# Chapter 5

# Summary and Future Plans

## 5.1 Summary

In this thesis we have demonstrated the effects of coupling of NV center hosted in a diamond nanocrystal with different types of nonlinear nanomechanical oscillators. Study in Chapter 2 is mainly focused on hybrid system which consist of a periodically driven nonlinear oscillator and a quantum spin-1/2 system and Chapter 3 discuss about coupling of kicked rotator and a NV center spin. In this chapter we have discussed both cases NV center as spin-1/2 and as spin-1 system. Chapter 4 describes about coupling of a coupled nonlinear oscillators and two NV center spin which is not directly coupled they are coupled through oscillators and oscillators are directly coupled. These results are concluded below, ideas for future work to build upon these findings are presented in section 5.2.

Coupling of a periodic driven nonlinear oscillator and NV center spin which is spin-1/2 system gives exciting phenomena. This type of NEMS hybrid system is studied in Chapter 2. In this study we analysed spin dynamics in the region where energy spectrum of mathematical pendulum is degenerate and non-degenerate depending on the height of the potential barrier. We move from degenerate to non-degenerate region by varying height of the potential barrier. We also explored effects of environment on the hybrid system. We investigated divergence $C(\rho(t)|\rho_d)$ which is quantifier of generation of coherence.

Later, in Chapter 3, study is mainly based on hybrid NEMS system which consist of quantum and classical system where classical part is a nanomechanical cantilever, and quantum part is an NV center spin, a quantum spin-1/2 system. Nonlinear nanomechanical cantilever works as a Kicked rotator which oscillates in regular as well as in chaotic regimes. Due to the coupling between spin-cantilever, we expected to see the effect of the oscillation of kicked rotator onto the spin dynamics. We have observed the effect of dynamical chaos on the quantum subsystem. It should be noted that the dynamical chaos is imposed on the cantilever dynamics through the kicking. In order to see this effect of dynamical chaos on quantum subsystem we explored Poincaré section of spin-dynamics and studied the Fourier power spectral-density of spin dynamics in the chaotic and regular regimes. It clearly shows dynamic stochasticity in chaotic regime. We also studied generation of coherence and quantum Poincaré recurrence in regular and chaotic regions. We observed signature of quantum chaos on quantum subsystem in only Fourier power spectrum. The reason may be the small Hilbert space of the quantum subsystem. Therefore we extended the Hilbert space of the quantum subsystem by considering the NV center spin as spin-1 system (a three-level system). We calculated Fourier power spectrum and level statistics for this three level system. We found that the Fourier power spectrum and nearest-neighbour level spacing distribution shows the essence of chaos when oscillator motion is in chaotic regime. The nearest-neighbour level spacing distribution is nearly a Gaussian distribution due to level repulsion in chaotic case while in regular case it is nearly a Poissonian distribution. We also explored feedback effects in spin-1/2 case which is not significant.

Subsequently in Chapter 4, we studied the spread of quantum correlations in a system which consists of a coupled oscillator and two NV center spins. For this, we calculated Out of time order correlation which is quantitative measure of spread of quantum correlation.

The system is considered in such a way that the oscillators are coupled directly to each other while NV center spins are coupled through the oscillators. Quantum correlation between NV center spins arises due to the quantum feedback exerted by first NV center spin to the first oscillator transferred to second oscillator due to direct coupling and then to the second NV center spin. The out of order correlation is considered as a quantifier of the quantum feedback. We calculated OTOC when the oscillator is oscillating in different dynamical regimes such as linear, nonlinear, driven linear and driven nonlinear regime. We see that in the all the cases whether oscillators are weakly connected or strongly connected, the OTOC remains zero which is indicating the absence of quantum feedback. We also explored another case with a quantum harmonic oscillator and two quantum NV center spins. We consider this case as a strong connectivity limit when the coupled oscillators behave as a single oscillator and NV center spins are attached at the opposite ends of quantum harmonic oscillator through magnetic tips. We have shown that in this quantum case even though the NV center spin are not directly coupled still OTOC, GME and concurrence are non-zero and in the classical limit of the oscillator, OTOC vanishes.

## 5.2 Future Plans

Further work is required to get more insight into hybrid NEMS. We plan to explore propagation of quantum information through a chain of NEMS with a nearest coupling. We also plan to consider a nonlinear chain using Fermi–Pasta–Ulam (FPU) model and explore extensively. We may ask questions: What will be the effect of the localized energy in the middle of FPU chain for propagation of quantum information through this localization region? Is there any connection between localization of the mechanical energy in parts of the chain and entanglement? What will be the role of environment in such a system?