## Chapter 3

## Bistatic radar scattering

### 3.1 Introduction

This chapter discussed the types of radar configurations, radar coordinate systems, how radar equations can be used to capture the fully polarimetric scattering response of the targets, and the advantages/disadvantages of the bistatic radar system. Any object seen by the antenna is called a target in the field of radar sensing. Since the modeling of the Bistatic Radar Cross-Section (BRCS) or bistatic scattering coefficient ( $\sigma_{p q}^{0}$ ) is an important quantity in radar scattering measurement. Therefore, the fundamental concept of deriving the BRCS or $\sigma_{p q}^{0}$ of the distributive target using bistatic radar system configuration is discussed in detail. The bistatic scattering coefficient is defined such that it is influenced by only the structural properties of the target, incidence/scattering angle, frequency, and polarization used in the measurement system.

### 3.2 Scattering Coordinate System

When a target is illuminated by the plane wave of an electric field, it absorbs part of the energy and scatters the rest. The magnitude and direction (polarization) of the scattered


Fig. 3.1 (a) Monostatic radar system and (b) Bistatic radar system
electric field are related to the incident field, structural properties of the target relative to the incident and scattered direction, and electrical properties of the target, such as permittivity and conductivity. Radar is an instrument that uses transmitter and receiver antennas to target illumination and measure scattered reflected energy, respectively. The configuration is shown in Figure 3.1(b) consists of two antennas (i.e., transmitter and receiver) at different locations, which is called bistatic radar. However, Figure 3.1(a) represents the monostatic radar configuration in which the transmitter and receiver antennas are co-located using duplexers. Since the monostatic radar configuration measures the backscattered energy toward the receiver antenna. Therefore, the monostatic configuration is widely known as the backscattering configuration.

Theoretical scattering calculations of BRCS or $\sigma_{p q}^{0}$ are basically performed in two major types of coordinates systems: (1) Forward Scattering Alignment (FSA) convention and (2) Back Scattering Alignment (BSA) convention. The plane incident electric fields and spherically scattered waves for both the conventions are defined in terms of local coordinate systems centered on the transmitting and receiving antennas, respectively.

### 3.2.1 Forward Scattering Alignment (FSA) convention



Fig. 3.2 FSA Convention

The FSA is also called as 'wave oriented' convention as shown in the Figure 3.2. In the FSA convention, the polarization unit vectors (i.e., $\hat{v}$ and $\hat{h}$ ) is specified to the propogation of wave vector $(\hat{k})$ direction. The coordinates $(\hat{k}, \hat{v}, \hat{h})$ of the FSA are selected to match with the standard spherical co-ordinate system (i.e., $\hat{R}, \hat{\theta}, \hat{\phi}$ ). Therefore, the unit vector in the FSA convention are defined as

For wave incident in direction $\hat{k}_{i}$,

$$
\begin{align*}
& \hat{k}_{i}=\sin \theta_{i} \cos \phi_{i} \hat{x}+\sin \theta_{i} \sin \phi_{i} \hat{y}-\cos \theta_{i} \hat{z}  \tag{3.1}\\
& \hat{h}_{i}=\frac{\hat{z} \times \hat{k}_{i}}{\left|\hat{z} \times \hat{k}_{i}\right|}=-\sin \phi_{i} \hat{x}+\cos \phi_{i} \hat{y}  \tag{3.2}\\
& \hat{v}_{i}=\hat{h_{i}} \times \hat{k}_{i}=-\cos \theta_{i} \cos \phi_{i} \hat{x}-\cos \theta_{i} \sin \phi_{i} \hat{y}-\cos \theta_{i} \hat{z} \tag{3.3}
\end{align*}
$$

For wave scattered in direction $\hat{k}_{s}$,

$$
\begin{align*}
& \hat{k_{s}}=\sin \theta_{s} \cos \phi_{s} \hat{x}+\sin \theta_{s} \sin \phi_{s} \hat{y}+\cos \theta_{s} \hat{z}  \tag{3.4}\\
& \hat{h_{s}}=\frac{\hat{z} \times \hat{k_{s}}}{\left|\hat{z} \times \hat{k_{s}}\right|}=-\sin \phi_{s} \hat{x}+\cos \phi_{s} \hat{y}  \tag{3.5}\\
& \hat{v_{s}}=\hat{h_{s}} \times \hat{k_{s}}=\cos \theta_{s} \cos \phi_{s} \hat{x}+\cos \theta_{s} \sin \phi_{s} \hat{y}-\sin \theta_{i} \hat{z} \tag{3.6}
\end{align*}
$$

### 3.2.2 Back Scattering Alignment (BSA) convention



Fig. 3.3 BSA Convention

In the BSA convention, the incident and scattered polarization vectors are defined with respect to the transmitter and receiver of the radar system compared to the FSA convention. In the BSA convention, as shown in Figure 3.3, the direction of propagation of the scattered field is opposite to the incident field. In addition, the backscattering leads to reversing the direction of one out of the two polarization vectors. Therefore, for the BSA convention, we have a relation $\hat{k_{r}}=-\hat{k_{i}}$; $\hat{v_{r}}=\hat{v_{i}}$; and $\hat{h_{r}}=-\hat{h_{i}}$, expressed below

For wave incident in direction $\hat{k}_{i}$,

$$
\begin{align*}
& \hat{k}_{i}=\sin \theta_{i} \cos \phi_{i} \hat{x}+\sin \theta_{i} \sin \phi_{i} \hat{y}-\cos \theta_{i} \hat{z}  \tag{3.7}\\
& \hat{h}_{i}=-\sin \phi_{i} \hat{x}+\cos \phi_{i} \hat{y}  \tag{3.8}\\
& \hat{v}_{i}=-\cos \theta_{i} \cos \phi_{i} \hat{x}-\cos \theta_{i} \sin \phi_{i} \hat{y}-\cos \theta_{i} \hat{z} \tag{3.9}
\end{align*}
$$

For wave backscattered in direction $\hat{k_{s}}$,

$$
\begin{align*}
& \hat{k_{r}}=-\left[\sin \theta_{r} \cos \phi_{r} \hat{x}+\sin \theta_{r} \sin \phi_{r} \hat{y}+\cos \theta_{r} \hat{z}\right]  \tag{3.10}\\
& \hat{h_{r}}=-\left[-\sin \phi_{r} \hat{x}+\cos \phi_{r} \hat{y}\right]  \tag{3.11}\\
& \hat{v_{r}}=\hat{h_{r}} \times \hat{k_{r}}=\cos \theta_{r} \cos \phi_{r} \hat{x}+\cos \theta_{r} \sin \phi_{r} \hat{y}-\sin \theta_{i} \hat{z} \tag{3.12}
\end{align*}
$$

In the geometric configuration of the bistatic radar system in the FSA convention, the forward scattering direction corresponds to $\theta_{r}=\pi-\theta_{i}$; and $\phi_{r}=\phi_{i}$. However, for monostatic radar systems in the BSA convention, the direction of backscatterd wave corresponds to $\theta_{r}=\theta_{i} ;$ and $\phi_{r}=\phi_{i}+\pi$.

### 3.3 Bistatic radar equation

Figure 3.4 depicts the geometric configuration of the bistatic radar system. The p-polarized incident plane wave of power $P_{p}^{i}$ at a range $R_{i}$ from the transmitter impinges on the distributive target. The $q$-polarized scatter spherical wave of power $P_{q}^{s}$ at a range $R_{s}$ is recorded at the receiver horn antenna. The range is defined from horn antennas to the center of the illuminated target of area (A) under the Radar Field Of View (RFOV). The power loss between the distributive target and horn antenna is neglected due to the higher penetrating power of microwave signals. The $p q$-polarized radar measures the co-polarized and cross-polarized scattering by the target (i.e., HH and VV scattering if $p=q=$ either $H$


Fig. 3.4 Geometric configuration of the bistatic radar system
or $V$; and HV and VH scattering if $p=H$ and $q=V$ ). If $\lambda$ is defined as a wavelength of the transmitted microwave signals, $G_{t}$ and $G_{r}$ are the gain of the transmitting and receiving antennas, then the power scattered by the target by taking medium characteristics in the form of Fresnels coefficient into account is given by Eq. 3.13.

$$
\begin{equation*}
P_{q}^{s}=\frac{P_{p}^{i} G_{t} G_{r} \lambda^{2} \rho_{0}}{(4 \pi)^{2}\left(R_{i}+R_{s}\right)^{2}} \tag{3.13}
\end{equation*}
$$

The power reflected from the distributive target along any direction is calibrated using the scattered power with the scattered power of the known target (i.e., flat conducting
aluminum plate) to find the target reflectivity.

$$
\begin{align*}
P_{q}^{s} & =\rho_{0} P_{q}^{s}(s t d)=\left|\Gamma_{p q}\right|^{2} P_{q}^{s}(s t d)  \tag{3.14}\\
P_{q}^{s}(s t d) & =\frac{P_{p}^{i} G_{t 0} G_{r 0} \lambda^{2}}{(4 \pi)^{2}\left(R_{i}+R_{s}\right)^{2}} \tag{3.15}
\end{align*}
$$

Where $P_{q}^{s}(s t d)$ is the reflected power from a perfectly flat aluminum sheet received by the receiver antenna. $\rho_{0}$ represents reflectivity and is defined in terms of the Fresnels reflection coefficient $\left(\Gamma_{p q}\right) . G_{t 0}$ and $G_{r 0}$ are the maximum gain of the transmitting and receiving antennas. The average power scattered from the distributed target in the far-field region can be obtained by integrating the bistatic scattered power over an illuminated area (A):

$$
\begin{equation*}
P_{q}^{s}=\iint_{A} \frac{P_{p}^{i} G_{t} G_{r} \lambda^{2}}{(4 \pi)^{3} R_{i}^{2} R_{s}^{2}} \sigma_{p q}^{0} d A \tag{3.16}
\end{equation*}
$$

Where $\sigma_{p q}^{0}$ is defined as $p q$-polarized bistatic scattering cross-section per unit area or bistatic scattering coefficient. The far-field implies the Fraunhofer distance criterion (i.e., $\mathrm{R}>\frac{2 L^{2}}{\lambda}$ ), where L and $\lambda$ are the largest lateral dimension of the horn antenna and wavelength used by the radar systems. The Fraunhofer distance condition might help in finding the minimum distance for obtaining scattering responses of the target while maintaining the maximum sensitivity. For the average and meaningful $\sigma_{p q}^{0}$ of the target, the radar system is externally calibrated using the radar cross-section of the known targets. Therefore, on dividing Equations 3.16 and 3.15 using $R=R_{i}=R_{s}$, we get

$$
\begin{align*}
& \rho_{0}=\frac{P_{q}^{s}}{P_{q}^{s}(s t d)}=\frac{\iint_{A} G_{t n} G_{r n} \sigma_{p q}^{0} d A}{\pi R^{2}}  \tag{3.17}\\
& \text { where, } G_{t n}=\frac{G_{t}}{G_{t 0}} ; \text { and } G_{r n}=\frac{G_{r}}{G_{r 0}} \tag{3.18}
\end{align*}
$$

Using Equation 3.17, On assuming the values of $\sigma_{p q}^{0}$ constant over 3-dB beam-width of the antenna, we have

$$
\begin{align*}
\sigma_{p q}^{0} & =\pi R^{2} \frac{\rho_{0}}{A}  \tag{3.19}\\
A & =\iint_{A} G_{t n} G_{r n} d A \tag{3.20}
\end{align*}
$$

The illuminated area of the distributive target is calculated using the geometric configuration of the bistatic radar system as shown in Figure 3.4. The expression for illuminated target area (A) or RFOV is given as

$$
\begin{equation*}
A=\frac{\pi}{2} R^{2} \tan \frac{\phi_{a z}}{2} \sin \frac{\phi_{e l}}{2}\left[\sec \left(\theta-\frac{\phi_{e l}}{2}\right)+\sec \left(\theta+\frac{\phi_{e l}}{2}\right)\right] \tag{3.21}
\end{equation*}
$$

Where, $\theta, \phi_{e l}$, and $\phi_{a z}$ are the look angle, elevation beam-width, and azimuth beamwidth of the horn antenna. On substituting the target illuminated area in Equation 3.19, the average value of the $\sigma_{p q}^{0}$ in dB of the distributed target, normalized to the RFOV of horn antenna, is expressed as

$$
\begin{equation*}
\sigma_{p q}^{0}(d B)=10 \log _{10}\left[\frac{2 \rho_{0} \cot \frac{\phi_{a z}}{2} \csc \frac{\phi_{e l}}{2}}{\sec \left(\theta-\frac{\phi_{e l}}{2}\right)+\sec \left(\theta+\frac{\phi_{e l}}{2}\right)}\right] \tag{3.22}
\end{equation*}
$$

On knowing the following parameters, such as the reflectivity of the target, elevation angle, azimuth angle, and look angle, the value of $\sigma_{p q}^{0}$ of the target can be computed using Equation 3.22.

### 3.4 Advantages of bistatic radar system

The geometric configuration of the bistatic radar systems introduces technical complications in synchronizing transmitter and receiver at different locations and is significantly
costlier than monostatic radar systems. However, the bistatic configuration has several potential advantages over monostatic configurations:
$\checkmark$ It has the potential to detect stealthy targets which are shaped to scatter energy in other directions from monostatic.
$\checkmark$ In many instances, the receiver is undetectable and safer. Therefore, it is difficult to find the bistatic radar receiver using electronic support methods.
$\checkmark$ Bistatic radar is becoming more appealing as the use of Unmanned Aerial Vehicles (UAVs) grows. For example, UAVs can only carry the receiver and leave the bulky, complex, and power-hungry transmitter behind.
$\checkmark$ Many traditional technical complications such as difficulties in synchronization and geolocation problems are now efficiently addressed using the Global Positioning System (GPS).
$\checkmark$ The bistatic radar system can obtain stronger scattered echoes from the target. The higher system design flexibility provides extra freedom that helps in retrieving enhanced information about the target for remote sensing applications.

