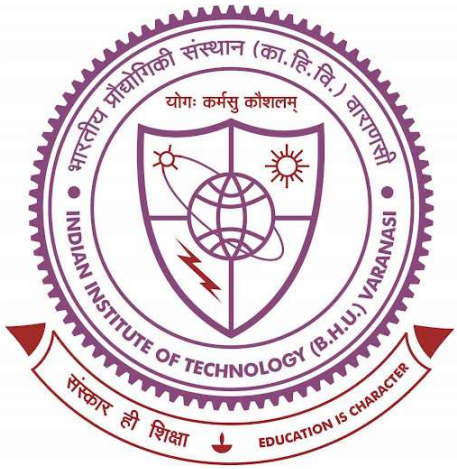
STUDY OF NON-LINEAR WAVE PROPAGATION PHENOMENA IN GAS DYNAMICS



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by

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Chapter 7

Summary and Future scope

7.1 Summary

The entire thesis is devoted for the study of the non-linear wave propagation and its properties in the various gas regime. The basic features of the propagating waves governed by quasi linear system of hyperbolic PDEs is studied by using the several analytical methods in different material medium. The effect of various differential constraints on the propagating waves and its behavior is analyzed. The following are the major highlights of this thesis:

- Investigation of evolutionary behavior of the of waves of finite and small amplitude in a non-ideal gas with the presence of dust particles.
- The study of effect of mass concentration of the dust particles, ratio of the density of solid particles to that of initial density of the medium, the relative specific heat and the effect of the non-ideal parameter, on the weak discontinuities for several geometry of flows in non-ideal dusty gasdynamics.

- Generalization of the theory of acceleration discontinuities and the study of various parameter effect on the disturbance propagated in a non-ideal dusty gas regime under the effect of magnetic field.

- Examination of the behavior of shock wave in two dimensional planar and axisymmetric non-ideal radiating gas flow under the influence of magnetic field.

- The construction of the solution of the Riemann Problem under certain conditions for the one-dimensional compressible hyperbolic system in dusty gas-dynamics under the influence of external forces.

The first chapter is introductory and gives an overview of work done in the field of non-linear wave propagation including their applications and methodology. The mathematical theory and their fundamental properties have also been briefly discussed. The basic features of the non-linear waves and its propagation is described. The physical properties of non-ideal gases, dusty gases, radiating gases used in this study are briefly reviewed. Some results, which we shall need in subsequent chapters, have also been included in this chapter.

The second chapter concerns with the study of progressive wave solution for the waves of finite and moderately small amplitude in the mixture of the non-ideal gas and dust particles governed by quasilinear hyperbolic system of PDEs. Using the progressive wave method, we derive the transport equation which provides the conditions of the shock formation, and equation to determine the shock strength. Also, it is shown that how the presence of the parameter of non-idealness and mass fraction of dust particles influence the shock formation and shock strength for the planar and non planar cases.

In third chapter, we have studied the various parameter effects on the propagation of weak discontinuities by using the method of characteristics. Analytical solutions of the quasi-linear system of hyperbolic PDEs are obtained and the evolutionary behavior of shock in the characteristic plane is examined. The general behavior of solutions to the Bernoulli equation, which determines the evolution of weak discontinuity in a non-linear system, is studied in detail. Also, we discuss the formation and distortion of compressive and expansive discontinuities under the effect of van der Waals parameter and small dust particles for planar and cylindrically symmetric flows. The comparison between planar flow and cylindrically symmetric flow is studied under the influence of non-idealness and mass fraction of dust particles.

The fourth chapter presents the study of the evolutionary behavior of plane and cylindrically symmetric acceleration discontinuities along the characteristic path under the effect of dust particles in a non-ideal magnetogasdynamic flow. It is explored how the dust particles, along with the non-ideal parameter, will influence the steepening or flattening of the propagating waves in magnetic and non-magnetic cases. The transport equation leading to the evolution of acceleration discontinuities is determined, which provides the relation for the occurrence of shock. The impact of non-idealness of the gas and dust on the evolutionary process of propagating waves for the magnetic and non-magnetic cases are discussed.

In the fifth chapter, influence of the magnetic field on the propagation of shock waves in radiation gasdynamics is analyzed by using the method of wavefront analysis. We examined the behavior of the waves propagated into the two-dimensional (2-D) steady supersonic magnetogasdynamic flow of non-ideal gas with radiation. The effect of non-idealness and thermal radiation and their consequences under the influence of magnetic field is studied and examined how the flow patterns of the disturbance vary with respect to the variation in the parameters of the flow. The

nature of the solution with respect to the Mach number is analyzed, and it is examined how the shock formation distance changes with an increase or decrease in the value of the Mach number.

In the sixth chapter, we have studied the Riemann Problem (RP) for a non-homogeneous system governing the one-dimensional compressible flow of dusty gas, where the external forces are assumed to be continuous function of time. The elementary wave solutions such as rarefaction wave, shock wave and contact discontinuity are rigorously determined. The effect of dust particles on the density, flow velocity and shock speed is examined, and their consequences on the solution of the Riemann Problem are discussed. Moreover, we discussed the condition for the existence of vacuum that appears in the solution of the Riemann Problem.

7.2 Future Scope

In this thesis, we addressed some problems concerned with the non-linear wave propagation governed by the Euler system of PDEs. One of the main contributions of our work is to study the behavior of propagating waves in various gas regime and discussed the effect of various parameters on the solution obtained by using some analytical methods. This section provides future work to consolidate the study presented in this thesis. Our study is restricted to a one-dimensional system of non-linear partial differential equations in gas dynamics. However, this analysis can be extended for two or higher dimensional non-linear partial differential equations in gas dynamics. The key areas that can be focused for future research are identified here. Future work concerns deeper analysis of particular mechanisms, new proposals to try different methods, or simply curiosity. There are some highlight of the proposed extensions of the work made in the thesis given as follows:

- In this thesis, we focused only on the one-dimensional wave propagation in various gas regime and the behavior of the waves affected by some parameters. One may study the wave interaction problems in various gas regime governed by quasilinear hyperbolic system of PDEs.

- In this thesis, we have studied the propagation of wave in dusty gases, where we neglected the velocity of the dust particles and the particles are chosen in such a way that the particles are of uniform size and there is no interaction between the dust particles. We can generalize this study for two phase flow, where we also consider the particle density.

- In this thesis, we have studied the structure of the solutions to the Riemann Problem. We only focused on the classical solutions. One may investigate the solution to the Riemann Problem for various system and determine the non-classical solutions.

- In this thesis, we have studied the one-dimensional Riemann Problem. We can extend it for two or three dimensional Riemann problem and study the various kind of solution to the two or three dimensional Riemann Problem.

- In the entire thesis, we have used some analytical methods to study the wave propagation phenomena. There are many numerical techniques available by which one can study this phenomena for various system of hyperbolic PDEs.

- There are certain problems related to the Generalized Riemann Problem (GRP) also an interesting area of research. We can use Differential Constraint Method to study the GRP for various models.

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