CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Micro and Nano-Electromechanical Systems and Applications

Micro and nano-electromechanical systems (MEMS/NEMS) are the devices that integrate electrical and mechanical functions of the systems at the micro and nanoscales. They consist of miniaturized electrical and mechanical apparatuses such as actuators, beams, sensors, pumps, resonators, and motors. These components convert one form of energy into another, which can be quickly and conveniently measured. The MEMS sensors commonly measure pressure, force, linear acceleration, rate of angular motion, torque, and flow. The MEMS/NEMS actuators provide the ability to manipulate physical parameters at the micro and nanoscale, and can employ eletrostatic, thermal, magnetic, piezoelectric, piezoresistive, and shape memory transformation methods. The MEMS structures such as micro-nozzles are used in atomizers, medical inhalers, fluid spray systems, fuel injection, and ink jet printers. The MEMS/NEMS inherently have a reduced size and weight for the function they carry out, but they can also carry advantages such as low power consumption, improved speed, increased function in one package, and higher precision.

In fact, MEMS/NEMS are enabling technologies that bring new functionalities with the potential to radically transform markets ranging from consumer products to national defense. The meteoric rise of the smartphone is an excellent example, in which MEMS/MEMS accelerometers, gyroscopes, microphones, displays, and radio frequency filters and oscillators provide functionality that has made the most sophisticated mobile phone from a decade ago. Furthermore, the micrometer and nanometer length-scales are particularly relevant to biological materials because they are comparable to the size of cells and molecules. The most popular structures utilized as MEMS/NEMS systems are resonators, which have a wide range of applications such as musical instruments, quartz watches, radio transmitters, and so on.

1.1.1 Micro and nanomechanical resonators

A resonators is a device or system that exhibits resonance or resonant behavior. It naturally oscillates with greater amplitude at some frequencies, called resonant frequencies, than at other frequencies. The resonant frequency of micro and nanomechanical resonators depend upon many factors, including geometry, structural material properties, stress, external loading, surface topography etc. The oscillations in a resonator can be either electromagnetic or mechanical (including acoustic). The resonators are used to either generate waves of specific frequencies or to select specific frequencies from a signal. The musical instruments use acoustic resonators that produce sound waves of specific tones. Another example is quartz crystals used in electronic devices such as radio transmitters and quartz watches to produce oscillations of very precise frequency.

With the rapid advancements of the micro and nanotechnologies in MEMS/NEMS, more and more micro and nanomechanical resonators have been developed, which are of interest to both the scientific community and engineering fields. Due to their small sizes and low weight, micro and nanomechanical resonators can oscillate at very high resonant frequencies, which provides them with a remarkable ability to perform both sensing and detection in advanced technological applications, including ultra-sensitive mass and force sensing, ultra-low-power radio frequency (RF) signal generation and timing, chemical and biological sensing, environmental control, and quantum measurement.

A physical system can have as many resonant frequencies as it has degrees of freedom; each degree of freedom can vibrate as a harmonic oscillator. Systems with one degree of freedom, such as a mass on a spring, pendulums, balance wheels, and LC tuned circuits have one resonant frequency. Systems with two degrees of freedom, such as coupled pendulums and resonant transformers can have two resonant frequencies. Various geometrical structures like cantilever and bridge beams, and plates are the most typical micro and nanomechanical resonators. These resonators are of very simple geometry and can easily be fabricated by using surface manufacturing techniques.

1.1.2 Quality factor of mechanical resonators

Energy of the resonator can be lost due to many physical mechanisms. The rate at which a resonator dissipates energy is defined as the quality factor of the resonator. Low loss of energy implies the high quality factor, and is important for all the applications of micro and nanobeam resonators. There are mainly three important sources from which a resonator losses the energy. The resonators can dissipate energy through the intrinsic dissipation mechanism, through the clamping to the substrate via elastic waves, and through surrounding medium. By improving the resonator's design and medium interconnection losses, one can minimize the clamping losses. The material friction and fundamental loss mechanisms, such as phonon-phonon interaction and thermoelastic damping, cause intrinsic losses. In fact, damping dilution under tensile stress reduces the effect of intrinsic loss of resonators and increases the quality factor up to several million even at room temperature.

The quality factor (Q) is defined as the ratio of stored energy in the system and dissipated energy by the system per cycle of vibration, i.e.,

$$Q = 2\pi \frac{Stored\,Energy}{Dissipated\,Energy}$$

In the most general definition, Q represents all the processes by which the vibration energy (average kinetic and potential energy per vibration period) of the resonator decays over time. A high quality factor (Q) is directly related to reduced motional impedance, improved stability and improved noise performance of the MEMS resonator. In light of these performance attributes, prediction of Q is specifically important for optimizing the function of many applications such as time reference oscillators and inertial sensors.

1.1.3 Energy loss mechanisms in resonators

Energy dissipation in mechanical resonators has been of significant interest to the scientific and engineering communities since the early 1950s. After more than 50 years, researchers continue to refine their understanding of dissipation mechanisms, still motivated towards a high demand for low-loss resonators in frequency selection, timing, and sensing applications.

In order to design high-performance mechanical resonators, the understanding of damping effects is important. Damping in micromechanical resonators represents all of the processes by which the energy associated with the vibration of the resonator (average of kinetic and potential energy over a complete cycle) decays over time. There are various processes through which the energy of the resonator can be lost. Some of them are as follows:

1.1.3.1 Air damping

When the resonators oscillate, it has to control the resistance of air trapped in the actuation gaps and those generated by the friction with air for the sides parallel to the vibration displacement. The dissipation of energy by the actuation gaps dominates when it exists. This type of energy loss mechanism can be kept away by packaging the resonator under vacuum.

1.1.3.2 Internal friction

It occurs by the imperfections in the material such as impurities, broken bonds, or dangling. This loss mechanism causes energy dissipation in the form of heat and its contribution depends on the material and fabrication technology of the systems.

1.1.3.3 Gas damping

Moving devices transfer energy to surrounding air or gas through their motion. The resulting gas damping plays a critical role in some devices, such as accelerometers, microphones, display mirrors and switches and contributes to the signal to noise ratio. While it is possible, in principle, to simulate gas damping with a general purpose fluid dynamics field solver, such a brute-force approach is generally not practical.

1.1.3.4 Anchor losses

Resonator anchors provide the mechanical connection to the rest of the system, as well as the electrical connections for biasing and sensing of the resonator. Unlike a free vibration in space, these anchor points also can provide a pathway for energy loss from the resonator to the surroundings. One important way to improve on the energy loss through the anchors is to minimize the forces applied to the anchors during the oscillation cycle.

1.1.3.5 Thermoelastic damping

The thermoelastic effect describes the temperature change that occurs due to the stretching or contracting of an elastic material. The thermally isolated elastic structure produces a temperature variation when it suffers pressure or tension. To be precise, the temperature of the elastic structure decreases when it is uniformly stretched. The drop in temperature is balanced by the increase in entropy, which is caused by the stress (since the process is reversible, the energy remains constant). Similarly, in compression, the elastic structure heats up. Under such ideal conditions, there is no energy loss, which implies no thermoelastic damping. However, in the real case, the elastic structure is always in a more complex normal mode, so there are regions of compression and extension. Depending on the timescale of the vibration, heat flows from the warmer parts of the structure to the cooler parts. Since the heat flow is an irreversible process, this heat flow is associated with the energy loss from the vibrational mode and the corresponding damping for the resonant mode. These type of energy losses that arise due to coupling between temperature field and elastic field is called thermoelastic damping (TED). TED can be reduced by careful design and placement of perforations in the vibrating devices, but such design can only be done with the help of accurate TED simulations.

TED is proportional to the energy dissipated due to the thermoelastic effect. The metric used to measure TED is a dimensionless quantity called the quality factor (Q). The theory linking bending mode oscillations of a resonating structure to TED is summarized as follows:

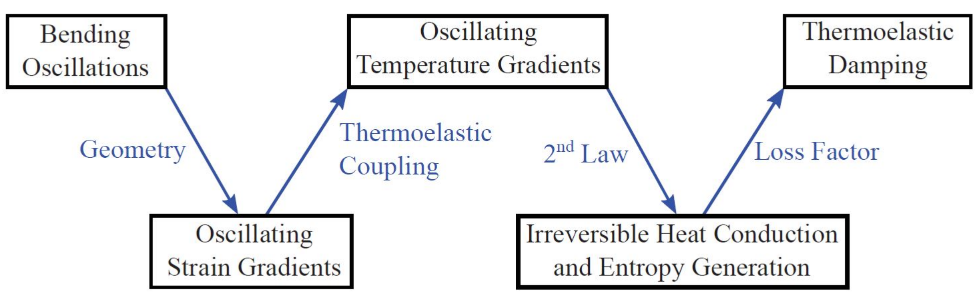

Figure 1.1.1: Flow chart depicting the theory behind thermoelastic damping.

However, the total energy dissipation in the resonator can be described as

$$\frac{1}{Q_{Total}} = \sum \frac{1}{Q_{Air}} + \frac{1}{Q_{Internal}} + \frac{1}{Q_{Gas}} + \frac{1}{Q_{Anchor}} + \frac{1}{Q_{TED}}$$

TED is one of the most important loss mechanisms for micro and nanobeam resonators among all the loss mechanisms. TED is totally based on the thermal and elastic nature of the body. Many investigations have shown the experimental evidences about TED as intrinsic losses for the flexural micro and nanobeam resonators. It is impossible to remove TED completely, however, it can only be reduced by design or fabrication of suitable resonators. In fact, TED is the keystone that affects the performance of the vacuum-operated micro and nanobeam resonators. For many resonant modes, TED determines the limit to the inverse quality factor (Q^{-1}). TED analysis in micro and nano resonators is therefore an important area of research.

TED problem was first studied by Zener in 1937. According to Zener's idea, the bending in the bean cause dilation of the opposite signs to be on the lower and upper halves. Also, an amount of heat passes through any medium, then the one side of the medium is compressed and heated and other side is expanded and closed. This means that the transverse temperature gradient is built up in the existence of finite thermal expansion. Therefore, a local heat current occurs and causes an increase in the entropy in the beam and dissipation of energy due to temperature gradient. The characteristic time τ_R of the beam is equalized by the temperature across the beam and the flexural period of the beam is ω^{-1}. Moreover, when

- $\tau_R \ll \omega^{-1}$, the vibration in the beam is isothermal and less amount of energy is dissipated.
- $\tau_R \gg \omega^{-1}$, the adiabatic conditions of beam influence with low dissipation of energy and similar to low frequency range.
- $\tau_R \approx \omega^{-1}$, a maximum of internal friction takes place due to out of phase of stress and strain.

According to the Zener's theory, the characteristic time of the beam is defined as

$$\tau_R(T) = \left(\frac{h}{\pi}\right)^2 \chi^{-1}(T)$$

where $\chi = \frac{k}{\rho C_v}$ is thermal diffusivity in which ρ and C_v denote the mass density and the specific heat at constant volume, respectively. The parameter k is thermal conductivity and h is the thickness of the beam.

The classical Fourier law of heat conduction was applied in Zener's theory. Therefore, there is no flow of heat perpendicular to the surface of the beam. Thus, TED in terms of inverse quality factor is defined by

$$Q^{-1} = \frac{ET_0\alpha_T^2}{C_v}\frac{\omega\tau_R}{1+\omega^2\tau_R^2}$$

in which α_T is thermal expansion coefficient, T_0 is reference temperature, and E is Young's modulus.

Afterwards, Lifshitz and Roukes (2000) developed the expression of the quality factor for TED in microbeam. Like Zener's theory, here also authors made their analysis on the basis of the classical Fourier law. They mainly analyzed the size-dependent effect of TED on the quality factor of microbeam resonator made with single-crystal material. The expression of the inverse quality factor for TED is given here in the form

$$Q^{-1} = \frac{ET_0\alpha_T^2}{C_v}\left\{\frac{6}{\xi^2} - \frac{6\left(sinh\xi + sin\xi\right)}{\xi^3\left(cosh\xi + cos\xi\right)}\right\}$$

where $\xi = \sqrt{\frac{\omega}{2\chi}}$. The suggested model of Lifshitz and Roukes (2000) reveals that there is a peak of TED that happens at the micrometer scale. It is also observed that when the height of the beam is more than $100\,\mu m$ or less than nanometer scale, the value of TED will decrease accordingly.

1.2 Classical and Non-Classical Continuum Theories

The classical theory of elasticity is primarily a theory for isotropic, linearly elastic materials subjected to small deformations. The classical elasticity theory is not suitable to capture the effect of the small size of the microstructures. In addition, it is appropriate to study the material behavior on a large scale. As the size of the study decreases, the accuracy by the classical theory diminishes and therefore its expectation of the material behavior on the micro and nanoscales does not match with experimental results. It is observed that the explanation behind this deviation is the critical effect of the microstructures.

Several experimental observations indicate that small-scale effect plays a considerable role in designing of the micro and nanomechanical systems (Faris et al., 2002; Fleck et al., 1994; McFarland and Colton, 2005; Stolken and Evans, 1998). Unfortunately, the classical continuum theory cannot accurately capture the size effect in such systems due to the lack of length-scale parameter. To remove this weakness, some non-classical continuum theories, such as couple stress theory (Toupin, 1964), strain gradient theory (Mindlin and Eshel, 1968; Lam et al., 2003), nonlocal elasticity theory (Eringen and Edelen, 1972; Eringen, 1983), modified couple stress theory (Yang et al., 2002) etc., have been proposed. The couple stress theory admits the possibility of asymmetric stress tensor since shear stress no longer have to be conjugate in order to ensure rotational equilibrium. The couple stress theory consisting of two or more material parameters is capable of capturing the small-scale effect in micron scale structures. However, tackling the problems of microstructures with two or more material length-scale parameters is challenging. Therefore, modified couple stress theory (MCST) has been proposed in order to solve corresponding problems by reducing the material length-scale parameters into a single material length-scale parameter. On other hand, nonlocal elasticity theory proposed by Eringen (1983) has been developed to capture the small-scale effects in microstructures at submicron scale. The stress tensor at a point in this theory is considered to be dependent on strains in a region near that point, which differs from the stress tensor described in classical (local) continuum theory. It is worth to be mentioned here that the nonlocal theory considers the long-range interatomic cohesive force, but not considered as one of the microstructure effects. However, the modified couple stress theory provides the advantages by proposing an equilibrium condition of moments of couples: symmetric couple stress tensor and by involving only one material length-scale parameter to capture size effects. By considering these conditions, the strain energy function depends only on the strain and symmetric part of the stress tensor. These non-classical theories became the center of active research in last few decades for understanding the behavior of small-scale structures.

In order to capture the size effect in the structures at micron and submicron scales, we employed solely the modified couple stress theory (MCST) and the nonlocal elasticity theory as non-classical theories in the current study. Therefore, we have concentrated only on these two elasticity theories.

1.2.1 Modified couple stress theory

The modified couple stress theory (MCST) developed by Yang et al. (2002) consists of only one internal material length-scale parameter in order to capture the size effect in microstructures. According to this theory, the total deformed strain energy (U) for a linear elastic body occupying the region V is given by

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\epsilon_{ij} + m^s_{ij}\chi^s_{ij}\right) dV$$

where

$$m^s_{ij} = 2\mu l^2 \chi^s_{ij}$$

In above equations, ϵ_{ij} mean the component of strain tensor $\boldsymbol{\epsilon}$ and χ^s_{ij} stands for the symmetric part of the rotation gradient tensor $\boldsymbol{\chi}$. The terms σ_{ij} and m^s_{ij} respectively are used to denote the component of the stress tensor $\boldsymbol{\sigma}$ and the component of the deviatoric part of the couple stress tensor $\boldsymbol{m}$.

The parameter l appeared in above equation is called as length-scale parameter that captures the effect of couple stress theory. It is worth noting here that if $l = 0$, the MCST reduces to the classical elasticity theory.

1.2.2 Nonlocal elasticity theory

The nonlocal elasticity theory proposed by Eringen's (Eringen, 1983; Eringen and Edelen, 1972) explains that the state of stress at a point inside a body is regarded to be a function of strains of all points in the neighboring regions of the point. According to this nonlocal elasticity theory, the constitutive relationship between classical and nonlocal stresses in differential form is described as follows (Eringen, 1983)

$$\sigma^{nl}_{ij} - (e_0 a)^2 \nabla^2 \sigma^{nl}_{ij} = \sigma^{l}_{ij}$$

where ∇^2 is the Laplacian operator. σ^l_{ij} represents the classical (local) stress and σ^{nl}_{ij} is the nonlocal stress. The quantity $e_0 a$ stands for the nonlocality effect or the size effect, in which e_0 and a denote the material constant and interior characteristic length, respectively.

1.3 Some Beam and Plate Theories

1.3.1 Euler-Bernoulli beam theory

The Euler-Bernoulli beam is named after Leonhard Euler and Daniel Bernoulli, who made the significant discoveries and gave this useful theory together in 1750. The Euler-Bernoulli beam theory is also known as the classical or engineer's beam theory. This theory covers the case for the small deflection of the beam subjected to a lateral load. This theory also explains the linear theory of elasticity, and gives an idea to calculate the deflection characteristic and supporting load of the beam.

The Euler-Bernoulli beam theory is applied to an analysis of thin beams, and neglects the effects of rotary inertia and shear deformation. When a beam is bent, one of the faces undergoes tension and the other undergoes compression. Somewhere in between these faces, there exists an axis, which does not experience any force called the neutral axis. The neutral axis is considered as a reference as it is easy to define the bending deformation. Further, in thin beams, one can assume that a line which is perpendicular to the neutral axis before deformation will remain perpendicular after the deformation. In other words, the shear strain can be neglected in thin beams. This assumption makes the analysis simple and is the basis of the Euler-Bernoulli beam theory.

The equation of motion for the transverse vibration of beams are in the form of fourth-order partial differential equation with two boundary conditions at each end. The equation of motion is of the form

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = f\left(x,t\right)$$

where w is the deflection of the beam, I is the bending moment, A is the cross-section of the beam, and $f\left(x,t\right)$ is the transverse load on the beam. It is worth mentioning here that for free vibration of the beam, the transverse load must be zero, i.e., $f\left(x,t\right) = 0$.

1.3.2 Timoshenko beam theory

Early in the 20th century, Stephen Timoshenko developed a theory about the beam named as Timoshenko beam theory. This theory of beam allows the analysis of the shear deformation and bending effects of thick beams. The resulting equation of the Timoshenko beam theory is of fourth order and there is a second order partial derivative as the Euler-Bernoulli beam theory.

In Timoshenko beam theory, the neutral axis is considered as a reference as it is easy to define the bending deformation. In thick beam, there exists a considerably shear strain, which should not neglected. Timoshenko beam theory holds a general nonzero shear strain and obtains the governing equations of bending. The equation of motion for Timoshenko beam of uniform length is given by

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - \rho I\left(1 + \frac{E}{kG}\right)\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\rho^2 I}{kG}\frac{\partial^4 w}{\partial t^4} = f\left(x, t\right)$$

where G denotes the shear modulus. The terms in above equation can be identified as follows: The first two terms are the same as those of the Euler–Bernoulli theory. The third term, $-\rho I(\partial^4 w/\partial x^2 \partial t^2)$, denotes the effect of rotary inertia. The last two terms, involving kG in the denominators, represent the influence of shear deformation. The Timoshenko beam model is suitable for describing the behavior of thick beams, sandwich composite beams, or beams subjected to high-frequency excitation when the wavelength approaches the thickness of the beam.

1.3.3 Kirchhoff plate theory

The Kirchhoff–Love theory of plates is a two-dimensional mathematical model that is used to determine the stresses and deformations in thin plates subjected to external forces and moments. This theory is an extension of Euler-Bernoulli beam theory and was developed in 1888 by Love (Love, 1888) using assumptions proposed by Kirchhoff. The theory assumes that a mid-surface plane can be used to represent a three-dimensional plate in two-dimensional form.

The following kinematic assumptions are made in this theory:

- Straight lines normal to the mid-surface remain straight after deformation
- Straight lines normal to the mid-surface remain normal to the mid-surface after

deformation

- The thickness of the plate does not change during a deformation.

The governing equations simplify considerably for isotropic and homogeneous plates for which the in-plane deformations can be neglected. In that case we are left with one equation of the following form

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial z^4}\right)+2\rho h\frac{\partial^2 w}{\partial t^2}+f\left(x,y,t\right)=0$$

where D is the bending stiffness of the plate and $f\left(x,y,t\right)$ is load on the plate. For a uniform plate of thickness $2h$

$$D=\frac{2h^3E}{3\left(1-\nu^2\right)}$$

in which ν denotes the Poisson ratio.

1.4 Thermoelasticity

Thermoelasticity is a branch of science that describes the behaviour of elastic bodies under the impact of non-uniform temperature and mechanical fields. Thermoelasticity is concerned with the effect of heat on elastic deformable bodies and vice-versa. Deformation refers to any changes in the shape and size of a body due to an applied force or change in temperature. When a load is applied to a body, it deforms. Before deformation, an internal force opposes the deformation and that applied force per unit area is called stress. A deformation differing with time leads a change in temperature field which produces strain in the body. Thus, the internal energy of the body becomes a function of the deformation and temperature. Thermoelasticity theory therefore describes the mutual interactions of mechanical strain and temperatures fields of the body. It relates two different and independently developed branch of sciences which are the theory of heat conduction and the theory of elasticity.

In various disciplines of science and technology, thermoelasticity theory has gained considerable interest from engineers and researchers due to its innumerous applications to multiple fields. Thermal stress analysis is significant in a variety of structural issues, such as high-speed plane manufacturing, designing of space vehicle, rocket and jet engine etc, nuclear reactor design, and so on. The thermoelasticity theory is also finding increasing use in a variety of engineering issues, such as developing material parts that can withstand abrupt thermal and mechanical loads and function at high temperatures.

The classical thermoelasticity theory has been widely used to study the thermal and elastic coupling involved in various thermoelastic problems. The classical uncoupled theory of thermoelasticity suffers from the drawback that the elastic changes have no effect on the temperature and vice-versa, and this theory also suffers from infinite speed of thermal signals. These are two drawbacks of this theory. To remove these drawbacks, Biot (1956) proposed a theory which was based on the ground of irreversible thermodynamics. This theory gives us an effective model to study the coupling effects of thermal and elastic fields. Also, this theory removes the first drawback inherent in the classical uncoupled theory. But, the paradox of infinite speed of thermal signal remains inherent in Biot's theory. Moreover, this inherent paradox is mainly due to heat transport equation, which is based on the classical Fourier law of thermal conduction and of parabolic type partial differential equation. However, the equation of motion is wave type.

Several efforts are being made for a long time to remove the drawbacks inherent in the classical coupled thermoelasticity theory developed by Biot. Accordingly, several non-conventional thermoelasticity theories came into existence. In this theory, the parabolic type heat conduction equations are replaced by the hyperbolic type equations, and admit wave-like thermal disturbance propagating with finite speed. The wave-like thermal signal is called as "second sound". The thermoelasticity theory that allows wave-like thermal signal is known as "thermoelasticity theory with second sound" or "generalized thermoelasticity theory".

Due to progress in laser pulse, nuclear reactors, particle accelerators, etc. that can provide heat pulses with a speedy time range, scientists pay appreciable curiosity towards the generalized theory of thermoelasticity. The development of these generalized theories are mainly based on the following three different approaches:

• Incorporating the concept of phase-lags/thermal relaxation parameters for constitutive variables in the Fourier law of heat conduction.

• Considering the effects of higher order terms of constitutive field variables in the formulation of the governing equations.

• Developing alternative formulation of the coupled theory by introducing new constitutive field variables in the derivation of governing equations.

The generalized thermoelasticity theories which are used in the current thesis are given below:

1.4.1 Lord-Shulman (LS) thermoelasticity theory

Lord and Shulman (1967) have proposed a generalized thermoelastic model suggesting the finite speed of heat propagation. This theory is also known as the extended thermoelasticity theory. The first modification in the Fourier's heat conduction theory has been suggested by Cattaneo (1958) and Vernotte (1958; 1961) by introducing the heat flux rate term in Fourier's law with a time relaxation parameter. Lord and Shulman (1967) have applied this modified Fourier's law of heat conduction (Catteneo-Vernotte law) and derived the first generalized coupled theory of thermoelasticity. The heat conduction law based on the LS thermoelasticity theory for the homogeneous and isotropic medium can be given as

$$q\left(\boldsymbol{r},t\right)+\tau_q\dot{q}\left(\boldsymbol{r},t\right)=-\left[k\nabla T\left(\boldsymbol{r},t\right)\right]$$

where T is temperature, k is thermal conductivity of the material, q is heat flux vector, and $\boldsymbol{r}$ is position vector. Here, the constant τ_q has a definite physical meaning and shows the time lag to set up the steady state heat conduction in a material volume when a temperature gradient is instantly imposed on the material. This time lag is known as the thermal relaxation time of the material.

A physical meaning of the above equation had been explained by Chester (1963). He suggested the value of relaxation time τ_q through the experimental work given as

$$\tau_q = \frac{3k}{\rho C_v \varphi_s^2}$$

where φ_s is the speed of ordinary sound. However, there is no theoretical model available to determine the relaxation time (Shiomi and Maruyama, 2006). The heat conduction equation of this theory is wave-type, and have finite speed of propagation of heat and elastic wave. According to this hyperbolic type differential equation, the thermal signals propagate with finite speed φ_T given as

$$\varphi_T = \sqrt{\frac{k}{\rho C_v \tau_q}},\ \tau_q \neq 0$$

1.4.2 Green-Naghdi (GN) thermoelasticity theory

In the 1990s, Green and Naghdi (1991; 1992; 1993) have followed a completely different approach to develop an alternative version of thermoelasticity theory. They modified Fourier's law by introducing a new constitutive variable in the theory of heat conduction, and developed their new thermoelasticity theory that is divided into three parts. These parts now referred to as the thermoelasticity theories of type I, II, and III. The linearized version of type-I model is identical with coupled theory of thermoelasticity, possessing the paradox of infinite speed of heat propagation. The type-II shows that there is no dissipation of thermal energy in the body because the internal rate of production of entropy is considered to be identically zero here. This model allows undamped thermoelastic waves in the thermoelastic body. Therefore, this model is known as the theory of thermoelasticity without energy dissipation. The type-III model includes the previous two models as special cases, shows dissipation of energy in general, and involves damped thermoelastic waves. In this model, the constitutive equations are obtained by starting reduced energy equation, where the thermal displacement gradient, in addition to the temperature gradient, is among the constitutive variables. For this model, the proposed heat conduction equation is of the form

$$q\left(\boldsymbol{r},t\right)=-\left[k\nabla T\left(\boldsymbol{r},t\right)+k^{*}\nabla\upsilon\left(\boldsymbol{r},t\right)\right]$$

where υ is termed as thermal displacement that satisfies the relation $\dot{\upsilon}=T$.

1.4.3 Dual-phase-lag (DPL) thermoelasticity theory

In order to consider the microscopic effects in ultra fast process of heat transport phenomenon, a new heat conduction model has been proposed by Tzou (1995a, 1995b). They attempted the Fourier law of heat conduction by introducing two time phase-lags, one for the heat flux vector and the other for the temperature gradient and called their theory as dual phase-lag (DPL) heat conduction theory. It is worth to note that while the classical Fourier law of thermal conduction is macroscopic in both space and time, and single-phase-lag model is microscopic in time, the DPL model is microscopic in both space and time.

In DPL model, the Fourier law is replaced by an approximation of the form

$$q\left(\boldsymbol{r},t+\tau_{q}\right)=-k\nabla T\left(\boldsymbol{r},t+\tau_{T}\right)$$

This equation implies that the conductive temperature gradient at a point r at time $t+\tau_T$ results in a heat flux at the same point at time $t+\tau_q$. The delay time τ_T is interpreted as that caused by the microstructural interactions such as phonon scattering or phonon-electron interactions, and is called the phase-lag of the temperature gradient. The other time delay term τ_q is interpreted as the relaxation time due to the fast transient effect of thermal inertia, and is called as the phase-lag of the heat flux. These two phase-lag are small and positive (Tzou, 1997). This DPL heat conduction theory has been extended to dual phase-lag thermoelasticity theory by Chandrasekharaiah (1998) who modified the basic governing equations of Biot's theory and employed the DPL heat conduction theory (Tzou, 1995a; 1995b).

1.4.4 Three-phase-lag (TPL) thermoelasticity theory

Roychoudhuri (2007) has further generalized the concept of phase-lag to Green-Naghdi thermoelasticity theory by incorporating three different phase-lag parameters in the constitutive relation for heat conduction given by Green and Naghdi (GN-III model). One additional phase-lag parameter is incorporated here for the gradient of thermal displacement, along with the incorporation of phase-lag parameters for the heat flux as well as temperature gradient terms. The modified heat conduction law corresponding to the TPL theory for the homogeneous and isotropic medium is expressed by

$$q\left(\boldsymbol{r}, t+\tau_q\right) = -\left[k\nabla T\left(\boldsymbol{r}, t+\tau_T\right) + k^{*}\nabla \upsilon\left(\boldsymbol{r}, t+\tau_\upsilon\right)\right]$$

where τ_υ is the thermal gradient vector. The TPL model plays a very important role in the problems related to nuclear boiling, exothermic catalytic reactions, phonon scattering, phonon-electron interactions, etc.

1.4.5 Thermoelasticity theory with a single delay term

Quintanilla (2011) has proposed some modifications to the three-phase-lag (TPL) model, and studied the well-posedness and spatial behavior of this newly proposed model. In his new formulation, the parameters are assumed to be $\tau_q = \tau_T$ and $\tau_0 = \tau_q - \tau_v > 0$. Now, the TPL model is reduced to a heat conduction model with a single delay term, $\tau_0 > 0$. the proposed constitutive relation by Quintanilla (2011) is of the form

$$q(\boldsymbol{r}, t) = -\left[k\nabla T(\boldsymbol{r}, t) + k^* \nabla \upsilon(\boldsymbol{r}, t - \tau_0)\right]$$

where the new parameter τ_0 is called as time delay parameter.

1.4.6 Moore-Gibson-Thompson (MGT) thermoelasticity theory

Recently, Quintanilla (2019) has proposed a new thermoelasticity theory named as Moore-Gibson-Thompson (MGT) thermoelasticity theory. In fact, this new proposed thermoelastic model is the generalization of Lord-Shulman (LS) model and Green-Naghdi (GN-III) model. The new heat conduction model in this theory has been taken in the following form:

$$q(\boldsymbol{r}, t) + \tau_q \dot{q}(\boldsymbol{r}, t) = -\left[k\nabla T(\boldsymbol{r}, t) + k^* \nabla \upsilon(\boldsymbol{r}, t)\right]$$

1.5 Literature Review

For the first time, the energy dissipation mechanism in microstructures caused by TED was observed by Zener (1937, 1938). He formulated an expression of TED in terms of the quality factor for a beam by applying the classical Fourier heat conduction equation. The Zener's theory works well for simple beams and has been experimentally verified by Candler (2003). Later on, Zener (1938) gave an experimental evidence about thermoelastic internal friction closely related to energy dissipation. Afterwards, Berry (1955) presented an experiment in view of Zener's theory for a brass material. The exact expression for the attenuation coefficients of thermoelastic vibrations was presented by Landau and Lifshitz (1959). However, they did not give a conscientious solution and derivation of the governing equations. An analysis about the thermally induced vibrations of the beams and plates was approximated by Boley (1972). Manolis and Beskos (1980) examined the effect of axial loads and damping on the vibration of the beams by neglecting the coupling effect between stress and temperature fields. Further, the vibration of the beam with step heat flux at the surface was analyzed by Massalas and Kalpakidis (1983) disregarding the inertial term to make the analysis simpler. Crawley and Van Schoor (1987) published an article on investigation of material damping taking the aluminum and metal matrix composites. Roszhardt (1990) analyzed TED in single crystal silicon micro resonators at the room temperature. Bishop and Kinra (1992) studied about the measurements of the flexural damping. Later on, Kinra and Milligan (1994) exhibited an analysis regarding the second law of thermoelastic damping. Burns et al. (1995) uncovered the pressure sensor of the scaled-cavity resonant microbeam. Givoli and Rand (1995) studied the effect of thermoelastic coupling in a rod and observed that the nature of dynamic response of the structure varies significantly as the frequency of the thermal loading of the rod is near the critical frequency. Hosaka et al. (1995) have shown the damping characteristics of microbeam oscillators. Mihailovich and MacDonald (1995) measured the mechanical loss of different micro-scaled-sized vacuum operated single crystal silicon resonators and recognized the dominant loss mechanism of the resonators. According to their examination, doping impurity losses, surface related losses, and support related losses are three possible sources of mechanical loss in the resonators. Cleland and Roukes (1996) studied about the fabrication of high frequency nano-sized mechanical resonators from the silicon crystal. Carr and Craighead (1997) also investigated the fabrication technique of nanoelectromechanical devices in single crystal silicon. Cleland and Roukes (1999) disclosed the external control of dissipation in a nanometer radio frequency mechanical resonators. The investigation of TED in silicon nitrate micro resonators was reported by Yasumura et al. (1999).

Lifshitz and Roukes (2000) extended the Zener's work and obtained exact solution of TED in terms of the quality factor utilizing the classical Fourier law of thermal conduction. They demonstrated the size-dependency of resonators and observed that the quality factor of resonators decreases even when they are built up from pure single crystal materials. Harrington et al. (2000) studied about the mechanical dissipation of micro-scaled single crystal gallium arsenide resonators that vibrate in torsion and flexural modes. They analyzed that the resonance frequency changes with the change in temperature. Kenny (2001) presented a review article on the study of nanometer-scale force sensing with MEMS devices. Copper and Pilkey (2002) provided a thermoelastic solution technique for beams with arbitrary quasi-static temperature distributions that create large transverse normal and shear stresses. This technique calculates the stress resultants and centroid displacements along a beam. Zhang et al. (2002) analyzed the effect of cubic nonlinearity on auto-parametrically amplified resonant MEMS mass sensor. Srikar and Senturia (2002) presented the closed-form expressions of TED to estimate an upper bound on the attainable quality factors of polycrystalline beam resonators with thickness much larger than the average grain size. Guo and Rogerson (2003) investigated the effect of thermoelastic coupling on a micro-machined resonator and observed that the frequency shift ratio caused by thermoelastic coupling is of the order of 10^{-3}, which is much larger than that of air damping. Zhang et al. (2003) gave analytical results that show air damping generally shifts the resonant frequency downward and degrades the quality factor, and that this effect increases as the dimension of the beam decreases. Duwel et al. (2003) presented the experimental study of TED in MEMS gyros, sensors, and actuators. Hao et al. (2003) gave an analytical model for support loss in clamped–free and clamped–clamped micromachined beam resonators with in-plane flexural vibrations. Wang et al. (2003) studied about the mechanical energy losses in micromachined silicon structures and observed that surface effects will become increasingly more important as the sizes of micromechanical devices continue to decrease. Datskos et al. (2004) examined the performance of micro cantilevers as uncooled infrared detectors with optical readout. Nayfeh and Younis (2004) derived analytical expressions for the quality factors of microplates with the help of perturbation method under electrostatic loading and residual stresses in terms of its structural mode shapes. Vengallatore (2005) studied thermoelastic damping in symmetric, three-layered, laminated, micromechanical Euler–Bernoulli beams using an analytical framework developed by Bishop and Kinra (1992). Norris and Photiadis (2005) analyzed TED in thin plates and uncovered that the thermal relaxation loss is inhomogeneous and depends upon the local state of vibrating flexure, specifically, the principal curvatures at a given point on the plate. Khisaeva and Ostoja-Starzewski (2006) examined TED in nanomechanical resonators using the generalized thermoelasticity theory with one relaxation time. Sun et al. (2006) established the governing equations of coupled thermoelastic problems based on the generalized thermoelastic theory with one relaxation time by using both the finite sine Fourier transformation method combined with Laplace transformation. Wong et al. (2006) analyzed TED of the in-plane vibration of thin silicon rings. Fang et al. (2007) published a review article on the advances in TED in micro and nanomechanical resonators. Pratap et al. (2007) studied about the squeeze film effects in MEMS and discussed the development of squeeze film flow modelling, tracking its routes to the air damped vibrating system studies in the early twentieth century. Hao (2008) investigated TED in the contour-mode vibrations of micro and nanoelectromechanical circular thin plate resonators. Prabhakar and Vengallatore (2008) evaluated TED in micromechanical resonators and gave an exact theory for TED with two dimensional heat conduction that enables a detailed evaluation of the accuracy of the quasi one dimensional theories. Wilson-Rae (2008) analyzed the dissipation mechanism that arises in nanomechanical beam structures due to the tunneling of mesoscopic phonons between the beam and its supports. Guo et al. (2009) investigated the coupled thermoelastic vibration characteristics of the axially moving beam and derived the equation of motion based on the equilibrium equation and the thermal conduction equation involving deformation term. Chandorkar et al. (2009) presented a formulation of TED based on entropy generation that accounts for heat transfer in three dimensions and obtained analytical closed form solutions for energy loss estimation in a variety of resonating structures. Sun and Saka (2010) analyzed TED in microscale plate resonators under different environmental temperature, plate dimensions and boundary conditions. Tunvir et al. (2010) studied the effects of hollow geometry on thermoelastic dissipation of tubular beam resonators of circular cross-section and derived the expression for the quality factor. Kim et al. (2010) investigated the quality factor for TED in rotating thin rings with in-plane vibration. Xu-Xia and Zhong-Min (2010) analyzed the thermoelastic coupling vibration characteristics of the axially moving beam with frictional contact. Sharma (2011) obtained the analytical expressions for TED and frequency shift of flexural vibrations in a transversely isotropic thermoelastic beam based on Euler–Bernoulli theory. Sharma and Grover (2011) derived the closed form expressions for the transverse vibrations of a homogeneous isotropic, thermoelastic thin beam with voids based on Euler–Bernoulli theory. They analyzed the effects of voids, relaxation times, thermomechanical coupling, surface conditions and beam dimensions on energy dissipation induced by TED in MEMS/NEMS resonators under clamped and simply supported conditions. Vahdat and Rezazadeh (2011) revealed the effects of residual and axial stresses on TED in microbeam resonators. In their study, a Galerkin based finite element formulation has been used to analyze TED for the first mode of vibration of the micro-beam resonator with both ends clamped and isothermal. Li et al. (2012) presented an analytical model for the TED in the fully clamped and simply supported rectangular microplates and obtained the quality factor by calculating the energy dissipated per cycle of vibration over the volume of the microplate. Guo et al. (2012) analyzed the TED in micro and nanomechanical resonators based on generalized thermoelasticity theory with dual-phase-lagging effect. Tunvir et al. (2012)

studied thermoelastic dissipation of micro and nanobeams of elliptical, triangular or arbitrary rectangular cross-section with accurate satisfaction of the surface thermal condition. Guo (2013) investigated thermoelastc dissipation of microbeam resonators in the framework of generalized thermoelasticity theory. Fang et al. (2013) presented an analytical solution for TED in the axisymmetric vibration of circular microplate resonators with two dimensional heat conduction equation. Subsequently, Sun et al. (2014) presented an analytical solution for TED in the axisymmetric vibration of laminated trilayered circular plate resonators. Guo et al. (2014) investigated TED in circular microplate resonators in the context of dual-phase-lag thermoelasticity theory. Tai et al. (2014) evaluated TED in torsion microresonators with coupling effect between torsion and bending. Fang and Li (2015) presented a simple analytical model for TED in microrings with two dimensional heat conduction over thermoelastic temperature gradients along the radial thickness and the circumferential direction. Ale and Mohammadi (2015) determined the effect of TED in nonlinear beam model of MEMS resonators by differential quadrature method. Youssef and Alghamdi (2015) also investigated TED in nanomechanical resonators based on two-temperature generalized thermoelasticity theory. Zenkour (2016) gave a thermoelastic model of TED for free vibration of a microbeam resting on pasternak's foundation via the Green-Naghdi thermoelasticity theory without energy dissipation. Zuo et al. (2016) investigated TED in bilayered microplate resonators and developed an analytical model in the form of an infinite series for TED in the bilayered fully clamped rectangular and circular microplates. Guo et al. (2016) evaluated TED in microbeam resonators using the generalized thermoelasticity theory based on the dual-phase-lag model. Zenkour (2017) used a model of nonlocal thermoelasticity theory of Green and Naghdi without energy dissipation to consider the vibration behavior of a nanomachined resonator. Fang et al. (2017) computed TED in the rectangular microplates with three dimensional heat conduction theory. Li et al. (2017) investigated TED in free vibrating functionally graded material (FGM) microbeams with rectangular cross-sections by assuming the material properties to be varied continuously in the thickness direction. Alghamdi (2017) presented analytical expressions for deflection, temperature change, and frequency shifts for thermoelastic vibration of microbeam resonators with voids in the frame work of dual-phase-lag hat conduction equation. Grover and Seth (2018) established the analytical expressions for TED and frequency shift of coupled dual-phase-lag generalized visco-thermoelastic thin beam under clamped and simply supported boundary conditions. Kumar et al. (2018) obtained a closed-form solution of TED for microbeam resonators under generalized thermoelasticity theory with three-phase-lag effect. Li et al. (2018) analyzed TED in functionally graded circular microplates based on classical plate theory and one-way coupled heat conduction equation. Parayil et al. (2018) presented a general model to accurately capture TED in Timoshenko beams with mid-plane stretching nonlinearity. Zhou et al. (2019) derived the analytical TED models for rectangular cross-sectional micro and nano ring resonators with heat conduction along the radial thickness direction and the circumferential direction in the context of non-Fourier theory of single-phase-lag model. Kumar and Kumar (2019) studied TED in microbeam resonators based on three-phase-lag thermoelastic model and derived the expressions for deflection and thermal moment by using the integral transform technique. Chen et al. (2019) formulated an analytical model for evaluating TED in micromechanical resonators based on the thermal energy method, in which thermal conduction in both thickness and axial directions are considered. Recently, Zhou et al. (2020) developed an analytical formula of TED in micro and nanobeam resonators with circular cross-section by adopting the non-Fourier theory of dual-phase-lag model. Li and Ma (2020) presented a theoretical investigation on the response of free vibration in microplates of functionally graded material (FGM) and investigated TED of the plate resonator. Yang et al. (2020) presented an analytical TED model based on the two dimensional heat conduction in the thickness and length directions in a bilayer microbeam with a rectangular cross-section.

Alghamdi (2020) analyzed the vibration of a visco-thermoelastic nanobeam of silicon nitride based on dual-phase-lag heat conduction model subjected to ramp-type thermal loading. Selim (2020) investigated the propagation of the longitudinal waves in a single-walled carbon nanotube considering the effects of TED. Abouelregal and Marin (2020a) studied about the size-dependent thermoelastic vibrations of nanobeams subjected to harmonic excitation and rectified sine wave heating. Youssef et al. (2021a) analyzed vibration of thermoelastic silicon nitride nanobeam based on Green-Naghdi theorem of type-II subjected to mechanical damage and ramp-type heat. Further, Youssef et al. (2021b) constructed the numerical analysis for thermoelastic homogeneous isotropic microbeams by using a generalized viscothermoelasticity theory with one relaxation time with variable thermal conductivity in the context of damage mechanics definition. Alharthi (2021) constructed a novel model by applying fractional order strain theory that introduces a thermal analysis of a thermoelastic, isotropic, and homogeneous nanobeam. Kaur and Singh (2021) studied TED in transversely isotropic thin circular Kirchhoff–Love plate and formulated mathematical model for time-harmonic displacement and temperature fields due to the Green and Naghdi theory of thermoelasticity of type III. Yang et al. (2021) gave a generalized methodology for TED in axisymmetric vibration of circular plate resonators covered by multiple partial coatings.

Above mentioned literature review is based on classical continuum theory. Considering small-scale effects observed in micro and nanostructures, some studies have been reported in recent years where the authors employed the non-classical continuum theories like, couple stress theory, modified couple stress theory, nonlocal elasticity theory, strain gradient elasticity theory. Anthoine (2000) solved the problem of the pure bending of a circular cylinder within the linear couple- stress theory. Park and Gao (2006) developed a new model for bending of Euler-Bernoulli beam using modified couple stress theory by employing a variational formulation based on the principle of minimum total potential energy. Gao and Park (2007) provided a variational formulation for a simplified strain gradient elasticity theory by using the principle of minimum total potential energy and its application to a pressurized thick-walled cylinder problem. Maranganti and Sharma (2007) studied about nonlocal elasticity considering surface energy effects and provided a detailed analysis for length-scales at which classical elasticity breaks down for various materials. Kong et al. (2008) solved the dynamic problems of Bernoulli–Euler microbeams analytically on the basis of modified couple stress theory considering simply supported and cantilever boundary conditions. Ma et al. (2008) developed a microstructure-dependent Timoshenko beam model using a variational formulation based on modified couple stress theory and Hamilton's principle. This new model contains a material length-scale parameter and can capture the size effect, unlike the classical Timoshenko beam theory. Park and Gao (2008) presented a variational formulation based on the principle of minimum total potential energy for the modified couple stress theory proposed by Yang et al. (2002), which leads to the simultaneous determination of the equilibrium equations and the boundary conditions. Kong et al. (2009) analyzed the static and dynamic behavior of Euler-Bernoulli microbeams utilizing the strain gradient elasticity theory. Tsiatas (2009) developed a new Kirchhoff plate model for the static analysis of isotropic microplates with arbitrary shape based on modified couple stress theory. The proposed model is capable of handling plates with complex geometries and boundary conditions. Asghari et al. (2010) investigated the size-dependent static and vibration behavior of microbeams made of functionally graded materials (FGMs) on the basis of modified couple stress theory in the elastic range. Wang et al. (2010) gave a formulation of a microscale Timoshenko beam model based on strain gradient elasticity theory, and derived the governing equations of motion and boundary conditions on the basis of Hamilton principle. Şimşek (2010) proposed analytical and numerical solution procedures for vibration of an embedded microbeam under action of a moving microparticle based on modified couple stress theory within the framework of Euler–Bernoulli beam theory. Ke and Wang

(2011) determined the size effect on dynamic stability of microbeams made of functionally graded materials (FGMs) based on modified couple stress theory and Timoshenko beam theory. Jomehzadeh et al. (2011) studied about the size-dependent vibration analysis of microplates based on modified couple stress theory. Najafi et al. (2012) evaluated TED and derived the expression of the quality factor in an electrostatically deflected microbeam resonator based on modified couple stress theory and hyperbolic heat conduction model. Ke et al. (2012) developed a Mindlin microplate model based on modified couple stress theory for the free vibration analysis of microplates. Natghi et al. (2012) have shown the effect of shear deformations of functionally graded microbeams based on modified couple stress theory. Rashvand et al. (2013) revealed the analysis on the size-dependent behavior of a capacitive circular microplate considering the variable length-scale parameter in view of modified couple stress theory. Ghayesh et al. (2013) investigated the nonlinear forced vibrations of a microbeam employing the strain gradient elasticity theory. Taati et al. (2014) presented an explicit formulation for coupled thermoelasticity addressing a Timoshenko microbeam based on strain gradient and non-Fourier heat conduction theories. Mohammad-Abadi and Daneshmehr (2014) studied about the buckling analysis of three different microbeam models; Euler–Bernoulli beam theory, Timoshenko beam theory, and Reddy beam theory utilizing modified couple stress theory. Rezazadeh et al. (2015a) proposed a TED model for a nonlocal nanobeam resonator based on GN-III theory and nonlocal elasticity theory. Further, Rezazadeh et al. (2015b) presented a detailed analysis of bias DC voltage effect on TED ratio in short nanobeam resonators based on nonlocal elasticity theory and dual-phase-lagging heat conduction model. Khanchehgardan et al. (2015) revealed the effect of mass diffusion on the damping ratio in microbeam resonators based on modified couple stress theory and the Euler-Bernoulli beam assumptions. Kakhki et al. (2016) established an analytical method to study on TED and dynamic behavior of microbeam resonators using modified coupled stress theory and the generalized theory of thermoelasticity with one relaxation time. Razavilar et al. (2016) investigated TED in rectangular microplate resonator using modified couple stress theory under plane stress condition. Yu et al. (2017) studied on the size-dependent damping of a nanobeam using nonlocal thermoelasticity and nonlocal elasticity theories which is an extension of Zener, Lifshitz, and Roukes' damping model. Kumar and Devi (2017) discussed about the response of thermoelastic functionally graded beam due to ramp type heating in modified couple stress with dual-phase-lag model. Hosseini (2018) reported a size-dependent analytical solution for nonlocal coupled thermoelasticity analysis in a heat affected MEMS/NEMS beam resonator based on Green–Naghdi theory. Borjalilou and Asghari (2018) analyzed small-scale effect of plates with TED utilizing modified couple stress theory and dual-phase-lag heat conduction model. Bostani and Mohammadi (2018) investigated TED in microbeam resonators in view of modified strain gradient elasticity theory and generalized thermoelasticity theory with one relaxation time. Borjalilou et al. (2019) also investigated TED in microbeams considering modified couple stress theory and dual-phase-lag heat conduction model. Borjalilou and Asghari (2019) revealed the analysis of TED in microbeams based on size-dependent strain gradient theory and generalized thermoelasticity theory with dual-phase-lag effects. Kumar et al. (2019) studied vibration in thermoelastic thin beam based on modified couple stress theory with three-phase-lag thermoelastic diffusion model subjected to thermal and chemical potential sources. Further investigations on TED in micro/nanobeam and plate resonators can also be found in the work recently reported by Hamidi et al. (2020), Abouelregal and Marin (2020b), Awrejcewicz et al. (2020), Borjalilou and Asghari (2021), Devi and Kumar (2020), Zhou and Li (2021), Zhao et al. (2021), Ge et al. (2021) and Shi et al. (2021).

1.6 Objective of the Thesis

The main objective of the present thesis is to study TED in vibrating micro and nanomechanical resonators under the impact of some recently developed generalized thermoelasticity theories. It also attempts to investigate the size-dependent vibrations of mechanical resonators considering MCST and Eringen's nonlocal elasticity theory. In order to analyze TED, beams and plates are considered as micro and nanomechanical resonators. The present investigations contain basically two main parts: analysis of TED by deriving the formula of the quality factor for beams and plates, and analysis of dynamic behavior of beams by finding analytical solution for deflection, temperature, and thermal moment.

The performance of the mechanical resonators is highly dependent on TED. The less value of TED offers better performance of the resonators. Therefore, the attempt is made for the analysis and minimization of TED in micro and nanobeam/plate resonators by deriving the expression of the inverse quality factor. The equations of motion are first derived and analytically solved by following frequency approach as well as entropy generation approach methods. Experimentally, it has been observed that TED is size-dependent. Therefore, in order to capture the size effect, MCST and Eringen's nonlocal elasticity theory are used in the current work. To understand the impact of thermal and elastic fields on TED, the generalized thermoelasticity theories are taken into account. Moreover, the effects of non-classical continuum theories and generalized thermoelasticity theories on TED are analyzed in detail.

This thesis also attempts to illustrate the dynamic behavior of thermoelastic micro and nanobeam resonators utilizing MCST and nonlocal elasticity theory in the framework of generalized thermoelasticity theories. In order to analyze the thermoelastic vibrations of beams, Euler-Bernoulli beam and Timoshenko beam models are adopted here. The equations of motion are first derived, and then analytically solved for simply supported beams using finite Fourier sine transform together with Laplace transform methods. The analytical solutions are obtained for deflection, temperature, and thermal moment by considering the case when a uniform load is applied to the upper surface of the beam. The effect of phase-lag time associated with generalized heat conduction equations on deflection, temperature, and thermal moment of beam are investigated. Further, the surface effects due to the curvature generated by the bending of the beam are also analyzed in the present work. Moreover, the impacts of small-scale parameter, phase-lag time, length, thickness, and surface of the beam on the vibrational responses of deflection, temperature, and thermal moment over time are thoroughly analyzed.