2 Bianchi Type-V Early Decelerating and Late Time Accelerating Cosmological Model with Perfect Fluid and Heat Conduction

2.1 Introduction

The purpose of this work is to investigate Bianchi type-V spatially homogeneous and anisotropic cosmological model when the source of gravitational field is a perfect fluid together with heat conduction. It is certainly of interest to study cosmologies with a richer structure, both geometrically and physically, than the standard perfect fluid models. Cosmological models, which are spatially homogeneous and anisotropic, play significant roles in the description of the universe in the early stages of its evolution. Bianchi I-IX spaces are very useful tools for constructing spatially homogeneous and anisotropic cosmological models. For anisotropic cosmological models, cosmologists generally consider Bianchi type-I space-time, which is the simplest generalization of flat FRW model. Bianchi type-V models are of

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particular interest since they are sufficiently complex as the Einstein tensor has off-diagonal terms, while at the same time they are simple generalization of negative curvature FRW model. The matter content in the universe is satisfactorily described by a perfect fluid. Several authors have investigated spatially homogeneous and anisotropic Bianchi type cosmological models in different physical contexts. It is worth mentioning that Bali and Meena (1998) have investigated anisotropic Bianchi type-I magnetized stiff-fluid model of the universe in general relativity. Bali and Kumawat (2008) have obtained some LRS Bianchi type-V bulk viscous tilted stiff cosmological models. As the matter content is not expected to attain thermal equilibrium in the early stages of evolution of the universe, it is evident that there would be heat flow in the universe.

The effect of heat flow in the evolution of the universe has been investigated by several authors such as Deng (1989), Mukherjee (1986), Novello and Reboucas (1978), Ray (1980), Reboucas and Lima (1981) , Reboucas (1982), Bradley and Sviestins (1984), Sviestins (1985) etc. Banerjee and Sanyal (1988) discussed Bianchi type-V cosmological model with viscosity and heat conduction and have shown that it is possible for dissipative models not to be in thermal equilibrium in their early stages. Coley (1990) investigated Bianchi type-V imperfect fluid cosmological models which contain both viscosity and heat conduction. Coley and Hoogen (1994) generalized the work of Coley and Dunn (1992) and have obtained Bianchi type-V model with both bulk viscous fluid and heat conduction. Subsequently, Bali and Sharma (2000) have investigated some tilted spatially homogeneous Bianchi type-I models filled with disordered radiation in the presence of heat conduction.

Singh (2007) presented Bianchi type-V models in the presence of perfect fluid and heat conduction. Shri Ram et al.(2008, 2009) have investigated Bianchi type-V models with perfect fluid together with heat conduction in Lyra's geometry and Saez-Ballester theory of gravitation respectively. In these works exact solutions of Einstein's field equations have been obtained by applying the variation law of Hubble's parameter in two types of cosmologies viz. the power law and exponential law cosmologies that yield constant value of deceleration parameter. In fact, the power-law and exponential law cosmologies can be used only to describe epoch based evolution of the universe because of the constancy of deceleration parameter. These cosmologies do not exhibit the transition of universe from deceleration to acceleration era.

The recent cosmological observations have confirmed that our universe is undergoing a late time accelerating expansion (Riess et al.(1998), Perlmutter et al.(1999), Bahcall (1999), Bennett et al.(2003), Spergel et al.(2003)). Cunha (2009) has provided the direct evidences caused for the present accelerating universe. Many cosmologists have suggested a number of ideas to explain the current accelerating universe such as scalar field models, exotic equation of state, modified gravity and the inhomogeneous cosmological models. It is believed that DE possesses negative pressure which tends to increase the rate of expansion of the universe (Peebles and Ratra (2003)). Kumar and Yadav (2011) have studied power-law and exponential law for Bianchi type-V space time with non interacting matter fluid and DE. Recently, Akarsu et al.(2013) have proposed a generalized form of the average scale factor of the space-time metric that leads to a mixture of power-law and exponential-law cosmologies in a unified way, called hybrid expansion law (HEL). Kumar (2013) has studied the dynamics of universe within the framework of a Bianchi type-V space time in the presence of a perfect fluid composed of non-interacting matter and dynamical DE and obtained exact solutions of Einstein's field equations by applying HEL for the average scale factor that yields power-law and exponential law cosmologies in special cases.

In this chapter, our aim is to derive a physically realistic Bianchi type-V cosmological model filled with perfect fluid and heat conduction. In Sec.(2.2), the metric and Einstein's field equations are presented. In Sec.(2.3), we apply HEL for the average scale factor to obtain exact solutions of the field equations which corresponds to an early decelerating and late time accelerating cosmological model. In Sec.(2.4), we discuss kinematical and physical behaviors of the derived model. Sec.(2.5) contains the concluding remarks.

2.2 The Metric and Field Equations

We consider the diagonal form of the spatially homogeneous and anisotropic Bianchi type-V metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx}(B^2 dy^2 + C^2 dz^2) \tag{2.1}$$

where A, B, C are functions of cosmic time t and m is a constant.

The average scale factor a and the volume scalar V of the metric (2.1) are defined by

$$V = a^3 = ABC. \tag{2.2}$$

The generalized mean Hubble's parameter H is given by

$$H = \frac{1}{3}(H_x + H_y + H_z) \tag{2.3}$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$ are the directional Hubble parameter in the direction of x, y, z respectively.

The kinematical parameters such as expansion scalar θ and shear scalar σ for Bianchi type-V metric are given as

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{2.4}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2. \tag{2.5}$$

The Einstein's field equations, in a system of units $8\pi G = c = 1$, are

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij}. \tag{2.6}$$

The energy momentum tensor T_{ij} of a perfect fluid with heat conduction is given in Eq.(1.9).

We assume that the heat flow is in x-direction only so that $h_i = (h_1, 0, 0, 0)$, h being a function of time. In comoving coordinates, the field equations (2.6) with Eq.(1.9) for the metric (2.1), can explicitly be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -p, \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -p, \tag{2.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -p, \tag{2.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = \rho, \tag{2.10}$$

$$m\left(\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=h_1. \tag{2.11}$$

The law of energy conservation equation $T^{ij}_{;j}=0$ leads to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{2m}{A^2}h_1. \tag{2.12}$$

From Eqs.(2.7)-(2.10), we obtain the energy density and pressure in terms of H, σ^2 and q as follows

$$\rho=3H^2-\sigma^2-\frac{3m^2}{A^2}, \tag{2.13}$$

$$p=H^2(2q-1)-\sigma^2+\frac{m^2}{A^2}. \tag{2.14}$$

In the next section, we follow the approach of Saha and Rikhvitsky (2006), Singh et al.(2008) to solve the field equations (2.7)-(2.11) in quadrature forms.

2.3 Solutions of Field Equations

Subtracting Eq.(2.7) from Eq.(2.8), Eq.(2.7) from Eq.(2.9) and Eq.(2.8) from Eq.(2.9) respectively, we get the following three relations:

$$\frac{A}{B}=d_1\exp\left(k_1\int\frac{dt}{a^3}\right), \tag{2.15}$$

$$\frac{A}{C}=d_2\exp\left(k_2\int\frac{dt}{a^3}\right), \tag{2.16}$$

$$\frac{B}{C}=d_3\exp\left(k_3\int\frac{dt}{a^3}\right) \tag{2.17}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. Going through further straightforward calculations, the metric functions A, B and C can be written explicitly as

$$A(t)=l_1 a\exp\left(\frac{X_1}{3}\int\frac{dt}{a^3}\right), \tag{2.18}$$

$$B(t) = l_2 a \exp\left(\frac{X_2}{3}\int\frac{dt}{a^3}\right), \tag{2.19}$$

$$C(t) = l_3 a \exp\left(\frac{X_3}{3}\int\frac{dt}{a^3}\right) \tag{2.20}$$

where

$l_1 = \sqrt[3]{d_1 d_2}$, $l_2 = \sqrt[3]{d_1{}^{-1} d_3}$, $l_3 = \sqrt[3]{(d_2 d_3)^{-1}}$,

$X_1 = k_1 + k_2$, $X_2 = k_3 - k_1$, $X_3 = -(k_2 + k_3)$

and the constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0 \qquad \text{and} \qquad l_1 l_2 l_3 = 1. \tag{2.21}$$

Several authors have presented the solutions of Eqs.(2.18)-(2.20) in power-law and exponential-law cosmologies in different physical contexts, by applying the special law of variation of Hubble parameter, proposed by Berman (1983). We consider the following ansatz for the average scale factor of the model (2.1) as

$$a(t) = k t^{\alpha} e^{\beta t}, \tag{2.22}$$

where $k > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are constants (Akarsu et al. (2013)). This generalized form of average scale is known as hybrid expansion law. We observe that HEL leads to power-law cosmology for $\beta = 0$ and to the exponential law cosmology for $\alpha = 0$. Thus, the case $\alpha > 0$ and $\beta > 0$ leads to a new cosmology arising from the HEL. Using Eq.(2.22) in Eqs.(2.18)-(2.20), we obtain the expressions of scale factors as

$$A(t) = l_1 k t^{\alpha} e^{\beta t} \exp\left\{\frac{-X_1}{3k^3}(3\beta)^{3\alpha-1}\gamma[1-3\alpha, 3\beta t]\right\}, \tag{2.23}$$

$$B(t) = l_2 k t^{\alpha} e^{\beta t} \exp\left\{\frac{-X_2}{3k^3}(3\beta)^{3\alpha-1}\gamma[1-3\alpha, 3\beta t]\right\}, \tag{2.24}$$

$$C(t) = l_3 k t^{\alpha} e^{\beta t} \exp\left\{\frac{-X_3}{3k^3}(3\beta)^{3\alpha-1}\gamma[1-3\alpha, 3\beta t]\right\} \tag{2.25}$$

where γ denotes the lower incomplete gamma function. For the scale factors A, B, C to be realistic, we must have $\alpha \leq 1/3$.

2.4 Physical and Kinematical Behaviors of the Model

The directional Hubble parameters and average Hubble parameter are obtained as

$$H_x = \frac{\alpha}{t} + \beta + \frac{X_1}{3(kt^{\alpha}e^{\beta t})^3}, \tag{2.26}$$

$$H_y = \frac{\alpha}{t} + \beta + \frac{X_2}{3(kt^{\alpha}e^{\beta t})^3}, \tag{2.27}$$

$$H_z = \frac{\alpha}{t} + \beta + \frac{X_3}{3(kt^{\alpha}e^{\beta t})^3}, \tag{2.28}$$

$$H = \frac{\alpha}{t} + \beta. \tag{2.29}$$

The shear scalar σ and expansion scalar θ have the values:

$$\sigma^2 = \frac{X_1^2 + X_2^2 + X_3^2}{18(kt^{\alpha}e^{\beta t})^6}, \tag{2.30}$$

$$\theta = 3\left(\frac{\alpha}{t} + \beta\right), \tag{2.31}$$

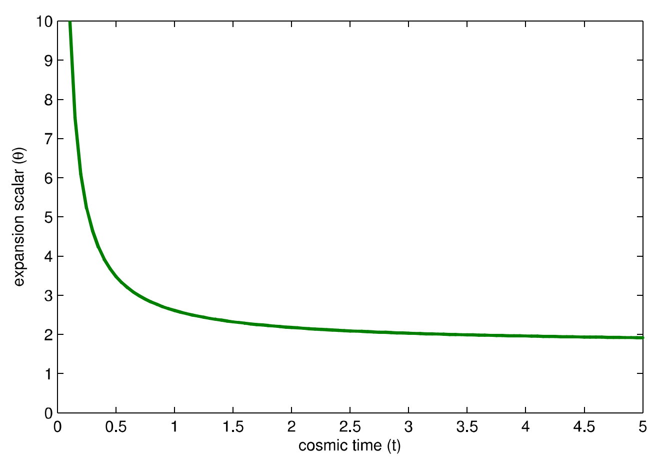

Figure 2.1: Variation of expansion scalar θ with time t for $\alpha = 0.29$, $\beta = 0.68$

Substituting the values of A, B and C in Eq. (2.11), the heat conduction vector component h_1 is calculated as

$$h_1 = \frac{mX_1}{(kt^{\alpha}e^{\beta t})^3}. \tag{2.32}$$

From Eq.(1.41), we obtain the anisotropic parameter as

$$A_m = \frac{(X_1^2 + X_2^2 + X_3^2)t^2}{27(\alpha + \beta t)^2 (k t^\alpha e^{\beta t})^6}. \tag{2.33}$$

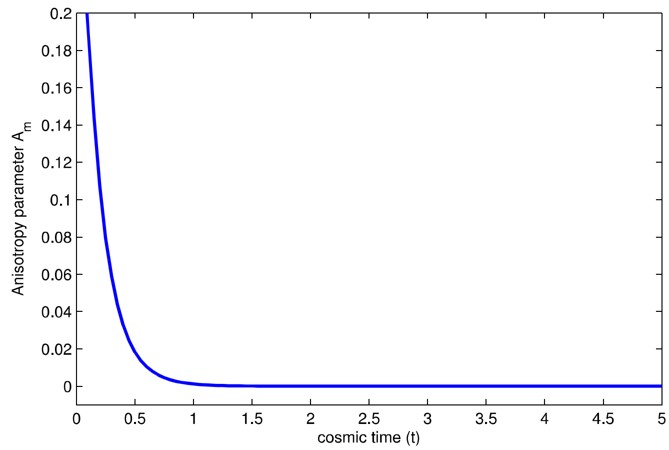

Figure 2.2: Variation of anisotropy parameter A_m with time t for $\alpha = 0.29$, $\beta = 0.68$, k=.45, $X_1 = -.05$, $X_2 = -.05$, $X_3 = 0.1$

The value of time-varying deceleration parameter is given as follows:

$$q = -1 + \frac{\alpha}{(\alpha + \beta t)^2}. \tag{2.34}$$

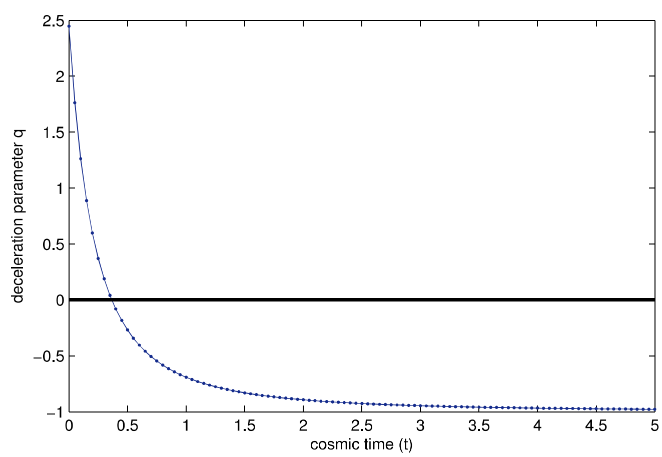

Figure 2.3: Variation of deceleration parameter q with time t for $\alpha = 0.29$, $\beta = 0.68$

Making use of Eqs.(2.29), (2.30) and (2.34) in Eqs.(2.13) and (2.14), we obtain the expressions for energy density ρ and pressure p as follows:

$$\rho = \frac{3(\alpha+\beta t)^2}{t^2} - \frac{X_1^2+X_2^2+X_3^2}{18(kt^{\alpha}e^{\beta t})^6} - \frac{3m^2}{l_1^2(kt^{\alpha}e^{\beta t})^2}\exp\left\{\frac{2X_1}{3k^3}(3\beta)^{3\alpha-1}\gamma[1-3\alpha, 3\beta t]\right\}, \tag{2.35}$$

$$p = \frac{-3(\alpha+\beta t)^2}{t^2} + \frac{2\alpha}{t^2} - \frac{X_1^2+X_2^2+X_3^2}{18(kt^{\alpha}e^{\beta t})^6} + \frac{m^2}{l_1^2(kt^{\alpha}e^{\beta t})^2}\exp\left\{\frac{2X_1}{3k^3}(3\beta)^{3\alpha-1}\gamma[1-3\alpha, 3\beta t]\right\}. \tag{2.36}$$

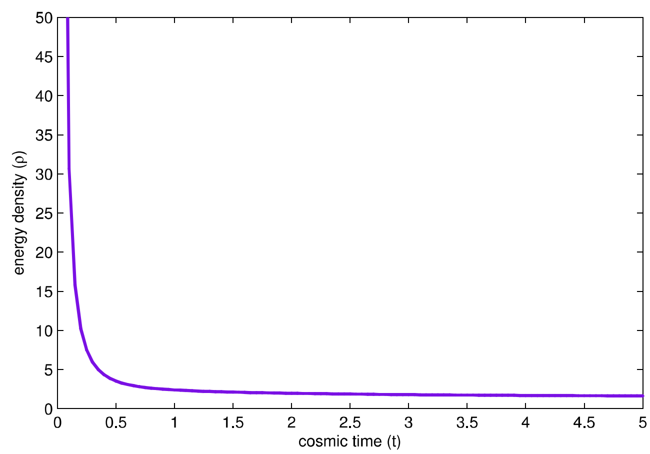

Figure 2.4: Variation of energy density ρ with time t for $\alpha = 0.29$, $\beta = 0.68$, k=.45, l=1.5, m=0.5, $X_1 = -0.05$, $X_2 = -0.05$, $X_3 = 0.1$

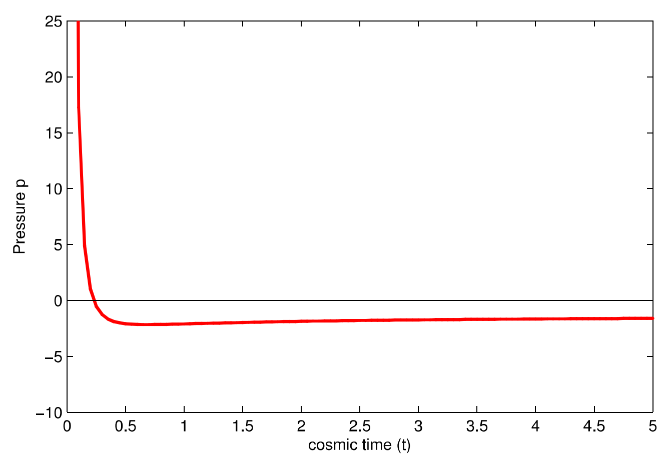

Figure 2.5: Variation of matter pressure p with time t for $\alpha = 0.29$, $\beta = 0.68$, k=0.45, l=1.5, m=0.5, $X_1 = -0.05$, $X_2 = -0.05$, $X_3 = 0.1$

We observe that the spatial volume V is zero at $t = 0$. At this epoch the energy density and matter pressure are infinite. Thus, the model has a big-bang singularity at $t = 0$. Form figs.(2.4) and (2.5), we observe that, as time increases, the energy density and pressure are gradually decreasing functions of time. The energy density attains the positive constant value, whereas matter pressure assumes negative constant value as $t \to \infty$, which shows that the universe is dominated by DE at late time causing the accelerated expansion of the model.

The variation of deceleration parameter q with time is shown in fig.(2.3). We observe that the universe evolves with variable deceleration parameter and the transition from deceleration to acceleration takes place at

$$t = \frac{\sqrt{\alpha} - \alpha}{\beta} \tag{2.37}$$

which restrict α in the range $0 < \alpha < 1$. As $t \to \infty$, $q \sim -1$ which shows the inflationary behavior of the universe at late time. This further indicates that the present-day universe is undergoing accelerated expansion.

The evolution of expansion scalar θ are shown in fig.(2.1). We see that expansion scalar is infinite at t=0 but as cosmic time t increases it attains a constant value 3β. The heat conduction vector is infinite at the initial singularity $t = 0$, which decreases as the cosmic time increases and ultimately dies out for large time. The fig.(2.2) indicates the variation of the anisotropy parameter A_m with cosmic time t. The anisotropy parameter decreases as time increases and ultimately decreases to zero as t tends to infinity. Hence, the model attains isotropy at late times which is in consistent with the recent observations that the universe is isotropic at large scale. For sufficiently large times, we find that $H \sim \beta$ which shows that the universe expands forever with the dominance of DE.

2.5 Conclusions

In this chapter, we have studied a spatially homogeneous and anisotropic Bianchi type-V cosmological model filled with perfect fluid together with heat conduction. Exact solutions of field equations have been presented by using a special form of scale factor referred to as the hybrid expansion law, being the mixture of power-law and exponential law cosmologies. The physical and kinematical behaviors have been also studied and analyzed in details. It is shown that the present model exhibits transition from deceleration to acceleration which is an essential feature of dynamic evolution of universe. The universe is accelerated expanding for late times with the dominance of DE. The universe is anisotropic for all finite time and becomes isotropic at late times.