

PREFACE

Gas dynamics is a particular branch of fluid dynamics evolved at the end of the 19th century from attempts to understand the fundamentals of high-speed compressible flow through nozzles and passages. It has grown with the development of high-speed flight and has become an area of research for physicists, chemists, applied mathematicians and astrophysicists. The term gas dynamics suggests the idea that the field is exclusively related to the state of gaseous matter. Historically, one of the most important areas of application of gas dynamics was also the theoretical treatment of detonation in gases, in particular the description of the fundamental properties of the detonation wave, a reactive shock wave and the unsteady motion of the detonation products. However, modern gas dynamics takes a wider, more general view than in the pioneering days and is nowadays concerned with the causes and effects arising from high-speed flows, steady and unsteady, viscous and non-viscous, conducting and non-conducting for which compressibility is physically significant.

A wave can be described as a disturbance that travels through a medium, accompanied by a transfer of energy, from one point to another but the particles in the medium are not displaced by the wave. They consist of vibrations around almost fixed locations. They are broadly classified by Whitham [1] as, dispersive waves and wave motions formulated mathematically as hyperbolic system of partial differential equations. In waves represented by hyperbolic PDEs, the disturbance is not felt by every point in the space at once unlike the dispersive waves where the disturbance is felt at once by all the particles of the medium. For the hyperbolic equations, every characteristic curve in the (x, t) space represents a moving wavelet, and the behavior of the solution on a characteristic curve corresponds to the idea that information is

carried by that wavelet [2].

In 1860, Bernhard Riemann published a study titled “On the propagation of plane air waves of finite amplitude”, the first mathematical analysis of the Euler equations of gas dynamics [3]. In that work, among many contributions, is a study of what came to be known as the Riemann problem which is a hyperbolic partial differential equation (PDE) with initial data consisting of two constant states separated by a discontinuity. In the construction of the solution to the Riemann Problem, there are certain kind of waves known as elementary waves (shock wave, rarefaction wave and contact discontinuity) described by the characteristics of the governing system, constitute the solution of the problem. It’s exact solution provides the real physical properties with several wave families and their propagation. Across a rarefaction wave the flow properties are continuous. The shock wave and contact discontinuity both are the discontinuous solutions but there is a slight difference between them. Across the shock, all the flow variables experience sudden change but in case of contact discontinuity the velocity and pressure will be continuous whereas the density, temperature, entropy etc. experience a sudden change.

The present thesis, embodies the results of researches carried out by me at the Department of Mathematical sciences, Indian Institute of Technology (BHU), Varanasi, during the period July 2017 to May 2022 under the supervision of Prof. L. P. Singh. The present thesis is devoted for the study of the propagation of non-linear waves in various gaseous medium governed by the quasilinear system of partial differential equations. The main purpose of this study is to understand the wave propagation and its evolutionary behavior under certain differential constraints by using some analytical methods such as progressive wave method, method of characteristic, method of wavefront analysis etc. Also, we have studied the Riemann Problem for the non-homogeneous system for a dusty gas under the influence of external force. The entire

thesis is divided into seven chapters as follows:

Chapter - 1 is introductory and gives an overview of work done in the field of non-linear wave propagation including their applications and methodology. The mathematical theory and their fundamental properties have also been briefly discussed. The basic features of the non-linear waves and its propagation is described. The physical properties of ideal gases, non-ideal gases, dusty gases are also briefly reviewed. Some results, which we shall need in subsequent chapters, have also been included in this chapter.

Chapter - 2 concerns with the study of progressive wave solution for the waves of finite and moderately small amplitude in the mixture of the non-ideal gas and dust particles governed by quasilinear hyperbolic system of PDEs. Using the progressive wave method, we derive the transport equation which provides the conditions of the shock formation, and equation to determine the shock strength. Also, it is shown that how the presence of the parameter of non-idealness and mass fraction of dust particles influence the shock formation and shock strength for the planar and non planar cases. We also discuss the consequence of variation in the value of the non-ideal parameter and the mass fraction of small solid dust particles on the evolutionary process of the shock.

In **Chapter - 3**, we have studied the various parameter effects on the propagation of weak discontinuities by using the method of characteristics. Analytical solutions of the quasi-linear system of hyperbolic PDEs are obtained and examined the evolutionary behavior of shock in the characteristic plane. The general behavior of solutions to the Bernoulli equation, which determines the evolution of weak discontinuity in a non-linear system, is studied in detail. Also, we discuss the formation and distortion of compressive and expansive discontinuities under the effect of van der Waals parameter and small particles for planar and cylindrically symmetric flow.

The comparison between planar flow and cylindrically symmetric flow is studied under the influence of non-idealness and mass fraction of dust particles. It is found that the compressive waves become shock after a certain lapse of time. The medium considered here is the mixture of van der Waals gas with small dust particles.

Chapter - 4 presents the study of the evolutionary behavior of plane and cylindrically symmetric acceleration discontinuities along the characteristic path under the effect of dust particles in a non-ideal magnetogasdynamic flow. Implications regarding the propagation of disturbances in planar and cylindrically symmetric flows have been shown. Using the characteristics of the governing quasilinear system as a reference coordinate system, we transform the fundamental equations and find the solution. It is explored how the dust particles, along with the non-ideal parameter, will influence the steepening or flattening of the propagating waves in magnetic and non-magnetic cases. The transport equation leading to the evolution of acceleration discontinuities is determined, which provides the relation for the occurrence of shock. The impact of non-idealness of the gas and dust on the evolutionary process of propagating waves for the magnetic and non-magnetic cases are discussed. The comparison between the flow patterns and distortion of the propagating waves for planar and cylindrically symmetric flows is demonstrated under the various parameter effects.

In **Chapter - 5**, influence of the magnetic field on the propagation of shock waves in radiation gasdynamics is analyzed by using method of wavefront analysis. We examined the behavior of the waves propagated into the two-dimensional (2-D) steady supersonic magnetogasdynamic flow of non-ideal gas with radiation. The transport equation is derived, which determines the condition for the shock formation. The effect of non-idealness and thermal radiation and their consequences under the influence of magnetic field is studied and examined how the flow patterns of the disturbance vary with respect to the variation in the parameters of the flow. It is found that the presence of magnetic field plays an essential role in the wave propagation

phenomena. The nature of the solution with respect to the Mach number is analyzed, and it is examined how the shock formation distance changes with an increase or decrease in the value of the Mach number. Also, the effect of non-idealness on the shock formation distance is elucidated and examined how the shock formation is affected by the increase in the value of the non-ideal parameter in the presence of magnetic field with thermal radiation.

In **Chapter - 6**, we have studied the Riemann Problem (RP) for a non-homogeneous system governing the one-dimensional compressible flow of dusty gas, where the external forces are assumed to be continuous function of time. The elementary wave solutions such as rarefaction wave, shock wave and contact discontinuity are rigorously determined. The effect of dust particles on the density and flow velocity and shock speed is examined, and their consequences on the solution of the Riemann Problem are discussed. Moreover, we discussed the condition for the existence of vacuum that appears in the solution of the Riemann Problem.

Lastly, in **Chapter - 7**, we have summarized the complete analysis of the work done in this thesis. Major contributions made in the thesis are briefly discussed followed by a discussion on the future scope.