CHAPTER 4

SIZE-DEPENDENT THERMOELASTIC DAMPING ANALYSIS IN NANOBEAM RESONATORS BASED ON ERINGEN'S NONLOCAL ELASTICITY AND MODIFIED COUPLE STRESS THEORIES

4.1 Introduction[1]

Thermoelastic damping (TED) has emerged as a critical issue in the modelling and design of micro and nanomechanical systems. Therefore, an extensive research interest is being devoted towards the reduction in TED for micro and nanomechanical systems. The present chapter intends to examine size effects on TED in nanobeam resonators using the simultaneous effect of modified couple stress theory (MCST) and Eringen's nonlocal elasticity theory, thus so-called modified nonlocal couple stress (MNCS) theory in the context of the recently proposed Moore-Gibson-Thompson (MGT) thermoelasticity theory. Some useful results obtained under the combined effects of MCST and nonlocal theory are reported here. Attia and Mahmoud (2016) revealed the investigation on modeling of nanobeams based on nonlocal couple stress elasticity and surface energy theories. Further, Attia and Mahmoud (2017) introduced an analytical computational model to simulate the effects of different simultaneous aspects on the behavior

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of nanobeams. The first one deals with the space nonlocality interaction and taking into account the microstructure effects, which has been formulated by using the nonlocal couple stress elasticity. The second factor deals with the memory-dependent effect and has been investigated in the framework of linear viscoelasticity theory. Ebrahimi and Barati (2018) developed analytical model based on nonlocal couple stress theory to investigate free vibration characteristics of functionally graded (FG) nanobeams considering exact position of neutral axis. The theory introduces two parameters based on nonlocal elasticity theory and modified couple stress theory to capture the size effects much accurately.

According to the literature survey, no analytical investigations on TED in nanobeam resonators have been done so far by combining Eringen's nonlocal elasticity theory and MCST in the context of the MGT heat conduction model. To fill this gap, and being motivated by the interesting results reported in above Refs., the present work investigates TED in nanobeam resonators incorporating nonlocal elasticity theory and MCST in the context of MGT heat conduction model. The frequency relation between natural and eigen frequencies is first derived by considering the isothermal case of nanobeams. Thereafter, a closed-form expression of size-dependent TED is obtained by following frequency approach method and using the concept of complex analysis to deal with the complex frequency. With the help of numerical results, the influences of nonlocal parameter and material length-scale parameter on TED are analyzed in a detailed manner. In addition, the effects of phase-lag time on TED associated with the MGT model is demonstrated. Furthermore, the obtained results under MNCS theory are compared with classical, MCST, and nonlocal elasticity theories. Present analysis is intended to be useful in the design of mechanical resonators at micron and submicron scales.

4.2 Problem formulation

4.2.1 TED in Euler-Bernoulli nanobeam resonators

In order to analyze the size-dependent TED, we consider a thermoelastic Euler-Bernoulli nanobeam of uniform length L $(0 \leq x \leq L)$, width b $(-b/2 \leq z \leq +b/2)$, and thickness h $(-h/2 \leq y \leq +h/2)$ as displayed in Figure 2.1.1.

Based on the Euler-Bernoulli beam theory and MCST, the components of rotation vector can be obtained as

$$\vartheta_y = -\frac{\partial w}{\partial x}, \quad \vartheta_x = \vartheta_z = 0 \tag{4.2.1}$$

Using Eq. (4.2.1) into Eq. (3.2.3), one can obtain

$$\chi_{xy} = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}, \quad \chi_{xz} = \chi_{zx} = \chi_{zy} = \chi_{yz} = 0 \tag{4.2.2}$$

Substituting Eq. (4.2.2) into Eq. (3.2.5), the non-zero component of couple stress tensor becomes

$$m_{xy} = -\mu l^2 \frac{\partial^2 w}{\partial x^2} \tag{4.2.3}$$

The coupled thermoelastic equation based on Euler-Bernoulli beam theory and MGT heat conduction equation can be derived as

$$\left(\frac{\partial}{\partial t} + \frac{k^*}{k}\right)\frac{\partial^2 \theta}{\partial y^2} = \frac{1}{\chi}\left(1 + \tau_q \frac{\partial}{\partial t}\right)\left(\Gamma \frac{\partial^2 \theta}{\partial t^2} - \frac{\triangle_E y}{\alpha_T}\frac{\partial^4 w}{\partial t^2 \partial x^2}\right) \tag{4.2.4}$$

4.2.2 Coupled thermoelastic equations based on modified nonlocal couple stress (MNCS) theory

Eringen's nonlocal elasticity theory (Eringen and Edelen, 1972; Eringin, 1983) explains that the state of stress at a point inside a body is regarded to be a function of strains of all points in the neighboring regions of the point. According to this nonlocal elasticity theory, the constitutive relationship between classical and nonlocal stresses in differential form is further described as follows (Eringen, 1983):

$$\sigma_{ij}^{nl} - (e_0 a)^2 \nabla^2 \sigma_{ij}^{nl} = \sigma_{ij}^{l} \tag{4.2.5}$$

Now, by following (Eringen and Edelen, 1972; Eringen, 1983; Attia and Mahmoud, 2016; Ebrahimi and Barati, 2018), the constitutive equations for modified nonlocal couple stress model based on Eringen's nonlocal elasticity theory can be written as

$$\sigma_{xx}^{nl} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}^{nl}}{\partial x^2} = \sigma_{xx} = -E\left(y\frac{\partial^2 w}{\partial x^2} + \alpha_T \theta\right) \tag{4.2.6}$$

$$m_{xy}^{nl} - (e_0 a)^2 \frac{\partial^2 m_{xy}^{nl}}{\partial x^2} = m_{xy} = -\mu l^2 \frac{\partial^2 w}{\partial x^2} \tag{4.2.7}$$

where σ_{xx}^{nl} and m_{xy}^{nl} respectively denote the nonlocal axial and couple stresses. The parameter $(e_0 a)^2$ is taken here as the nonlocal parameter that determines the size effect (Attar et al., 2021). Above relations imply that when the parameter a is ignored, the above equations reduce to the constitutive equations for the MCST and in addition, the case when the parameter l is further ignored, it reduces to the case of classical (local) Euler-Bernoulli beam theory.

From Euler-Bernoulli beam theory, the bending moments of stress (σ_{xx}^{nl}) and couple stress (m_{xy}^{nl}) are derived as (Toupin, 1964; Kumar, 2016)

$$M_m = b \int_{-h/2}^{+h/2} m_{xy}^{nl} dz$$

$$M_\sigma = b \int_{-h/2}^{+h/2} z\sigma_{xx}^{nl} dz \tag{4.2.8}$$

Therefore, the total resultant bending moment M of the nanobeam in the present context, where the modified couple stress theory and nonlocal elasticity theory are applied simultaneously in the frame of MGT theory, can be calculated as (Kumar, 2016)

$$M = M_m + M_\sigma \tag{4.2.9}$$

From Eqs. (4.2.6)-(4.2.9), we get

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} - E\alpha_T M_T - 2\mu l^2 A\frac{\partial^2 w}{\partial x^2} \tag{4.2.10}$$

where $I = bh^3/12$ is the moment of inertia and $A = bh$ is the cross-section area of nanobeam. The thermal moment M_T in above equation is given by Eq. (3.1.6). The equilibrium equation of transverse vibration of nanobeam in the absence of transverse load for Euler-Bernoulli beam theory is given by

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 w}{\partial t^2} \tag{4.2.11}$$

Using Eq. (4.2.10) into Eq. (4.2.11), the coupled governing equation of motion is derived as

$$\left(EI + 2\mu l^2 A\right) \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2}{\partial t^2} \left(1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2}\right) w + E\alpha_T \frac{\partial^2 M_T}{\partial x^2} = 0 \tag{4.2.12}$$

This is a more extended form of the equation of motion for an Euler-Bernoulli nanobeam that takes into consideration the effects of nonlocal elasticity and modified couple stress theories for MGT model. If nonlocal and modified couple stress theories are neglected, the above equation will be simplified to the classical Euler-Bernoulli beam motion equation as follows:

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} + E\alpha_T \frac{\partial^2 M_T}{\partial x^2} = 0 \tag{4.2.13}$$

4.3 Solution of the problem

4.3.1 Solution for temperature field

In this subsection, we calculate the temperature field function $\theta_0\left(x, y\right)$ of nanobeam. Substituting Eq. (2.1.11) into Eq. (4.2.4), one can obtain

$$\frac{\partial^2 \theta_0\left(x, y\right)}{\partial y^2} + p^2 \Gamma \theta_0\left(x, y\right) = \frac{p^2 \triangle_E y}{\alpha_T} \frac{\partial^2 W_0}{\partial x^2} \tag{4.3.1}$$

where

$$p_3 = \frac{\xi \eta_3}{h} - i \frac{\xi c_2}{h \eta_3}, \quad \xi = \sqrt{\frac{\omega}{2\chi}}, \quad \eta_3 = \sqrt{c_1 + \sqrt{c_1^2 + c_2^2}} \tag{4.3.2}$$

$$c_1 = \Gamma \frac{\left(\frac{k^*}{k} + \tau_q \omega^2\right) \omega}{\left[\left(\frac{k^*}{k}\right)^2 + \omega^2\right]}, \quad c_2 = \Gamma \frac{\left(1 - \tau_q \frac{k^*}{k}\right) \omega^2}{\left[\left(\frac{k^*}{k}\right)^2 + \omega^2\right]} \tag{4.3.3}$$

The general solution of Eq. (4.3.1) is obtained as

$$\theta_0(x,y) = C_1\, sin\,(p_3 y) + C_2\, cos\,(p_3 y) + \frac{\triangle_E y}{\alpha_T \Gamma}\frac{\partial^2 W_0}{\partial x^2} \tag{4.3.4}$$

where C_1 and C_2 are arbitrary constants that are determined by applying thermal boundary conditions on nanobeam. We consider that the upper and lower surfaces of the beam are thermally isolated, i.e. heat transfer to the surroundings from the surfaces of the beam is negligible. The boundary conditions are therefore assumed as $\partial\theta/\partial y = 0$ at $y = \pm h/2$. Hence, above equation becomes

$$\theta_0(x,y) = \frac{\triangle_E}{\alpha_T \Gamma}\left[y - \frac{sin(p_3 y)}{p_3\, cos\left(\frac{p_3 h}{2}\right)}\right]\frac{\partial^2 W_0}{\partial x^2} \tag{4.3.5}$$

Inserting Eq. (4.3.5) into Eq. (3.1.6), the thermal moment expression is derived as

$$\begin{aligned} M_T &= e^{i\omega t} b \int_{-h/2}^{+h/2} \theta_0(x,y)\, y dz \\ &= \frac{\triangle_E I e^{i\omega t}}{\alpha_T \Gamma}\left[1 + \frac{12}{(p_3 h)^2} - \frac{24}{(p_3 h)^3} tan\left(\frac{p_3 h}{2}\right)\right]\frac{\partial^2 W_0}{\partial x^2} \end{aligned} \tag{4.3.6}$$

4.3.2 Solution for quality factor

In order to obtain the expression of the inverse quality factor as defined in previous section, we substitute Eqs. (2.1.11) and (4.3.6) into Eq. (4.2.12) to obtain

$$\frac{EI}{\rho A}\left[1 + \frac{2\mu l^2 A}{EI} + \frac{\triangle_E}{\Gamma}\{1 + f(\omega)\}\right]\frac{\partial^4 W_0}{\partial x^4} + \omega^2 (e_0 a)^2 \frac{\partial^2 W_0}{\partial x^2} - \omega^2 W_0 = 0 \tag{4.3.7}$$

in which the complex function $f(\omega)$ is expressed as

$$f(\omega) = \frac{24}{(p_3 h)^3}\left[\frac{p_3 h}{2} - tan\left(\frac{p_3 h}{2}\right)\right] \tag{4.3.8}$$

To obtain the frequency relation of nanobeam, the isothermal state of nanobeam is considered. In this case, Eq. (4.3.7) reduces to

$$\frac{EI}{\rho A}\left[1 + \frac{2\mu l^2 A}{EI}\right]\frac{\partial^4 W_0}{\partial x^4} + \omega_0^2 (e_0 a)^2 \frac{\partial^2 W_0}{\partial x^2} - \omega_0^2 W_0 = 0 \tag{4.3.9}$$

where ω_0 is the frequency of nanobeam in isothermal case. The general solution of above equation is

$$W_0(x) = D_1\, sin(\beta_1 x) + D_2\, cos(\beta_1 x) + D_3\, sinh(\beta_2 x) + D_4\, cosh(\beta_2 x) \tag{4.3.10}$$

where D_i, $i = 1-4$ are unknown integrating constants. From Eqs. (4.3.9) and (4.3.10), the isothermal frequency ω_0 can be derived as

$$\omega_0 = \beta_1^2 \sqrt{\frac{EI\left[1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2\right]}{\rho A\,(1 + (e_0 a)^2 \beta_1^2)}} = \beta_2^2 \sqrt{\frac{EI\left[1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2\right]}{\rho A\,(1 - (e_0 a)^2 \beta_2^2)}} \tag{4.3.11}$$

We assume that the end points $x = 0$ and $x = L$ of nanobeam are simply supported, the transverse displacement and bending moment are zero. Thus, the boundary conditions can be stated as (Rao, 2019)

$$W_0(0) = \frac{\partial^2 W_0}{\partial x^2}(0) = W_0(L) = \frac{\partial^2 W_0}{\partial x^2}(L) = 0 \tag{4.3.12}$$

Substituting Eq. (4.3.10) into Eq. (4.3.12), we get

$$sin(\beta_1 L) = 0 \tag{4.3.13}$$

From above equation, one can obtain $\beta_1 L = n\pi$, $n = 1, 2, 3...$

Comparing Eq. (4.3.7) with its isothermal Eq. (4.3.9), the relation between ω and ω_0 can be obtained in the following form:

$$\omega = \omega_0 \sqrt{1 + \frac{\triangle_E}{\Gamma}\left[\frac{1 + f(\omega)}{1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2}\right]} \tag{4.3.14}$$

Note that the relaxation strength $\triangle_E$ is very small in comparison to unity, i.e. $\triangle_E \ll 1$. Therefore, taking Taylor series expansion of above equation up to first order and neglecting the higher order terms, we get

$$\omega = \omega_0 \left\{1 + \frac{\triangle_E}{2\Gamma}\left[\frac{1 + f(\omega)}{1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2}\right]\right\} \tag{4.3.15}$$

In view of the fact that the effect of thermoelastic coupling is very less, $f(\omega)$ can be approximated by $f(\omega_0)$ (Lifshitz and Roukes, 2000; Kumar et al., 2018). Thus, above equation further can be written as

$$\omega = \omega_0 \left\{1 + \frac{\triangle_E}{2\Gamma}\left[\frac{1 + f(\omega_0)}{1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2}\right]\right\} \tag{4.3.16}$$

Separating the real and imaginary parts of ω with the help of $p_3 = \frac{\xi\eta_3}{h} - i\frac{\xi c_2}{h\eta_3}$, we have

$$Re(\omega) = \omega_0 \left[1 + \frac{\triangle_E}{2\Gamma\left(1 + 24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2\right)}\left\{1 + \frac{6\left(\eta_3^2 - \frac{c_2^2}{\eta_3^2}\right)}{\xi^2\left(\eta_3^2 + \frac{c_2^2}{\eta_3^2}\right)^2} -12\frac{\left(\left(\eta_3^3 - \frac{3c_2^2}{\eta_3}\right) sin(\xi\eta_3) + \left(3\eta_3 c_2 - \frac{c_2^3}{\eta_3^3}\right) sinh\left(\frac{\xi c_2}{\eta_3}\right)\right)}{\xi^3\left(\eta_3^2 + \frac{c_2^2}{\eta_3^2}\right)^3\left(cos(\xi\eta_3) + cosh\left(\frac{\xi c_2}{\eta_3}\right)\right)}\right\}\right] \tag{4.3.17}$$

$$Im\left(\omega\right)=\frac{12\omega_0\triangle_E}{\Gamma\left(1+24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2\right)}\left[\frac{c_2}{\xi^2\left(\eta_3^2+\frac{c_2^2}{\eta_3^2}\right)^2}\right.$$
$$\left.-\frac{\left(\left(3\eta_3c_2-\frac{c_2^3}{\eta_3^3}\right)sin\left(\xi\eta_3\right)-\left(\eta_3^3-\frac{3c_2^2}{\eta_3}\right)sinh\left(\frac{\xi c_2}{\eta_3}\right)\right)}{\xi^3\left(\eta_3^2+\frac{c_2^2}{\eta_3^2}\right)^3\left(cos\left(\xi\eta_3\right)+cosh\left(\frac{\xi c_2}{\eta_3}\right)\right)}\right] \tag{4.3.18}$$

Inserting Eqs. (4.3.17) and (4.3.18) into Eq. (2.1.12) and neglecting the higher terms of $\triangle_E$, the expression of TED in terms of the inverse quality factor in the context of combined effects of modified nonlocal couple stress and MGT (MNCS-MGT) thermoelasticity theories is derived as

$$Q^{-1}=\left|\frac{24\triangle_E}{\Gamma\left(1+24\left(\frac{\mu}{E}\right)\left(\frac{l}{h}\right)^2\right)}\left[\frac{c_2}{\xi^2\left(\eta_3^2+\frac{c_2^2}{\eta_3^2}\right)^2}\right.\right.$$
$$\left.\left.-\frac{\left(\left(3\eta_3c_2-\frac{c_2^3}{\eta_3^3}\right)sin\left(\xi\eta_3\right)-\left(\eta_3^3-\frac{3c_2^2}{\eta_3}\right)sinh\left(\frac{\xi c_2}{\eta_3}\right)\right)}{\xi^3\left(\eta_3^2+\frac{c_2^2}{\eta_3^2}\right)^3\left(cos\left(\xi\eta_3\right)+cosh\left(\frac{\xi c_2}{\eta_3}\right)\right)}\right]\right| \tag{4.3.19}$$

This is the required TED solution in terms of the inverse quality factor of nanobeam resonators, that takes into consideration of the effects of modified couple stress and Eringen's nonlocal elasticity theories in the framework of MGT thermoelastic equations.

4.4 Results and discussion

This section aims to illustrate the present problem with numerical results and analyze the thermoelastic damping (TED) behavior in nanobeam resonators employing Eringen's nonlocal elasticity theory and MCST simultaneously in the framework of recently proposed Moore-Gibson-Thompson (MGT) thermoelasticity theory. The material of the nanobeam are considered to be Silicon. The amount of TED is presented in terms of the inverse quality factor (Q^{-1}) scaled by $\triangle_E$ i.e., $Q^{-1}/\triangle_E$ for a silicon simply supported nanobeam resonator. The influences of nonlocal parameter (e_0a), material length-scale parameter (l), and phase-lag time (τ_q) on the quality factor with respect to normalized frequency (ξ) and thickness (h) of nanobeam resonator are investigated in depth. For a detailed analysis of the current results under MNCS theory, we compare the obtained results with the results of classical, nonlocal, and MCST theories. At a fixed reference temperature $T_0 = 293\,K$, the material properties of Silicon nanobeam are listed below (Lifshitz and Roukes, 2000)

$E = 169\,GPa$, $\rho = 2330\,kg/m^3$, $C_v = 713\,J/kgK$, $\alpha_T = 2.59 \times 10^{-6}K^{-1}$, $v = 0.22$, $k = 156\,W/mK$, $k^* = 156\,W/mKs$.

The aspect ratios of length to thickness of nanobeam is assumed to be fixed as $L/h = 20$ in which $h = 10\,nm$. Furthermore, the value of phase-lag time is set to $\tau_q = 1 \times 10^{-11}s$ except in that Figures where its influence on TED is shown. It is worth noting here that the results are obtained for the first mode of vibrations.

4.4.1 Validation of results

In order to compare and explain the obtained results of current modified nonlocal couple stress (MNCS) theory, we consider the different elasticity theories as special cases of MNCS theory, such as classical theory ($e_0a = 0$, $l = 0$), nonlocal elasticity theory ($e_0a \neq 0$, $l = 0$), and MCST ($e_0a = 0$, $l \neq 0$) theory. The nonlocal parameter e_0a and the material length-scale parameter l are fixed here to $1\,nm$. Figures 4.4.1 (a) and 4.4.1 (b) respectively depict the variation of TED as the functions of normalized frequency and nanobeam thickness under four different theories such as MNCS theory, Eringen's nonlocal elasticity theory, MCST, and classical theory. It is observed that TED estimated by the MNCS theory is lower than the TED predicted by the MCST, nonlocal, and classical theories. It is further noticed that the amplitude of TED curve under MNCS theory is lower than that under the classical theory and MCST, and close to nonlocal elasticity theory. However, the MNCS theory shows a lower value of TED in comparison to nonlocal, classical, and MCST theories. In addition, the quality factor as calculated by using MNCS theory is larger than that obtained using other theories. It is also observed that the variation of TED as a function of nanobeam thickness under MNCS and nonlocal elasticity theories is more significant than the variation of TED as a function of normalized frequency. Thus, it is concluded that the modelling of nanobeam resonators at submicron scale under MNCS theory gives a lower rate of energy dissipation as compared to modelling under nonlocal, MCST or classical theories.

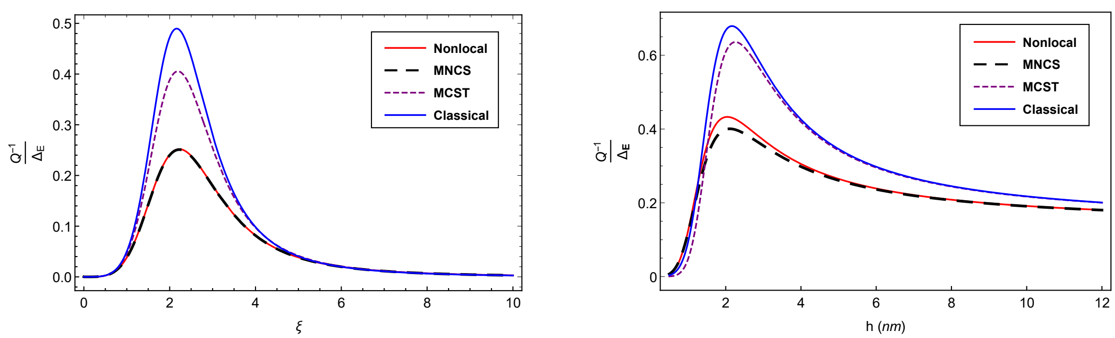

Figure 4.4.1: Variation of TED (Q^{-1}/Δ_E) under MNCS, nonlocal elasticity, MCST, and classical theories.

4.4.2 Effects of nonlocal parameter on TED

In this subsection, the focus is placed on the influences of nonlocal parameter (e_0a) on TED as the functions of normalized frequency and thickness of nanobeam resonator under MNCS theory by fixing the material length-scale parameter as $l = 1nm$. The graphs are plotted for various values of nonlocal parameter i.e. $e_0a = 1\,nm$, $1.5\,nm$, and $2\,nm$. Figure 4.4.2 (a) shows the variation of TED as a function of normalized frequency of nanobeam. It is clearly seen that the TED curve first increases with the increase of frequency to attain a maximum value, and then declines. At a specific frequency, TED curve reaches a peak, implying that the quality factor is lowest, resulting in more energy dissipation in the vibrating nanobeam resonators. It is further observed that nonlocal parameter has a significant impact on TED, although the critical frequency to show the peak value of TED does not get affected with the variation of nonlocal parameter. The Figure clearly indicates that TED reduces as the nonlocal parameter approaches the submicron scale and rises as the nonlocal parameter approaches the micron scale. Thus, the quality factor is increasing at submicron scale under Eringen's nonlocal elasticity theory.

Figure 4.4.2 (b) illustrates the variation of TED as a function of nanobeam thickness for different values of nonlocal parameter. It is noted here that when the nonlocal parameter increases, TED curve shifts, but the peak values of TED stay approximately the same. Hence, a significant effect of nonlocal parameter on TED has been observed as a function of nanobeam thickness.

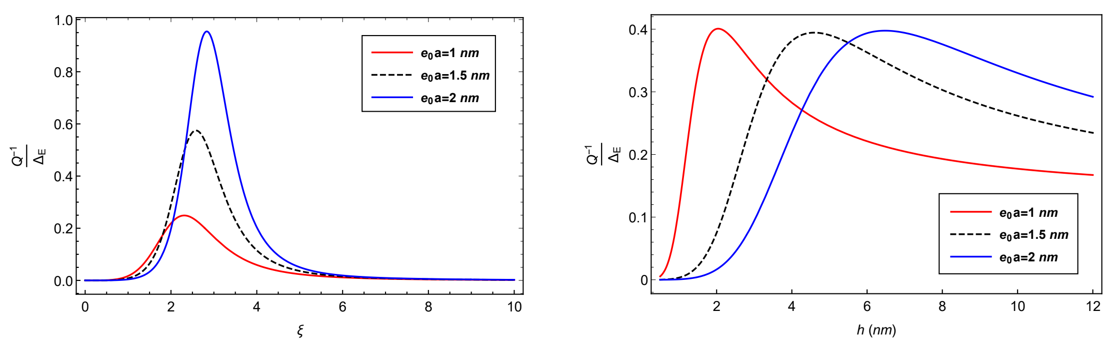

Figure 4.4.2: Effects of nonlocal parameter on TED by fixing material length-scale parameter as constant.

4.4.3 Couple stress effect on TED

This subsection explores the effects of material length-scale parameter (l) on TED versus normalized frequency and thickness of nanobeam resonator under MNCS theory by taking nonlocal parameter constant as $e_0a = 1nm$. The material length-scale parameter is assumed here to be $l = 1\,nm$, $1.5\,nm$, and $2\,nm$. Figure 4.4.3 (a) displays the variation of TED versus normalized frequency of nanobeam resonator. It is noted that when the material length-scale parameter approaches the submicron scale, the peak value of TED curve rises. In other words, the quality factor of nanobeam resonator decreases as l reaches to submicron scale. In this case, it is concluded that the vibration of nanobeam resonator will last for a smaller duration due to the increase of energy dissipation for larger length-scale parameter.

The impact of material length-scale parameter on TED as a function of nanobeam thickness is shown in Figure 4.4.3 (b). According to this result, a significant effect of l on TED against nanobeam thickness has been observed. It should be noted that the amplitude of TED decreases as l approaches to micron scale implying that the quality factor of nanobeam resonator increases. Thus, it is concluded from theses two Figures that modelling of nanobeam resonators at micron scale under modified couple stress theory offers better performance than modelling at submicron scale.

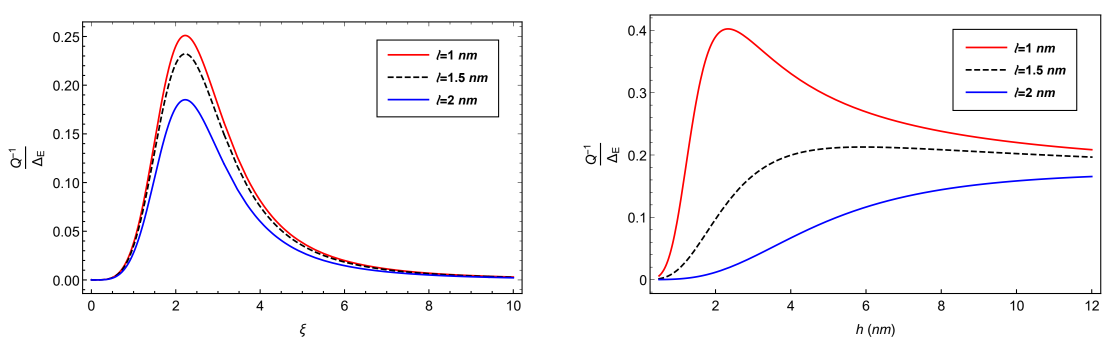

Figure 4.4.3: Effects of material length-scale parameter on TED by fixing nonlocal parameter as constant.

4.4.4 Influence of phase-lag parameter

The current size-dependent thermoelastic model involves phase-lag parameter (τ_q) due to MGT heat conduction equation. Therefore, in order to determine the model's prediction on TED, it is required to investigate the impacts of this parameter on TED in the present context. It is worth noting that the phase-lag time has no influence on TED as a function of nanobeam thickness, therefore, it is omitted here. Fig. 4.4.4 shows the effects of phase-lag time on TED as a function of normalized frequency of nanobeam resonator under MNCS theory. The size-dependent parameters under these two theories are set to be $e_0a = 1\,nm$ and $l = 1\,nm$. The phase-lag time is assumed as $\tau_q = 0\,s$, $1 \times 10^{-11}s$, $1.5 \times 10^{-11}s$, and $2 \times 10^{-11}s$. Note that $\tau_q = 0$ is the case when MGT thermoelastic model reduces to the Green-Naghdi thermoelastic model of type III (GN-III heat conduction model) (Green and Naghdi, 1991; 1992; 1993). It is observed from the Figure that TED increases as phase-lag time increases. In other words, the quality factor of nanobeam resonator will reduce for larger value of phase-lag time. This implies that the modelling of nanobeam resonator under MGT thermoelastic model is more efficient for smaller phase-lag time considering MNCS theory. Clearly, it is seen from the Figure that GN-III model predicts a lower value of TED as compared to MGT model. Thus, the nanobeam resonator modelled under MNCS and GN-III theory will give better performance than modelled under MGT theory. This is an important observation of the present chapter.

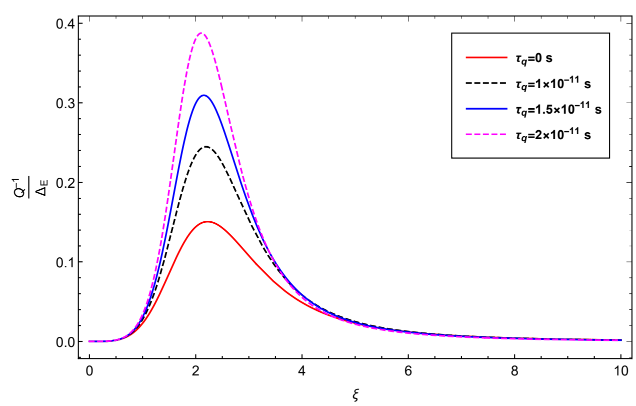

Figure 4.4.4: Effects of phase-lag time on TED under MNCS theory.

4.5 Conclusion

In this study, a closed-form expression for estimating the amount of TED in terms of the inverse quality factor in nanobeam resonators has been presented by taking the small-scale effects into account. The size-dependent expression for TED is formulated by combining Eringen's nonlocal elasticity theory and the modified couple stress theory, which is so-called modified nonlocal couple stress (MNCS) theory, in the framework of Moore-Gibson-Thompson (MGT) model. The impacts of some characteristic parameters on TED such as nonlocal parameter, material length-scale parameter, and phase-lag time are investigated in depth. In order to examine the current findings, other existing models like, classical, nonlocal, and MCST theories are adopted as special cases of MNCS theory. The concluding remarks of the present investigation can be highlighted as follows:

- The modified nonlocal couple stress (MNCS) theory estimates less values of TED as compared to MCST, nonlocal, and classical theories at nanoscale.
- When the nonlocal effect is taken into account, it results lower values of TED at submicron scale.
- The amount of TED predicted by MCST increases at submicron scale, whereas it decreases at micron scale.
- A significant effect of phase-lag time on TED has been observed. Also, TED may decrease for smaller phase-lag time parameter.
- For materials with smaller phase-lag time parameters, the modeling of nanobeam resonator with combined effects of MNCS and MGT can result better performance.

According to the above conclusions, the present work may be useful in modelling and designing the nanoscale devices with high performance at micron and submicron scales. It is also expected that this work will provide a good understanding about the size-dependent TED of nanobeam resonators for further investigation by simultaneously taking modified couple stress and Eringen's nonlocal elasticity theories into account.