Table of contents

List of figures **xxi**

Nomenclature **xxxiii**

1 Introduction **1**

1.1 Overview 1

1.2 Importance of the study of nonlinearity in NEMS and MEMS 6

1.3 Origin of nonlinearity in NEMS resonators 7

1.3.1 Nonlinearity due to external potential and geometry 9

1.4 Nitrogen vacancy (NV) center spin 10

1.4.1 Structure of NV center spin in Diamond 13

1.5 Quantum mechanical oscillators 14

1.6 Kicked rotator 18

1.7 Two coupled nanomechanical resonators 22

1.7.1 Experimental Setup of coupled nanomechanical resonators 23

1.7.2 Theoretical Modeling of coupled nanomechanical resonators . . . 23

1.8 Chaos 26

1.8.1 Limitations of chaos theory 27

1.9 Open quantum system 28

1.9.1 Markovian Lindblad Master Equation 29

1.9.2 Dynamical Map 31
1.9.3 Non-Markovian Behavior of Quantum Processes in Open Systems 33
1.10 Entanglement 36
1.11 Scrambling in a quantum system 39
1.12 Outline of the Thesis 41

2 Generation of coherence in an exactly solvable nonlinear nanomechanical system 43

2.1 Introduction 43
2.2 Theoretical modeling 45
2.2.1 Classical cantilever dynamics 46
2.3 Quantum cantilever dynamics 50
2.4 Quantum spin dynamics of NV center 52
2.5 Dissipation 60
2.5.1 Markovian Lindblad master equation 60
2.5.2 Fluctuations due to the spin bath 64
2.6 Multilevel dynamics 69
2.7 Unitary generation of coherence 72
2.8 Conclusion 79

3 Hybrid quantum-classical chaotic NEMS 81

3.1 Introduction 81
3.2 Dynamics of the cantilever 85
3.3 Spin-1/2 system attached to the cantilever 88
3.3.1 Evolved in time wave function 88
3.3.2 Expectation values of the NV spin components 91
3.3.3 Quantum coherence 92

3.3.4 Quantum Poincaré recurrence 96
3.3.5 Level statistics for spin-1/2 case 99
3.4 Dynamics of a three-level NV system 99
3.5 Statistical average over various I_0 and θ_0 104
3.6 Feedback Effect 107
3.7 Conclusions 110

4 Scrambling and quantum feedback in a nanomechanical system 113
4.1 Introduction 113
4.2 Model 117
4.3 Results and discussion 121
4.3.1 Analytical solution in the absence of feedback 121
4.3.2 Autonomous case 123
4.3.3 External driving 126
4.4 Inherently quantum case: Non-zero OTOC, geometric measure of entanglement and concurrence 127
4.5 Conclusions 132

5 Summary and Future Plans 133
5.1 Summary 133
5.2 Future Plans 135

References 137

Appendix A Generation of coherence in an exactly solvable nonlinear nanomechanical system 155
S-I Matrix elements in G_-, G_0 and G_+ regions 155
S-II Eigenvectors in G_- region 161

S-III Coefficients of density matrix in G_- region 162

S-IV Calculation of $d|c_1(t)|/dt$ 163

S-V Reduced density matrix $\rho_s(t)$ 164

S-VI Eigenvalues and eigenvectors related to section 2.7 165

Appendix B Hybrid quantum-classical chaotic NEMS 169

S-I Normalization condition for the wave function represented in Eq.(3.16) when $G_n\{\varphi\}$ is diagonal. 169

S-II Normalization condition for the wave function represented in Eq.(3.16) when $G_n\{\varphi\}$ is non-diagonal. 171

S-III Expectation value of $\langle\sigma_\alpha\rangle$, $\alpha = x,y,z$ 175

S-IV Elements of density matrix for Eq.(3.19) 176

S-V Normalization constants for eigenstates of Floquet operator for Eq.(3.26) and matrix elements for Eq.(3.28) 178

Appendix C Scrambling and quantum feedback in a nanomechanical system 181

S-I Appendix: Calculation of thermally averaged OTOC C_ρ 181

List of figures

1.1 (Color online) Nanomechanical resonator of the dimensions $(L,W,T) = (3000,300,30)nm$ attached to a single NV center electron spin with the help of magnetic tip made up of ferromagnetic material at the end of the resonator. The distance between NV center electron spin and magnetic tip is $25nm$ in absence of external driving. Microwave and laser fields are used to manipulate and measure the spin states. Range of Microwave frequency from 100 - 1000 GHz and Nd:YAG laser can be used. 8

1.2 (Color online) Nitrogen vacancy (NV) center in a diamond lattice. 11

1.3 (Color online) Spin dynamics in the NV center in diamond. The transition between ground and excited state is spin conserving. The spin polarization comes from decay via the intermediate singlets state from $m_s = \pm 1$ to $m_s = 0$. 12

1.4 (Color online) Level diagram of the driven NV center in the electronic ground state. 13

1.5 The Phase space plot of Kicked rotator in (a) the regular regime $K = 0.3$ (Red) where the phase space is covered by two different phase trajectories: open hyperbolic and some part of closed elliptic, and (b) the chaotic regime at $K = 1.5$ (Green) where the entire phase space is covered by a chaotic sea. 22

1.6 The figure is taken from Sotiris C. Masmanidis et al. science.1144793 (2007) only to depict a case of Nanomechanical piezoelectric actuation. (A) Shows a GaAs cantilever with embedded pin diode structure. (B) Setup for measurement used by Masmanidis et al. The bias T allows both dc and ac signals to be applied. The measurement are performed by Masmanidis et al. at room temperature and a pressure of 5 millitorrs. (C) Frequency response curve near the resonance of a diode-embedded cantilever under 0 dc bias. 24

1.7 (Color online) A total system divided into the system of interest, "System", and the environment. 29

1.8 Schematics representation of Entanglement. 38

2.1 Energy spectrum $E_n(l)$ of Mathieu-Schrödinger equation with varying barrier height l. The region where curves are split is called G_0 and the merging points define the boundaries of the G_- and G_+ subgroups. The energy spectrum corresponding to Mathieu function $|ce_n(l,\varphi)\rangle$ is described by Mathieu characteristic $a_n(l)$, and the energy spectrum corresponding to Mathieu function $|se_n(l,\varphi)\rangle$ is described by Mathieu characteristic $b_n(l)$. Barrier height l is in units of $\frac{U}{\omega'}$. 50

2.2 (a) Average transverse spin component $\langle\sigma_x\rangle$, (b) average transverse spin component $\langle\sigma_y\rangle$, and (c) average longitudinal spin component $\langle\sigma_z\rangle$, plotted for the bipartite system $\hat{\rho}_{AB}$ in the region G_0 for different quantum numbers $n = 2,3,4$. In all the figures blue (solid), orange (solid) and violet (dashed) lines represent $n = 2, l = 3.855$, $n = 3, l = 7.535$ and $n = 4, l = 10.785$ cases, respectively. The values of the barrier heights l are chosen to be in G_0 region for the given n. The interaction strength between the nonlinear oscillator and the NV spin is taken to be $Q = 0.5$. Time is in the units of ω_0^{-1}, which is of the order of nanoseconds. 52

2.3 (a) The amplitude and (b) frequency of $\langle\sigma_x\rangle$ with respect to barrier height l for different values of n. The quantum numbers n and l are chosen such that the mathematical pendulum remains in G_0 region. In all the figures, red (solid), violet (Dot-Dashed) and blue (dashed) lines represent $n = 2$, $n = 3$ and $n = 4$ cases, respectively. The dots in the figures represent the points which are considered in Fig. 2.2. Barrier height l is in units of $\frac{U}{\omega'}$. . 55

2.4 (a) Average transverse spin component $\langle\sigma_x\rangle$, (b) average transverse spin component $\langle\sigma_y\rangle$, and (c) average longitudinal spin component $\langle\sigma_z\rangle$, plotted as a function of time for the bipartite system $\hat{\rho}_{AB}$ in the region G_- for different quantum numbers $n = 2,3,4$. In all the figures blue (solid), orange (solid) and violet (dashed) lines represent $n = 2, l = 0.1$, $n = 3, l = 0.57$ and $n = 4, l = 1.585$ cases, respectively. In all the cases the coupling constant Q is equal to 0.5. The values of barrier heights l are chosen to be in the region G_- for the given n. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. 58

2.5 Purity for the hybrid system of NV center and nonlinear oscillator. (a) Behavior of purity for a damping constant $\gamma = 0.01$ at an arbitrary interaction strength Q. The blue (solid), red (solid), yellow (solid) and purple (solid) lines represent $Q = 0.5$, $Q = 5$, $Q = 10$ and $Q = 25$ cases, respectively. (b) For an interaction strength $Q = 0.5$ but at arbitrary damping constant γ. The blue (solid), red (solid), yellow (solid) and purple (solid) lines represent $\gamma = 0.01$, $\gamma = 0.02$, $\gamma = 0.03$ and $\gamma = 0.04$ cases, respectively. In G_0 region for the quantum state $n = 4$. The barrier height $l = 10.785$ corresponds to the region G_0 in the vicinity of the transition into the region G_-. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. . 61

2.6 Entropy of the hybrid system of NV center and nonlinear oscillator in the G_0 region for the quantum state $n = 4$. The barrier height $l = 10.785$ corresponds to the region G_0. The coupling strength $Q = 0.5$ at arbitrary damping constant γ. The blue (solid), red (solid), yellow (solid) and purple (solid) lines represent $\gamma = 0.01$, $\gamma = 0.02$, $\gamma = 0.03$ and $\gamma = 0.04$ cases, respectively. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds and entropy is in the unit of 'bits'. 62

2.7 Longitudinal spin component $\langle\sigma_z\rangle$ of the hybrid system of NV center and nonlinear oscillator (a). The blue (solid), red (solid) and yellow (solid) lines represent $Q = 1$, $Q = 5$ and $Q = 25$ cases, respectively. For different damping coefficient γ and fixed coupling strength $Q = 0.5$ (b). The blue (solid), red (solid), yellow (solid) and purple (solid) lines represent $\gamma = 0.01$, $\gamma = 0.02$, $\gamma = 0.03$ and $\gamma = 0.04$ cases, respectively. Both cases are in G_0 region for the quantum state $n = 4$ is considered. The barrier height $l = 10.785$ corresponds to the region G_0 in the vicinity of the transition to the G_- region. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. 63

2.8 $F(N,t)$ is plotted for different values of number of reservoirs N and time t. Weak system-reservoir coupling is considered with parameters $\tau = 1$, $g = \frac{3\tau}{8}$ and $\delta = 0.2$. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. 68

2.9 Purity of the system coupled with N reservoirs within a weak coupling regime with parameters $\tau = 1$ and $g = \frac{3\tau}{8}$, $\delta = 0.2$ (solid), $\delta = 0.5$ (dotted) for $c_0(0) = 0$ and $c_1(0) = 1$. The red, blue and black lines represent $N = 2,3,4$ cases, respectively. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. 69

2.10 (a) Transverse spin component $\langle\sigma_x\rangle$, (b) transverse spin component $\langle\sigma_y\rangle$, and (c) longitudinal spin component $\langle\sigma_z\rangle$ plotted as function of time for the bipartite system $\hat{\rho}_{AB}$ in the region G_0 for different multilevel quantum states $n = 2,3,4$. The blue (solid), orange (solid) and violet (dashed) lines represent $n = 2, l = 3.855$, $n = 3, l = 7.535$ and $n = 4, l = 10.785$ cases, respectively. The values of barrier heights l are chosen to be in the region G_0 in the vicinity to the transition into the region G_-. The interaction strength between nonlinear oscillator and NV spin is $Q = 0.5$. Time t is in the units of ω_0^{-1}, which is of the order of nanoseconds. 72

2.11 The Coherence $\mathscr{C}(\rho(t)|\rho_d)$ plotted for the bipartite system in the region G_0 for different quantum states $n = 2,3,4$ and corresponding barrier height l in G_0 region . The red (solid), violet (Dotted-Dashed) and blue (Dashed) lines represent $n = 2$, $n = 3$ and $n = 4$ cases, respectively. The parameter used for the plot is $p_1 = 0.9$ and $p_2 = 0.1$, $\Delta\omega_0 = 0.8$ and $\omega_0 = 1$, $Q = 5$. Barrier height l is in units of $\frac{U}{\omega'}$. 77

2.12 The quantum distance R_{2n+1} from the classically chaotic region plotted as a function of the barrier height l. The values of barrier heights l are chosen from the region G_0 in the vicinity to the transition into the region G_-, *i.e.*, in the vicinity to classical homoclinic tangle. Barrier height l is in units of $\frac{U}{\omega'}$. 79

3.1 The Phase space plot of cantilever's dynamics constructed through the recurrence relations Eq. (3.6) in (a) the regular regime $K = 0.5$ (Blue) where the phase space is covered by two different phase trajectories: open hyperbolic and some part of closed elliptic, and (b) the chaotic regime at $K = 10$ (Gray) where the entire phase space is covered by a chaotic sea. Topologically different phase trajectories are bordered by separatrix line. The values of parameters are: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$ 83

3.2 Poincaré sections for $(\langle\sigma_x\rangle, \langle\sigma_z\rangle)$ and $(\langle\sigma_y\rangle, \langle\sigma_z\rangle)$ in the regular regime at $K = 0.5$ ((a) and (b)) and chaotic regime at $K = 10$ ((c) and (d)) . The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units are $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$, $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. 90

3.3 Fourier Power spectrum density for expectation values of $\sigma_{x,y,z}$ in the regular regime ((a), (c) and (e)) at $K = 0.5$ (Blue), and in the chaotic regime ((b), (d) and (f)) at $K = 10$ (Gray). The parameters used for the plot are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. 93

3.4 Spin dynamics for $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$ in the regular regime at $K = 0.5$ (see (a), (c) and (e)) and $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$ in the chaotic regime at $K = 10$ (see (b), (d) and (f)). The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$, $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 94

3.5 The quantum coherence in (a) the regular regime at $K = 0.5$ (Blue) and (b) the chaotic regime at $K = 10$ (Gray), for the following values of the parameters $m = 1, g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of parameters are $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, T=10$\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 95

3.6 Quantum Poincaré recurrence as a function of time (*i.e.*, number of kicks) in (a) the regular regime at $K = 0.5$ (Blue) and (b) the chaotic regime at $K = 10$ (Gray). The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of parameters: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, T=10$\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 96

3.7 Histogram plot of level statistics of Hamiltonian $\hat{H}_n$ in the regular regime at $K = 0.5$ (Blue) and in the chaotic regime $K = 10$ (Gray). A reference plot for poissonian statistics (Orange) is also shown. For this plot we have taken upto 1000 kicks to get the ensemble. The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, T=1, $\alpha = \pi/2$. The values of parameters: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_0^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, T=10$\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. 98

3.8 Fourier Power spectrum density for the components $S_{x,z}$ in the regular regime ((a) and (c)) at $K = 0.5$ (Blue), and in the chaotic regime ((b) and (d)) at $K = 10$ (Gray). The parameters used for the plot are $m = 1$, $g = 1$, $\Omega = 1$, $\delta = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. 100

3.9 Spin dynamics for the components S_x and S_z for Spin-1 case in the regular regime at $K = 0.5$ ((a) and (c)) and in the chaotic regime at $K = 10$ ((b) and (d)). The parameters are $m = 1$, $g = 1$, $\Omega = 1$, $\delta = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$, $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 101

3.10 Histogram plot of level statistics of Hamiltonian $\hat{H}_n = \hat{H}_{NV} + g\hat{V}_{c,NV}$ for spin-1 system (a) in the regular regime at $K = 0.5$ (Blue) and (b) in the chaotic regime $K = 10$ (Gray). A reference plot for Poissonian statistics (Orange) and Gaussian statistics (Red) is also shown. 1000 kicks are considered. The parameters are $m = 1$, $g = 1$, $\Omega = 1$, $\delta = 1$, $\omega_r = 0.2$, T=1, $\alpha = \pi/2$. The values of parameters: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_0^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, T=$10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. . . 103

3.11 Statistical average of spin dynamics (Spin-1/2 system) for $\langle\langle\sigma_x\rangle\rangle$, $\langle\langle\sigma_y\rangle\rangle$ and $\langle\langle\sigma_z\rangle\rangle$ in the regular regime at $K = 0.5$ ((a), (c) and (e)) and in the chaotic regime at $K = 10$ ((b), (d) and (f)). For calculating statistical average of spin dynamics (Spin-1/2 system) we have taken 15 different sets of (I_0, θ_0). The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$, $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 105

3.12 Statistical average of Fourier Power spectrum density (Spin-1/2 system) for $\langle\langle\sigma_x\rangle\rangle$, $\langle\langle\sigma_y\rangle\rangle$ and $\langle\langle\sigma_z\rangle\rangle$ in the regular regime ((a), (c) and (e)) at $K = 0.5$ (Blue), and in the chaotic regime ((b), (d) and (f)) at $K = 10$ (Gray). For calculating Statistical average of Spin dynamics(Spin-1/2 system) we have taken 15 different sets of (I_0, θ_0). The parameters used for the plot are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic case $\varepsilon = 0.003$ and for the regular case $\varepsilon = 0.0003$. 106

3.13 The Phase space plot of cantilever's dynamics constructed through the recurrence relations Eq. (3.37) with feedback in ((a) and (b)) the regular regime $K = 0.5$ (Blue) where the phase space is covered by two different phase trajectories: open hyperbolic and some part of closed ellipse.The parameters for fig.3.13(a) are $m = 1$, $g = 0.1$, $\omega_0 = 10$, $\omega_r = 0.2$, $T = 1$. The parameters for fig.3.13(b) are $m = 1$, $g = 0.01$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$ $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for the regular case $\varepsilon = 0.0003$. 108

3.14 Spin dynamics with feedback (Solid) and without feedback (Dashed) for $\langle\sigma_z\rangle$ in the regular regime at $K = 0.5$ (see (a)) and $\langle\sigma_z\rangle$ in the chaotic regime at $K = 10$ (see (b)). The parameters are $m = 1$, $g = 1$, $\omega_0 = 1$, $\omega_r = 0.2$, $T = 1$, $\alpha = \pi/2$. The values of the parameters in the real units: $K = \varepsilon I_0 T \frac{6\pi\mu}{m^2\omega_r^2}$, $\mu = \frac{\omega_r^2 m}{2a_0^2}$, $I_0 = \frac{m}{2}x_0^2\omega_r$, $m = 6\times10^{-17}$Kg, $x_0 = a_0 = 5\times10^{-3}$m, $T = 10\mu$s, $\omega_r = \omega_0 = 2\pi\times5\times10^6$Hz, for chaotic $\varepsilon = 0.003$ and for regular $\varepsilon = 0.0003$. 109

4.1 (Color online) Schematics of two NV electron spins coupled via coupled oscillators. Oscillators are coupled to each other directly while NV electron spins are not. 116

4.2 Autonomous linear oscillators and weak connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0$, $\xi = 0$, $\gamma = 0$, $g = 1$, $K = 0.1$, $\alpha = \pi/3$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 118

4.3 Autonomous linear oscillators and strong connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0$, $\xi = 0$, $\gamma = 0$, $g = 1$, $K = 10$, $\alpha = \pi/3$. Time is in units of ω_0^{-1}, which is of the order of nanoseconds. 119

4.4 Autonomous nonlinear oscillators and weak connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0$, $\xi = 1$, $\gamma = 0$, $g = 1$, $K = 0.1$, $\alpha = \pi/3$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 122

4.5 Autonomous nonlinear oscillators and strong connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0$, $\xi = 1$,$\gamma = 0$, $g = 1$, $K = 10$, $\alpha = \pi/3$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 124

4.6 Driven nonlinear oscillators and weak connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0.5$, $\xi = 1$,$\gamma = 0.15$, $g = 1$, $K = 0.1$, $\alpha = \pi/3$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 125

4.7 Driven nonlinear oscillators and strong connectivity regime: Position and phase space plots in (a) and (b), and spin dynamics and OTOC in (c) and (d). Inset in (d) shows two-point time ordered correlation. The values of the parameters are $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0.5$, $\xi = 1$,$\gamma = 0.15$, $g = 1$, $K = 10$, $\alpha = \pi/3$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 127

4.8 Plot of average energy versus time for (a) the oscillator and (b) the NV spin, in the strong connectivity regime. The values of the parameters are: $\omega_0 = 1.5$, $\omega_1 = 1.0$, $\omega_2 = 1.5$, $F_1 = F_1 = 0$, $\xi = 0$, $\gamma = 0$, $g = 1$, $K = 10$, $\alpha = \pi/3$. The modulation depth of energies $\delta\langle H_{NV}\rangle = 0.5$, $\delta H_0 = 1.5$. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds. 128

4.9 Plot of thermally averaged OTOC vs time at fixed Temperature(T). The parameters used are $g = 1$, $\omega_0 = 3$, $\omega = 2$, $T = 100$, $n = 10$(red), $n = 100$(Green), $n = 1000$(Gray), $n = 10000$(blue). For very large n the oscillations die out. Time t is in units of ω_0^{-1}, which is of the order of nanoseconds and Temperature is in the units of ω/K_B where K_B is Boltzmann constant. 131