

7 Some Kaluza- Klein Cosmological Models in f(R,T) Gravity Theory *

7.1 Introduction

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimension beyond those four observed so far. Higher dimensional theories of Kaluza-Klein(KK)-type have been considered to study some aspects of early Universe (Chodos and Detweiler, 1980; Freund, 1982; Shafi and Wetterich, 1984; Sahdev, 1984). In such KK theory it has been assumed that the extra dimension form a compact manifold of very small size undetectable at present day energies. Thus, in such higher dimensional theories one would expect that at the grand unification scale the word manifold has more than one dimension. The Kaluza-Klein theory is attractive because it has an elegant presentation interms of geometry. In certain sense, it looks just like ordinary gravity in free space, except that it is phrased in five dimensions instead of four. Kaluza (1921)and (Klein, 1926a.,b) attempted to unify gravitation and electromagnetism. An interesting possibility known

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as the cosmological reduction process is based on the idea that at very early stage all dimensions in the universe are comparable. Later, the scale of the extra dimension becomes so small as to be unobservable by experiencing contraction. Such cosmological models were investigated by Forgacs and Horvath (1979). Guth (1981); Alvarez (1983) observed that during the contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario. Gross and Perry (1983) have shown that the five-dimensional Kaluza-Klein theory of unified gravity and electromagnetism admits soliton solutions. Further, they explained the inequality of the gravitational and inertial masses due to the violation of Birkoffs theorem in Kaluza-Klein theories, which is consistent with the principle of equivalence. Appelquist and Chodos (1983) claimed through solution of the field equations that there is an expansion of four-dimensional space-time while fifth dimension contracts to the unobservable Plankian length scale or remains constant as needed for the real universe.

Recent observations of type Ia Supernovae (SNe Ia) at red shift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be accelerating behavior, attributed to “Dark Energy” with negative pressure. These observations (Chaterjee, 1992; Frieman and Waga, 1998; Ozer and Taha, 1987; Carvalho et al., 1992; Ratra and Peebles, 1988), strongly favour a significant and positive value of Λ . A number of models for dark energy to explain the late-time cosmic acceleration without the cosmological constant has been proposed, for example, a canonical scalar field, so-called quintessence, a non-canonical scalar field such as phantom, tachyon scalar field motivated by string theories, and a fluid with a special equation of state (EoS) called as Chaplygin gas. Nojiri and Odintsov (2003a,b) have presented a review of various modified gravities which have considered as gravitational alternative for dark energy. Nojiri and Odintsov (2004.) proposed that dark energy may become over standard matter due

to universe expansion. Carroll et al. (2004) explained the presence of late time cosmic acceleration of the universe in $f(R)$ gravity and proposed that dark energy model for specific $\frac{1}{R}$ modified gravity. Allemandi et al. (2005) discussed the dark energy dominance cosmic acceleration in first order Palatini formalism. There also exists a proposal of holographic dark energy. One of the most important quantity to describe the features of dark energy models is the equation of state parameter (EoS) ω , which is the ratio of the pressure p to the energy density ρ of dark energy, defined as $\omega = \frac{p}{\rho}$. There are two ways to describe dark energy models. One is a fluid description and the other is to describe the action of a scalar field theory. In both description, we can write the gravitational field equations, so that we can describe various cosmologies, e.g., the Λ CDM model, in which ω is a constant and exactly equal to -1 , quintessence model, where ω is a dynamical quantity and $-1 < \omega < -\frac{1}{3}$, and phantom model, where ω also varies in time and $\omega < -1$. This means that one cosmology may be described equivalently by different model descriptions discussed by Bamba et al. (2012). In view of the late time acceleration of the universe and the existence of dark energy and dark matter, several modified theories of gravitation have been proposed as alternative to Einstein's theory. Noteworthy amongst them is the $f(R)$ gravity theory. Nojiri and Odintsov (2006a) developed the general scheme for modified $f(R)$ gravity reconstruction from any realistic FRW cosmology. They have shown that the modified $f(R)$ gravity indeed represents the realistic alternative to general relativity, being more consistent in dark epoch. Nojiri and Odintsov (2006b) developed a general programme for unification of matter -dominated era with acceleration epoch for scalar -tensor theory or dark fluid. Nojiri and Odintsov (2007) have reviewed various modified gravities considered as gravitational alternative for dark energy. They have considered the version of $f(R)$, $f(G)$ or $f(R,G)$ gravity, model with non-linear gravitational coupling or string inspired model with Gauss-Bonnet-dilaton coupling in the late universe. Nojiri and Odintsov (2011),

have studied f (R) gravity in different context. Bertolami et al. (2007)proposed a generalization of f(R) theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m . Shamir (2010), proposed a physically viable f(R) gravity model, which show the unification of early time inflation and the late time acceleration.

In this chapter, we present some new classes of five dimensional Kaluza-Klein cosmological models in the presence of a perfect fluid source in f(R,T) gravity theory. The chapter is organized as follows: In Sect. 7.2, we revisit the field equations presented by Reddy et al. (2012a). We then derive algorithms for generating new solutions of the field equations in Sect.7.3 . In Sect.7.4, starting with solution of Reddy et al. (2012a), we obtain some solutions of the field equations which represent accelerating cosmological models. The physical and kinematical properties of the models are also discussed. Conclusions are given in Sect.7.5. .

7.2 Metric and Field Equations

We consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\Psi^2 \quad (7.1)$$

where A(t) and B(t) are the scale factors. The fifth coordinate Ψ is taken to be space-like. The field equations in f(R,T)theory of gravity for the function f(R,T), which is given in (5.2), when the matter source is perfect fluid (1.8), are given by Harko et al. (2011b).

The field equations (5.5) for the metric (7.1) in comoving coordinates lead to the following equations

$$3 \left(\frac{\dot{A}}{A} \right)^2 + 3 \frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\lambda)\rho - p\lambda, \quad (7.2)$$

$$\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A} \right)^2 + 2 \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -(8\pi + 3\lambda)p + \rho\lambda, \quad (7.3)$$

$$3 \frac{\ddot{A}}{A} + 3 \left(\frac{\dot{A}}{A} \right)^2 = -(8\pi + 3\lambda)p + \rho\lambda. \quad (7.4)$$

Here an overhead over dot denotes ordinary differentiation with respect to time t .

For the metric (7.1), the spatial volume V and the average scale factor a are given by

$$V = a^4 = A^3 B \quad (7.5)$$

where a is the scale factor.

The mean Hubble parameter H has the expression

$$H = \frac{\dot{a}}{a} = \frac{1}{4} \left(3 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \quad (7.6)$$

where $H_x = H_y = H_z = \frac{\dot{A}}{A}$ and $H_\Psi = \frac{\dot{B}}{B}$ are directional Hubble parameters.

The scalar expansion θ and shear scalar σ are given by

$$\theta = \frac{3}{4} \left(3 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right), \quad (7.7)$$

$$\sigma^2 = \frac{3}{4} \theta^2. \quad (7.8)$$

In next Sect. we follow Hajj-Boutros (1986a) to derive algorithms for generating

new solutions of the field equations of KK -type perfect fluid cosmological models within the framework of $f(R,T)$ gravity theory .

7.3 Generating Technique

From Eqs. (7.3) and (7.4) we obtain

$$\frac{2\ddot{A}}{A} + 2\frac{\dot{A}^2}{A^2} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = 0. \quad (7.9)$$

To treat Eq. (7.9), we introduce new functions R and S given by

$$R = \frac{\dot{A}}{A}, S = \frac{\dot{B}}{B}, \quad (7.10)$$

By use of (7.10), Eq. (7.9), becomes

$$2\dot{R} + 4R^2 - 2RS - \dot{S} - S^2 = 0. \quad (7.11)$$

The nonlinear equation. (7.11) can be treated as a Riccati equation in R or S.

If we treat Eq. (7.11) as a Riccati equation in R, it can be linearized by means of change of function

$$R = R_0 + \frac{1}{X}. \quad (7.12)$$

where R_0 is a particular solution of Eq. (7.11) . Using (7.12) in Eq. (7.11), we obtain

$$\dot{X} + (S - 4R_0)X = 2 \quad (7.13)$$

Eq. (7.13) is linear first-order differential equation which has the general solution given

by

$$X = \frac{A_0^4}{B} \left(\int 2 \frac{B}{A_0^4} dt + k_1 \right), \quad (7.14)$$

k_1 being an integration constant. From Eqs. (7.12) and (7.14), we obtain after integration

$$A = A_0 k_2 \exp \left[\frac{dt}{\frac{A_0^4}{B} \left(\int 2 \frac{B}{A_0^4} dt + k_1 \right)} \right] \quad (7.15)$$

where k_2 being another constant. Hence, from metric function $[A_0, B]$ we can generate new function $[A, B]$ where (A) is given by Eq.(7.15) and B remains invariable.

If (7.11) is regarded as a Riccati equation in S, we can be linearized it by the change of function

$$S = S_0 + \frac{1}{Y}. \quad (7.16)$$

where S_0 is a particular solution of (7.11).

Introducing (7.16) into Eq. (7.11), we obtain

$$\dot{Y} - (2R + 2S_0)Y = 1 \quad (7.17)$$

Eq. (7.17), on integration, gives

$$Y = A^2 B_0^2 \left(\int \frac{dt}{A^2 B_0^2} + k_3 \right) \quad (7.18)$$

where k_3 being a constant . From Eqs. (7.17) and (7.18), we obtain

$$B = B_0 k_4 \exp \left[\int \frac{dt}{A^2 B_0^2 \left(\int \frac{dt}{A^2 B_0^2} + k_3 \right)} \right] \quad (7.19)$$

where k_4 is another constant of integration. Thus, from the couple $[A, B_0]$ we can generate $[A, B]$ where B is given by Eq.(7.19) and A remains invariable

Reddy et al.(2012a) have presented the solutions of the field equations (7.2)-(7.4) in $f(R,T)$ gravity theories has given by the metric

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - [kt]^{\frac{2m}{k}} d\Psi^2 \quad (7.20)$$

where $k = \frac{m^2 + 2m - 3}{m - 1}$, $m \neq 1$. Starting with this metric, we now generate new solutions of the field equations (7.2)-(7.4) by applying the generating techniques (7.15) and (7.19)

7.4 Model I

To apply our generation technique (7.19) to the metric (7.20), we take

$$A = (kt)^{\frac{1}{k}}, B_0 = (kt)^{\frac{m}{k}}$$

Then, performing the integration in (7.19), the new metric function B is obtained as

$$B = k_4 (k)^{\frac{m}{k}} [t]^{\frac{k^2 - 2m^2 - 2m + km}{k(k - 2m - 2)}} \quad (7.21)$$

by putting $k_3 = 0$. Hence the metric of our new solution can be written in the form

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - \left\{ k^{\frac{m}{k}} t^{\frac{k^2 - 2m^2 - 2m + km}{k(k - 2m - 2)}} \right\}^2 d\Psi^2 \quad (7.22)$$

For the model (7.22) the physical and kinematical parameters are given by

$$H = \frac{1}{4mkt(k-2m-2)} [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \quad (7.23)$$

$$\theta = 3H = \frac{3}{4mkt(k-2m-2)} [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \quad (7.24)$$

$$\sigma^2 = \frac{3}{4}\theta^2 = \frac{3}{4} \left(\frac{3}{4mkt(k-2m-2)} [k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k] \right)^2. \quad (7.25)$$

$$V = k^{\frac{m+3}{k}} t^{\frac{k^2-2m-2m+mk}{k(k-2m-2)} + \frac{3}{k}}. \quad (7.26)$$

The deceleration parameter q is defined in (1.31) which has the value given by

$$q = \frac{4k}{3(k-2m-2) + k^2 - 2m^2 - 2m + mk} - 1. \quad (7.27)$$

The pressure and energy density are obtained as

$$p = \left[\frac{\lambda[k(k-2m-2)E_1 - (8\pi + 3\lambda)E_2m^2]}{(8\pi + 4\lambda)(8\pi + 2\lambda)m^2k^2(k-2m-2)t^2} \right], \quad (7.28)$$

$$\rho = \left[\frac{3\left(1 + \frac{k^2-2m^2-2m+mk}{k(k-2m-2)}\right)}{m^2t^2(8\pi + 3\lambda)} + \frac{\lambda^2(k(k-2m-2) - \frac{(8\pi+3\lambda)}{\lambda}E_2m^2)}{(8\pi + 2\lambda)(8\pi + 3\lambda)(8\pi + 4\lambda)m^2k^2t^2} \right] \quad (7.29)$$

where

$$E_1 = k(k - 2m - 2) + 3m(k^2 - 2m^2 - 2m + mk),$$

$$E_2 = [2 - m + (2m - 1)(k^2 - 2m^2 - 2m + mk)](k(k - 2m - 2)) \\ + (k^2 - 2m^2 - 2m + mk)$$

From the above results we observed that the model has initial singularity at $t=0$ if $k > 2(m + 1)$ which leads to $m < 1$. We see that θ, σ, H, p and ρ have infinite value at the initial singularity $t=0$. These parameters are decreasing function of time which tend to zero for large time. Since $\frac{\sigma^2}{\theta^2} \neq 0$, the model is anisotropic throughout the evolution of the universe. We also find that the deceleration parameter q is negative, which corresponds to an accelerating model of the universe in five-dimensional Kaluza-Klein theory.

7.5 Model II

We apply formula (7.15) for the metric (7.20) to generate the new function A by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (k)^{\frac{m}{k}} [t]^{\frac{k^2 - 2m^2 - 2m + km}{k(k - 2m - 2)}}$$

Then, after integration, we obtain

$$A = k_2 (k)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(mk-4k)(k-2m-2))} + \frac{1}{k}} \quad (7.30)$$

assuming $k_1 = 0$. The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left\{ \left(k \right)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(km-4k)(k-2m-2))} + \frac{1}{k}} \right\}^2 (dx^2 + dy^2 + dz^2) - \left(k^{\frac{m}{k}} t^{\frac{k^2-2m^2-2m+mk}{k(k-2m-2)}} \right)^2 d\Psi^2. \quad (7.31)$$

The metric (7.31) represents the five-dimensional Kaluza-Klein cosmological model in f(R,T) gravity theory with the following physical and kinematical parameters.

$$V = k^{\frac{m+3}{k}} [t]^{\frac{3k(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(m-4)k(k-2m-2))} + \frac{k^2-2m^2-2m+mk}{k(k-2k-2)}}, \quad (7.32)$$

$$H = \left[\frac{1}{8t(m(k^2-2m^2-2m+mk)) + km(k-2m-2) - 4k^2(k-2m-2)^2} \right] \cdot [((m-4)k(k-2m-2)(1+(k-2m-2)((m-4)k+3mk^2)))] \quad (7.33)$$

$$+ [(k^2-2m^2-2m+km)(2km(k-2m-2) + 2m^2(k^2-2m^2-2m+km))],$$

$$\theta = \left[\frac{3}{8t(m(k^2-2m^2-2m+mk)) + km(k-2m-2) - 4k^2(k-2m-2)^2} \right] \cdot [((m-4)k(k-2m-2)(1+(k-2m-2)((m-4)k+3mk^2)))] \quad (7.34)$$

$$+ [(k^2-2m^2-2m+km)(2km(k-2m-2) + 2m^2(k^2-2m^2-2m+km))],$$

$$\sigma^2 = \frac{3}{4}\theta^2, \quad (7.35)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -[1 + (E_3^3 E_4)^{\frac{1}{2}}], \quad (7.36)$$

$$p = - \left[\frac{(16\pi + 3\lambda)(E_3^2 + E_3 E_4) + (8\pi + 3\lambda)(E_4^2 - (E_3 + E_4))}{t^2(8\pi + 4\lambda)(8\pi + 2\lambda)} \right], \quad (7.37)$$

$$\rho = \left[(3E_3E_4) - \frac{\lambda}{(8\pi + 4\lambda)(8\pi + 2\lambda)}(16\pi + 3\lambda)(E_3^2 + E_3E_4) + (8\pi + 3\lambda)(E_4^2 - (E_3 + E_4)) \right] \cdot \left[\frac{1}{t^2(8\pi + 3\lambda)} \right] \quad (7.38)$$

where

$$E_3 = \left[\frac{k^2(k - 2m - 2)}{2(m(k^2 - 2m^2 - 2m + mk) + mk(k - 2m - 2) - 4k(k - 2m - 2))} + \frac{1}{k} \right],$$

$$E_4 = \frac{k^2 - 2m^2 - 2m + mk}{k(k - 2m - 2)}$$

From the Fig.(7.5), it is clear that the model (7.31) represents a five-dimensional Kaluza-Klein accelerating cosmological model . The other physical and kinematical behaviors of the model are same as model I.

7.6 Model III

We now use formula (7.15) to generate new metric function A by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (kt)^{\frac{m}{k}}$$

Then performing integration in (7.15) ,we obtain

$$A = k_2 k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2 - 4k + m)} + \frac{1}{k}}, \quad (7.39)$$

assuming $k_1=0$ Then the metric (7.1) can be written in the form (7.40)

where k_5 is integration constant.

The metric can be written as

$$ds^2 = dt^2 - \left(k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k+m)} + \frac{1}{k}} \right)^2 (dx^2 + dy^2 + dz^2) - (kt)^{\frac{2m}{k}} d\Psi^2 \quad (7.40)$$

The model (7.40) represents the five-dimensional Kaluza-Klein cosmological with perfect fluid in f(R,T) gravity theory . The physical and the kinematical parameters of the model (7.40) are given as follows:

$$V = (k)^{\frac{m+3}{k}} [t]^{\frac{3+m(2m^2-5k+2m)}{2k(m^2-4k+m)}}, \quad (7.41)$$

$$H = \frac{1}{4tk} (3kE_5 + m), \quad (7.42)$$

$$\theta = 3H = \frac{3}{4kt} (3kE_5 + m), \quad (7.43)$$

$$\sigma^2 = \frac{27}{64t^2k^2} (3kE_5 + m)^2, \quad (7.44)$$

$$q = -1 + \left[\frac{8(m^2 - 4k - m)k}{t(3k^2m + 2m(m^2 - 4k + m))} \right], \quad (7.45)$$

$$p = \left[\frac{E_5((8\pi + 3\lambda)k^2 + 3\lambda km^2) - (8\pi + 3\lambda)(E_5(2mk - E_5) + m(m - k)) - (8\pi + 2\lambda)k^2 E_5^2}{(8\pi + 4\lambda)(8\pi + 2\lambda)k^2 t^2} \right], \quad (7.46)$$

$$\rho = \left[\frac{1}{(8\pi + 4\lambda)(8\pi + 3\lambda)(8\pi + 2\lambda)k^2 t^2} \right] \cdot [((3E_5^2(E_5 - 1)^2 k^2 + 3E_5 m)(8\pi + 4\lambda)(8\pi + 2\lambda))] \\ + [\lambda E_5(8\pi + 3\lambda)k^2 + 3\lambda k m^2 - \lambda(8\pi + 3\lambda)(m - k)m - (8\pi + 2\lambda)k^2 E_5] \quad (7.47)$$

where

$$E_5 = \frac{k}{2(m^2 - 4k + m)} + \frac{1}{k}.$$

For the metric (7.40) the spatial volume is zero at $t=0$ if $k < \frac{m(m+1)}{4}$. The physical and kinematical properties same as perfect fluid Model I

7.7 Model IV

We use the formula (7.19) for the metric (7.40) to generate the new function B by setting

$$A = k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k+m)} + \frac{1}{k}}, B = (kt)^{\frac{m}{k}}.$$

Then, from Eq.(7.19) , the new function B is obtained as:

$$B = k_4 k^{\frac{m}{k}} [t]^{\frac{mk(m^2-4k+m)}{km^2+2(m^2-4k+m)+(3k+2m)(m^2-4k+m)+mk(m^2-4k+m)}} \quad (7.48)$$

taking $k_3=0$. The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left[k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k-m)} + \frac{1}{k}} \right]^2 (dx^2 + dy^2 + dz^2) \\ - \left[(k)^{\frac{m}{k}} [t]^{\frac{km(m^2-4k+m)}{km^2+2(m^2-4k+m)(1+m+3km)}} \right]^2 d\Psi^2 \quad (7.49)$$

The metric (7.49) represents five-dimensional Kaluza-Klein cosmological model in f(R,T) gravity with the following physical and kinematic parameters in the model.

$$V = k^{\frac{m+3}{k}} \left[t^{\frac{3km}{2(m^2-4k+m)} + \frac{3}{k} + \frac{m}{k} + \frac{(km(m^2-4k+m))}{km^2+(2+m(4+k))(m^2-4k+m)}} \right], \quad (7.50)$$

$$H = \frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)}, \quad (7.51)$$

$$\theta = 3H = 3 \left[\frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)} \right], \quad (7.52)$$

$$\sigma^2 = \frac{27}{256} \left[\frac{(3(km^2 + 1) + 2m(m^2 - 4k + m)E_6)}{8tm(m^2 - 4k + m)} \right]^2, \quad (7.53)$$

$$q = -\left(1 - \frac{1}{E_7}\right), \quad (7.54)$$

$$\begin{aligned} p = - \left[\frac{1}{4m^2(m^2 - 4k + m)^2 t^2 (8\pi + 4\lambda)(8\pi + 2\lambda)} \right] & \cdot [((km^2 + 1)^2(16\pi + 3\lambda) + ((32\pi + 6\lambda)E_6) \\ & + [(km^2 + 1)m(m^2 - 4k + m)]) + [(8\pi + 3\lambda)m(m^2 - 4k + m)(km^2 + 1)] \\ & + [(8\pi + 3\lambda)E_6(E_6 - 1)m(m^2 - 4k + m)]], \end{aligned} \quad (7.55)$$

$$\rho = \left[\frac{3(km^2 + 1)^2 E_6(km^2 + 1)}{4m^2(m^2 - 4k + m)^2 t^2 (8\pi + 3\lambda)} \right] - \left[\frac{\lambda}{(8\pi + 4\lambda)(8\pi + 3\lambda)(8\pi + 2\lambda)t^2 4m^2(m^2 - 4k + m)^2} \right]$$

$$\cdot \left[((km^2 + 1)^2(16\pi + 3\lambda) + (32\pi + 6\lambda)E_6(km^2 + 1)m(m^2 - 4k + m)) \right]$$

$$+ \left[(8\pi + 3\lambda)E_6(E_6 - 1)m(m^2 - 4m + m) \right]$$

(7.56)

where

$$E_6 = \frac{m}{k} + \frac{m^2 - 4k + m}{(km^2 + (2 + 4km + m)(m^2 - 4k + m))},$$

$$E_7 = \left[\frac{3km}{2(m^2 - 4k + m)} + \frac{m}{k} + \frac{km(m^2 - 4k + m)}{km^2 + (m^2 - 4k + m)(2 + 4m + km)} \right].$$

We observe that the spatial volume of the model (7.49) is zero at $t=0$ and increases with time if $k < \frac{m(m+1)}{4}$. Therefore the model has a point type singularity at $t=0$ where θ , σ^2 , H , p and ρ diverge. These parameters are decreasing function of time and ultimately tend to zero for large time. The negative pressure, as shown by Fig.(7.11), indicates that the model is accelerating.

7.8 Conclusions

The higher dimensional cosmological models are of considerable importance because of the underlying idea that cosmos in early stages of evolution might have had a higher dimensional era. The extra space reduces to a volume with the passage of time, which is beyond the ability of experimental observation at the moment Reddy (2009). It is well known that Kaluza-Klein models represent the cosmos in its early stages of evolution. In the present work, we have derived algorithms for generating new solutions of the field

equations with a perfect fluid for a five dimension Kaluza-Klein space-time within the framework of $f(R,T)$ gravity theory proposed by Harko et al. (2011b). Starting from the model obtained by Reddy et al. (2012a), we have presented new cosmological models of the present-day accelerating universe. These models are expanding, shearing and accelerating which have point-type singularity at $t=0$. All the physical and kinematical parameters, being infinite at the initial singularity, are decreasing functions of time which ultimately tend to zero for large time. The anisotropy in the cosmological models are maintained throughout the passage of time.

Nojiri and Odintsov (2003a) studied a modify theory of gravity where the universe interns inflates, decelerates and then accelerates in early times, radiation dominated era. Our models are similar to the case of five dimensional $f(R)$ gravity except the decelerating behavior in the presence of a perfect fluid source discussed by Huang et al. (2010) and Agmohammadi, et al. (2009). It has been observed that in the five dimensional $f(R)$ and $f(R,T)$ gravity theories, the expansion and contraction of the extra dimension could result in the present accelerated expansion of other spatial dimensions. This is possible by cosmic re-collapse of the universe in the finite future. It follows that the the present accelerating models of the universe are consistent with the recent observation of type-Ia supernovae (Perlmutter et al., 1999; Riess et al., 1998).

7 Some Kaluza- Klein Cosmological Models in $f(R,T)$ Gravity Theory

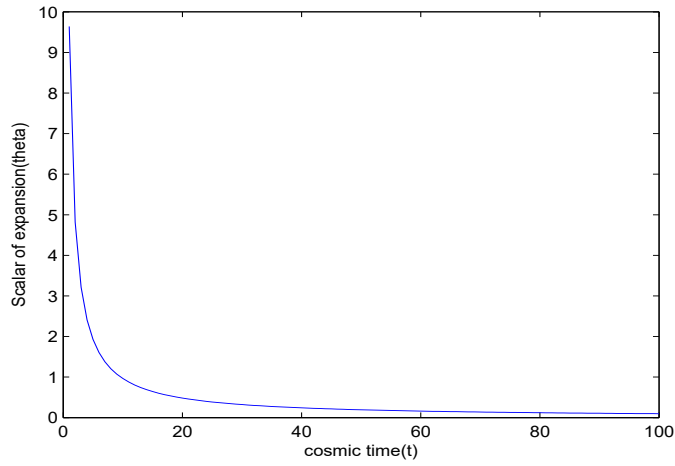


Figure 7.1: The plot of scalar expansion θ verses cosmic time t , $m=0.5;\lambda=1;k=1$;

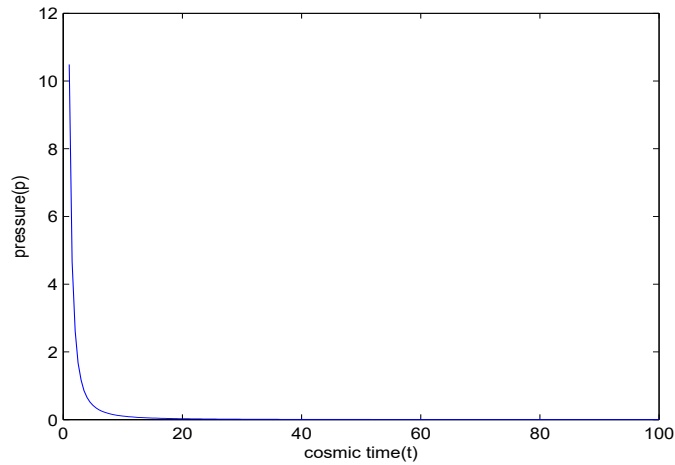


Figure 7.2: The plot of pressure p verses cosmic time t , $m=0.5;\lambda=1;k=1$;

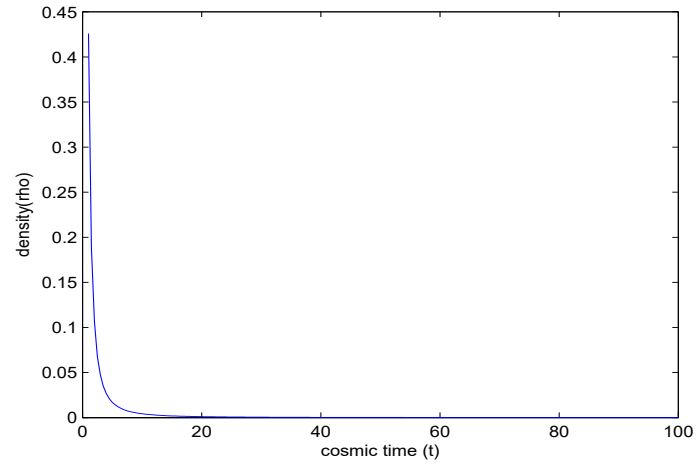


Figure 7.3: The plot of density ρ verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

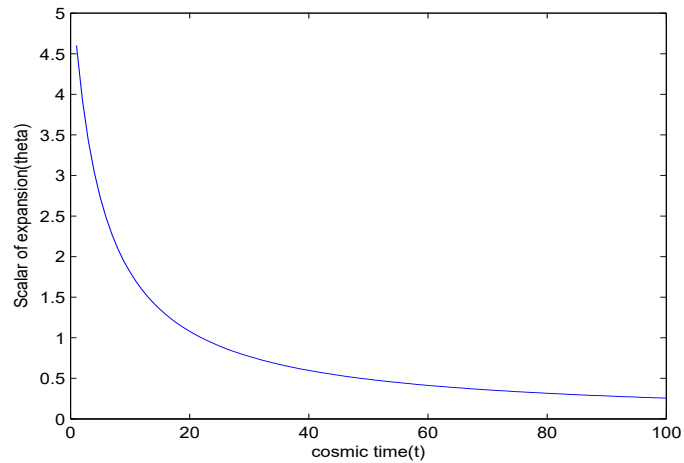


Figure 7.4: The plot of θ verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

7 Some Kaluza- Klein Cosmological Models in $f(R,T)$ Gravity Theory

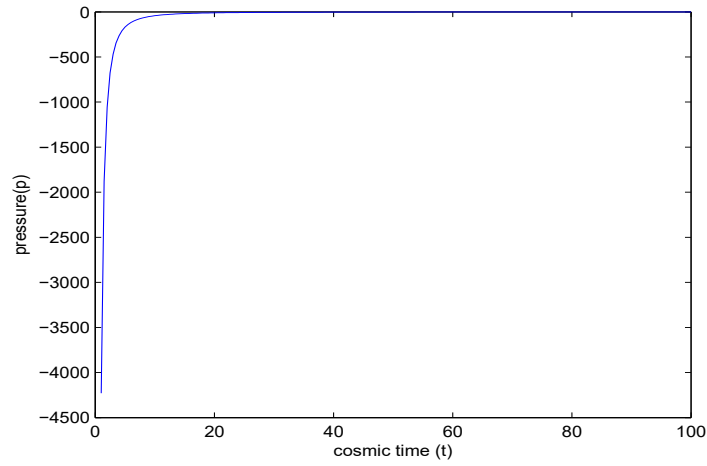


Figure 7.5: The plot of pressure p verses cosmic time t , $m=0.5;\lambda=1;k=1$;

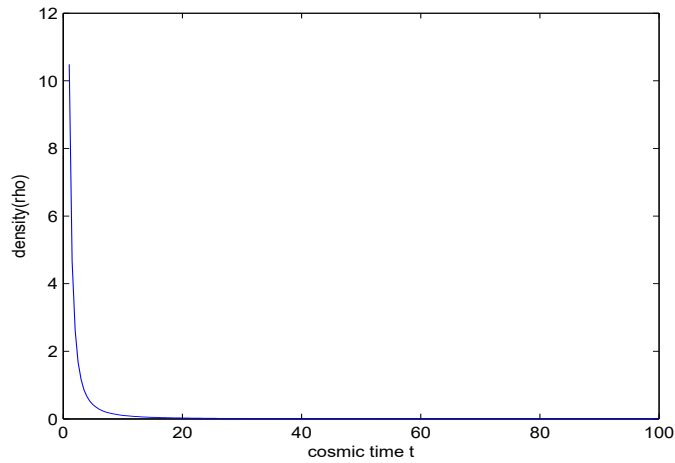


Figure 7.6: The plot of density ρ verses cosmic time t , $m=0.5;\lambda =1;k=1$;

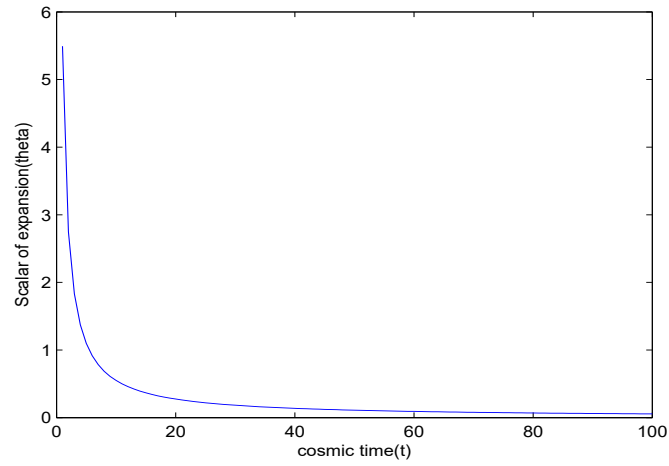


Figure 7.7: The plot of scalar expansion θ verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

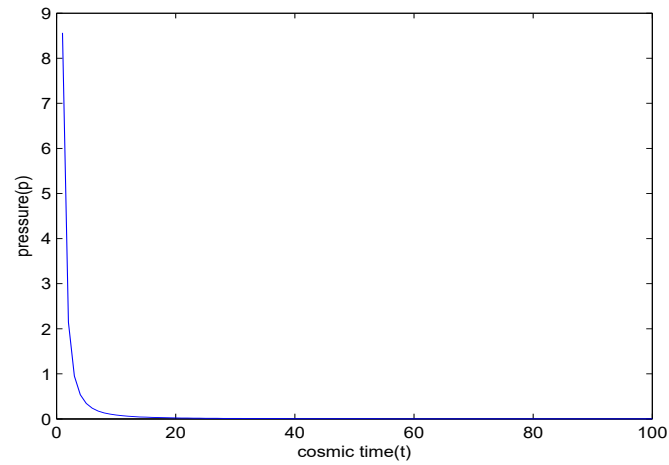


Figure 7.8: The plot of pressure p verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

7 Some Kaluza- Klein Cosmological Models in $f(R,T)$ Gravity Theory

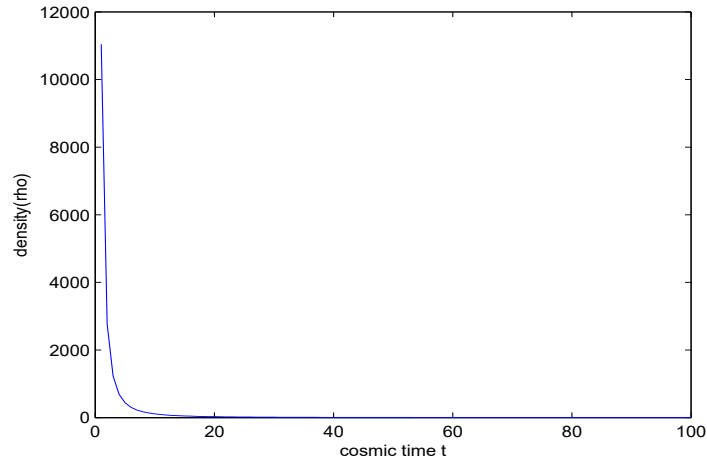


Figure 7.9: The plot of density ρ verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

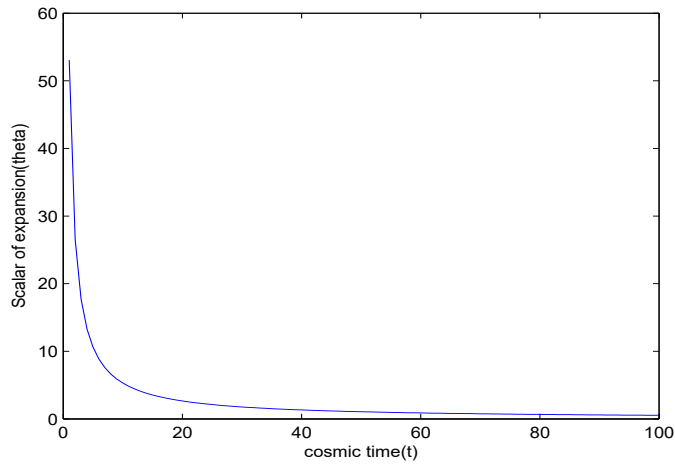


Figure 7.10: The plot of Scalar of expansion θ verses cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

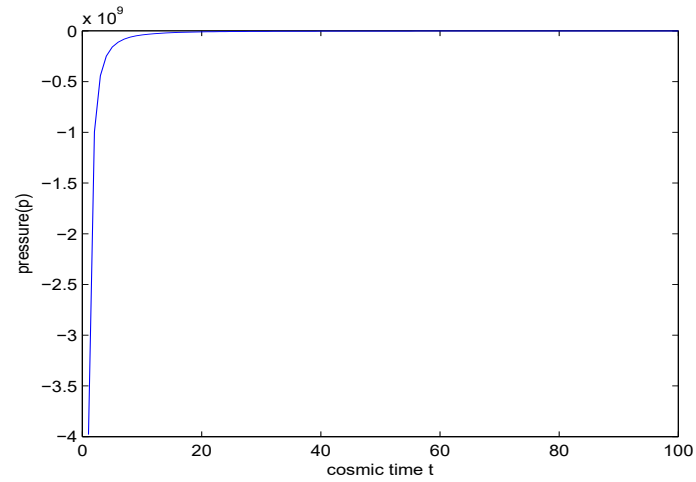


Figure 7.11: The plot of pressure p versus cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

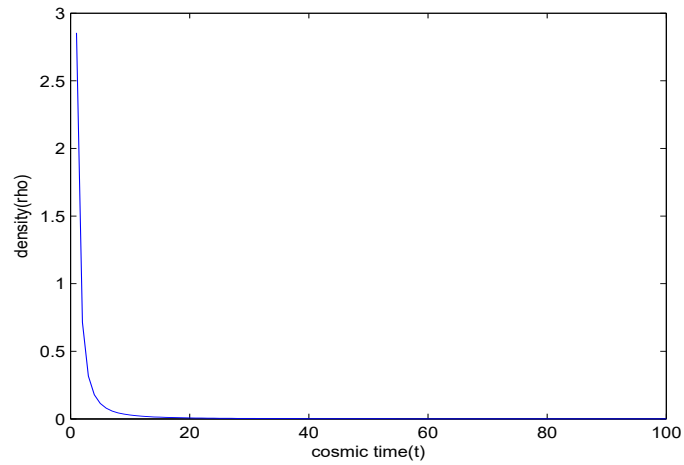


Figure 7.12: The plot of density ρ versus cosmic time t , $m=0.5$; $\lambda=1$; $k=1$;

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