. The explicit expressions for deflection and thermal moment of a Timoshenko microbeam considering MCST and DPL heat conduction model are derived using Hamilton's principle. The obtained results in the present context are compared to the corresponding results obtained in the contexts of other existing models such as LS and classical Fourier heat conduction models. The obtained findings reveal that as the length-scale parameter is increased, the frequency responses of deflection and thermal moment become faster. For small time range, the responses of thermal moment and deflection under DPL, LS, and Fourier heat conduction models converge and diverge from each other for large time range. The classical theory offers a lower rate of energy dissipation as compared to MCST in the context of DPL heat conduction. The frequency response of the LS and Fourier thermoelastic models is faster than that of the DPL model, resulting in more energy dissipation. Further, the size-effects on the frequency response are observed prominently when the DPL model is employed.

Lastly, the **Chapter 6** considers to analyze the TED in microplate resonators. Here the study is performed by employing the three phase-lag (TPL) heat conduction model. In order to capture the size-effect in microplate, MCST is taken into consideration. A well known Kirchhoff's plate model as microplate resonator is taken here. The expression of the quality factor for TED is obtained following complex frequency approach method. The variations of TED as functions of the normalized frequency, microplate thickness, and length-scale parameter have been investigated. The effect of phase-lag parameters on TED has also been discussed. The results of the present model are compared to the existing results of the classical continuum theory. The predicted results show that the quality factor utilizing MCST is higher than the classical continuum theory. With the increase of the material length-scale parameter, the quality factor increases significantly from the classical continuum theory. It is demonstrated that the results of the present model diverge from the classical continuum theory when the thickness of microplate resonators is close to material length-scale parameter. There is a significant effect of boundary condition on the TED of microplate resonators. However, the peak values of TED are approximately the same for simply supported and clamped-clamped boundary conditions when the normalized frequency is fixed. With the increase in the values of phase-lag time, the quality factor decreases significantly as a function of normalized frequency. Also, MCST with small values of phase-lag time can increase the quality factor of microplate resonators with a smaller thickness.

7.2 Future Scope

Based on the advancements of modern technologies, micro and nano-electromechanical systems (MEMS/NEMS) have a variety of applications in the fields of science and engineering. The interdisciplinary nature of MEMS/NEMS rely on design, engineering, and manufacturing expertise from a wide and diverse range of technical areas including integrated circuit fabrication technology, mechanical engineering, materials science, electrical engineering, chemistry and chemical engineering, as well as fluid engineering, optics, instrumentation and packaging. The complexity of MEMS/NEMS is also seen in the extensive range of markets and applications that incorporate such devices. MEMS/NEMS are used in a variety of systems, including consumer electronics, automotive, medical, communication, and defence. Accelerometers for airbag sensors, microphones, projection display chips, blood and tyre pressure sensors, optical switches, analytical components such as lab-on-chip, biosensors, and many more items are examples of MEMS devices in use today.

To develop the aforementioned MEMS/NEMS systems with high sensitivity and fast response, a thorough understanding of the energy dissipation mechanisms in such systems is required. The relevance of the issue may be seen in the literature's gradual development of elasticity theories toward reducing energy dissipation in such systems. MEMS/NEMS devices are extremely tiny in size. Therefore, analyzing the small-scale effects of such devices becomes critical. Furthermore, TED is one of the most critical energy loss causes that is completely dependent on the material qualities. It is worth to mention here that TED is the intrinsic material damping. It appears in the systems due to the coupling between elastic and temperature fields during vibration. Therefore, the thermoelasticity theory plays an important role in order to design systems with high quality factor. The non-classical theories and extended thermoelasticity theories are used to explain thermoelastic vibration of MEMS/NEMS devices in the literature. However, in the context of additional extended areas such as electro-thermoelasticity, visco-thermoelasticity, magneto-thermoelasticiy, and piezo-thermoelasticity, further modification of non-classical theories is required. Work carried out in this direction is very very limited. Hence, there is further scope to investigate the thermoelastic responses of MEMS/NEMS devices by employing non-classical and such extended thermoelasticity theories.