6 Bianchi types I and V Bulk Viscous Fluid Cosmological Models in f(R,T) Gravity Theory *

6.1 Introduction

The simplest model of the expanding universe is well represented by Friedmann-Robertson-Walker models which are spatially homogeneous and isotropic. These models in some sense are good global approximation of the present day universe, but it is unreasonable to assume that the regular expansion predicted by these models are also suitable for describing the early stages of evolution of the universe. The aim of modern cosmology is to study the past history, the present state and future evolution of the universe. Recent observational data indicate that our universe is accelerating (Riess et al., 1998; Perlmutter et al., 1997). Also, observations such as cosmic microwave background radiation (Spergel et al., 2003) and large scale structure (Tegmark, et al, 2004) provide indirect evidence for the late time accelerated expansion of the universe.This acceleration is explained in terms of the so-called dark energy.

In view of the late-time acceleration of the universe and the existence of the dark

*Contents of this chapter have been published in Central European Journal of Physics (2014), 12, 744, Springer energy and dark matter, several modified theories of gravitation have been proposed as alternative to Einstein's general theory of relativity . Noteworthy among them is the cosmologically important f(R) gravity theory. It has been shown that f(R) gravity theory is indeed a realistic alternative to general relativity, being consistent in dark epoch. It has been suggested that cosmic acceleration could be achieved by replacing the Einstein's -Hilbert action of general relativity with a general function of Ricci scalar R. Nojiri and Odintsov (2006a)developed a general program for unification of matter -dominated era with accelerated epoch for scalar -tensor theory or dark fluid. Nojiri and Odintsov (2007)presented an extensive review of modified gravity theories which is considered as gravitational alternative for dark energy . Bertolami et al. (2007) have proposed a generalization of f(R) gravity theory by including in the theory an explicit coupling of an arbitrary function of Ricci scalar R with the matter Lagrangian density. Shamir (2010) has also proposed a physically viable f(R) gravity model, presenting the unification of early time inflation and late time acceleration, Shamir and Jhangeer (2013) investigated static plane symmetric vacuum solutions in f(R) gravity for (n+1) dimensional space time. Recently, Adhav (2012b) studied Bianchi type-III cosmological model in f(R) theory of gravity in the presence of cosmic strings.

The role played by viscosity and the consequent dissipative mechanism in cosmology have been discussed by several authors (Misner, 1967, 1968; Murphy, 1973) . The heat represented by the large entropy per baryon in the microwave background provides a useful clue to the early universe and a possible explanation for this huge entropy per baryon is that it was generated by physical dissipative processes acting at the beginning to evolution. These dissipative processes may indeed be responsible for the smoothing out if initial anisotropics (Weinberg, 1967). Misner (1967, 1968) suggested that neutrino viscosity acting in the early era might have considerably reduced the present anisotropy of the black-body during the process of evolution. Bulk viscosity appears as the only dissipative phenomenon occurring in FRW models and plays a significance role in getting accelerated expansion of the universe known as inflationary phase, as discussed by Sheykhi (2010). Murphy (1973) has obtained a zero curvature FRW type model in the presence of bulk viscosity alone, which exhibits an interesting property that the big-bang singularity appears in the infinite past. Roy and Tiwari (1983) presented some plane symmetric solutions to Einstein's field equations representing inhomogeneous cosmological models with viscous fluid and constant bulk viscosity. Syzlowski and Heller (1983) constructed models of the universe filled with interacting matter and radiation including bulk viscosity dissipation. Mohanty and Pradhan (1983) obtained a class of exact non-static solution in closed elliptic Robertson-walker space-time filled with viscose fluid in the presence of attractive scalar field. Goenner and Kowalewsky (1987) developed a method for obtaining irrotational anisotropic viscous fluid solutions of Bianchi type-I with barotropic equation of state. Banerjee and Sanyal (1988) presented an irrorational Bianchi type V model under the influence of both shear and bulk viscosity together with heat flow . Coley (1990); Coley and Hoogan (1994), while generalizing the work of Coley and Tupper (1984), studied diagonal Bianchi type-V imperfect fluid models with both viscosity and heat condition with and without the cosmological term. .Bali and Meena (2002) have investigated tilted cosmological models filled with disordered radiation of perfect fluid and heat flow. Tilted Bianchi type I cosmological model for perfect fluid distribution in presence of magnetic field is investigated by Bali and Sharma (2003). Also, Bali and Anjali (2004) presented Bianchi type-I bulk viscous fluid string dust magnetized cosmological models in general relativity. Adhav et al. (2007) studied Bianchi type-III anisotropic cosmological models with varying Λ. Baghel and Singh (2012) considered spatially homogeneous and anisotropic Bianchi type-V space-time with a bulk viscous fluid source, and time varying gravitational constant G and cosmological term Λ .Several authors have discussed the role of bulk viscosity in early evolution of the universe in different physical contexts.

Harko et al. (2011b) developed another modification of Einstein's gravity theory, known as f(R,T) gravity theory, wherein the gravitational Lagrangian is an arbitrary function of Ricci scalar R and the trace T of the energy-momentum tensor T_{ij}. It is to be noted that the dependence from T may be induced by exotic imperfect fluid or quantum effects. They have derived the field equations of f(R,T) gravity by varying the action of the gravitational field equations with respect to the metric tensor and have presented physically realistic model with the special choice of the function f(R,T). Subsequently, several authors viz.Myrzakulov (2012); Adhav (2012a); Reddy et al. (2012b); Chaubey and Shukla (2013); Ram et al. (2013), . Samanta (2013) studied Kantowski -Sachs space time cosmological model filled with perfect fluid matter in f(R, T) gravity. Further, Reddy et al. (2012a); Ram and Priyanka (2013), have investigated five dimensional Kaluza-Klein cosmological models filled with perfect fluid in f(R,T) gravity theory. Naidu et al. (2013) investigated Bianchi type -V bulk viscous string cosmological model in f(R,T) gravity theory. Reddy et al. (2013) considered a LRS Bianchi type II space-time and obtained the solutions of field equations with cosmic string and bulk viscous fluid within the framework f(R,T) theory of gravity. Recently, Ahmed and Pradhan (2013) investigated a to study cosmological model in f(R,T) gravity of Bianchi type-V by assuming $f(R,T) = f_1(R) + f_2(\mathrm{T})$, Chakraborty (2013) formulated an alternative f(R,T) gravity theory and the dark energy problem. Recently, Sharif and Zubair (2012b) studied Bianchi type-I anisotropic models in f(R,T) gravity theory.Sahoo et al. (2014) considered an axially symmetric space -time in the presence of a perfect fluid source within the framework of f(R,T) gravity theory. Mishra and Sahoo (2014)investigated Bianchi type VIh cosmological model filled with perfect fluid within the framework of f(R,T) gravity theory. The spatially homogeneous and totally anisotropic Bianchi type-II cosmological solutions of massive strings in the presence of the magnetic field in f(R,T) theory of gravity have been studied by Sharma and Singh (2014). Singh and Singh (2014) presented the cosmological viability of reconstruction of an alternative gravitational theory, namely the modified f(R,T) gravity theory.

Motivated by the above studies, we investigate new classes of spatially homogeneous Bianchi type I and V bulk viscous fluid cosmological models in f(R,T) theory of gravity. We also discuss some physical and kinematical features of the cosmological models.

6.2 Field Equations

We assume the cosmic matter is represented by the energy-momentum (1.9) tensor of an imperfect bulk viscous fluid (1.10) satisfying a linear equation of state

$$p = \epsilon\rho, \;\; 0 \leqslant \epsilon \leqslant 1. \tag{6.1}$$

Here p is the equilibrium pressure, ρ is the energy density of matter, ζ is the coefficient of bulk viscosity and u^i is the flow vector of the fluid satisfying $u_i u^i = 1$. The semicolon stands for covariant differentiation. On thermodynamical grounds bulk viscosity coefficient ζ is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. However, correction to the thermodynamical pressure p due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution does not change fundamentally by the inclusion of viscous term in the energy-momentum tensor.

The field equations in f(R,T) gravity theory with the particular choice of the function f(R,T) given in (5.2) when the matter source is a bulk viscous fluid, are given by Reddy et al. (2013):

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f^{'}(T)T_{ij} + \left[2\bar{p}f^{'}(T) + f(T)\right] g_{ij} \tag{6.2}$$

We further take (5.6). This f(R,T) gravity model is equivalent to a cosmological model with an effective cosmological constant $\Lambda \propto H^2$, where H is the Hubble function (Poplawski, 2007). It is also interesting to note that generally for this choice of f(R,T) the gravitational coupling becomes effective and time dependent coupling, of the form $G_{eff} = G \pm 2f^{'}(T)$. Thus the term 2f(T) in the gravitational action modifies the gravitational interaction between matter and curvature replacing G by a running gravitational coupling parameter.

6.3 Bianchi type-I Model

We consider a spatially homogeneous Bianchi type-I metric given as:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2 \tag{6.3}$$

where A, B and C are cosmic scale functions.

For the metric (6.3), the field equations (1.9), (5.2) and (6.2), in comoving coordinates, lead to the following set equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \tag{6.4}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \tag{6.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \lambda\rho - (8\pi + 3\lambda)\bar{p}. \tag{6.6}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=(8\pi+3\lambda)\rho-\lambda\bar{p}, \tag{6.7}$$

These are four highly non-linear equations in six unknowns A,B, C, ρ , $\bar{p}$ and ζ. Therefore to find a consistent solution of these equations,we shall need meaningful assumptions either on the cost of physics or simply for mathematical convenience. Subtracting Eq. (6.5) from Eq.(6.4), Eq. (6.6) from (6.5), Eq.(6.6) from Eq.(6.4) and integrating the resulting equations, we obtain

$$\frac{B}{A}=d_1\exp\left(c_1\int\frac{dt}{a^3}\right), \tag{6.8}$$

$$\frac{C}{B}=d_2\exp\left(c_2\int\frac{dt}{a^3}\right), \tag{6.9}$$

$$\frac{A}{C}=d_3\exp\left(c_3\int\frac{dt}{a^3}\right) \tag{6.10}$$

where c_1, c_2, c_3 and d_1 , d_2 , d_3 integration constants which satisfy the relation

$$c_1+c_2+c_3=0, \quad d_1d_2d_3=1. \tag{6.11}$$

From Eqs. (6.8)-(6.10), we can obtain the scale factors A, B and C metric functions explicitly as

$$A=ap_1\exp\left(q_1\int\frac{dt}{a^3}\right), \tag{6.12}$$

$$B=ap_2\exp\left(q_2\int\frac{dt}{a^3}\right), \tag{6.13}$$

$$C = a p_3 \exp\left(q_3 \int \frac{dt}{a^3}\right) \tag{6.14}$$

where

$$p_1 = (d_1^{-2} d_2^{-1})^{\frac{1}{3}}, \;\; p_2 = (d_1 d_2^{-1})^{\frac{1}{3}}, \;\; p_3 = (d_1 d_2^2)^{\frac{1}{3}} \tag{6.15}$$

and

$$q_1 = -\frac{2c_1 + c_2}{3}, \quad q_2 = \frac{c_1 - c_2}{3}, \quad q_3 = \frac{c_1 + 2c_2}{3}. \tag{6.16}$$

The constants p_1, p_2 , p_3 and q_1, q_2, q_3 satisfy the relations

$$p_1 p_2 p_3 = 1, \quad q_1 + q_2 + q_3 = 0. \tag{6.17}$$

It is obvious that we determine the scale factors A, B, C from Eqs. (6.12)-(6.14) the average scale factor a(t) is known.

For constructing physically relevant cosmological models, the Hubble parameter and deceleration parameter (DP) play important roles. It has been the common practical to use a constant DP.Berman (1983); Berman and Gomide (1988) proposed a law of variation of Hubble parameter in FRW model that yields a constant value of DP, which subsequently leads to power-law and exponential forms of the average scale factor. The recent observations of SNe Ia (Riess et al., 1998; Perlmutter et al., 1997) indicate that the universe is presently accelerating while there was decelerated expansion in the past, and the universe undergoes transition from decelerated expansion to accelerated expansion and vice-versa at present. Therefore, in general DP is expected to be not a constant but rather a function of time. Some authors proposed time-dependent forms of DP and derived differential form of the average scale factor of the model .However, some authors further choose the average scale factor and then deduce the time-dependent DP, Eq. (1.31) can also be written as (5.15)

The Fig. (1) depicts the behavior of the deceleration parameter with time.

Here we use the form of a(t) given in Eq. (5.16) to determine the scale factors A,B and C from Eqs. (6.12)-(6.14). If we use the value of a(t) in Eqs. (6.12) -(6.14) the integration is rather difficult. Therefore, we take $\delta = 0$ and $\beta = \frac{3}{2}$ in Eq.(5.16) so that

$$a(t) = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}}. \tag{6.18}$$

Substituting Eq. (6.18) in Eqs. (6.12)-(6.14) and integrating, we obtain expression for the metric functions as

$$A = p_1 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_1 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right], \tag{6.19}$$

$$B = p_2 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_2 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right], \tag{6.20}$$

$$C = p_3 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_3 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right]. \tag{6.21}$$

For the model represented by metric functions in (6.19)- (6.21), the energy density ρ and the bulk viscous pressure $\bar{p}$ are given by

$$\rho = \frac{1}{9(8\pi+2\lambda)(8\pi+4\lambda)(t^2+\frac{2\alpha}{3})^2}[t^2[(8\pi+3\lambda)(12+18q_1^2) - (q_1^2+q_3^2)(144\pi+64\lambda) \\ -18\lambda(q_2+q_3)] - \lambda(8t+12\alpha(q_2+q_3))], \tag{6.22}$$

$$\bar{p} = \frac{1}{9\lambda(t^2+\frac{2\alpha}{3})^2}[(18(q_1^2+q_2^2+q_3^2)-12+\frac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}(12+18q_1^2(8\pi+3\lambda))$$
$$-(q_2^2+q_3^2)(144\pi+64\lambda)-18(q_2+q_3))t^2-\frac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}\lambda(8t+12\alpha(q_2+q_3))]. \tag{6.23}$$

The Figs. (6.2) and (6.3) depict the behavior of energy density and bulk viscous pressure with cosmic time respectively. Using the barotropic equation of state parameter to obtain coefficient of bulk viscosity

The Coefficient of bulk viscosity from Eqs. (6.1) and (6.23), is obtained as

$$\zeta = \frac{t}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})} * [\epsilon\lambda(8\lambda+3\lambda)(12+18q_1^2)-(q_1^2+q_2^2)$$
$$(144+64\lambda)-18\lambda(q_2+q_3)-(8\pi+3\lambda)(12+18q_1^2(8\pi+3\lambda))-(q_2^2+q_3^2)*$$
$$(144\pi+64\lambda)-18(q_2+q_3)]-\frac{1}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)t(t^2+\frac{2\alpha}{3})} \tag{6.24}$$
$$*[\epsilon t^2(8t+12\alpha(q_2+q_3))+18(8\pi+2\lambda)(8\pi+4\lambda)(q_1^2+q_2^2+q_3^2)$$
$$-12(8\pi+2\lambda)(8\pi+4\lambda)+(8\pi+3\lambda)\lambda(8t+12\alpha(q_2+q_3))]$$

Fig. (6.4) shows behavior of bulk viscosity coefficient with comic time. For the model 1 the energy density conditions $\rho+p\geq 0$ and $\rho+3p\geq 0$ are identically satisfied as shown, in the Fig (5)

We now discuss the physical and kinematical behaviors of Bianchi type-I cosmological model with metric functions given by Eqs.(6.19)- (6.21). The directional Hubble parameters and the average Hubble parameter are given by

$$H_1 = \frac{2t}{3\left(t^2 + \frac{2\alpha}{3}\right)}\left(3q_1 + 1\right), \tag{6.25}$$

$$H_2 = \frac{2t}{3\left(t^2 + \frac{2\alpha}{3}\right)}\left(3q_2 + 1\right), \tag{6.26}$$

$$H_3 = \frac{2t}{3\left(t^2 + \frac{2\alpha}{3}\right)}\left(3q_3 + 1\right), \tag{6.27}$$

$$H = \frac{2t}{\left(t^2 + \frac{2\alpha}{3}\right)}. \tag{6.28}$$

The expansion scalar, shear scalar and mean anisotropic parameters are found as

$$\theta = 3H = \left(\frac{6t}{t^2 + \frac{2\alpha}{3}}\right). \tag{6.29}$$

$$\sigma^2 = \left(\frac{2t^2}{(t^2 + \frac{2\alpha}{3})^2}\right)\left(q_1^2 + q_2^2 + q_3^2\right). \tag{6.30}$$

$$A_m = \frac{1}{3}\left(q_1^2 + q_2^2 + q_3^2\right). \tag{6.31}$$

Figs. (6.6), (6.7) and (6.8) depict the variation of H, θ and σ respectively . We observe that the model has no initial singularity. These parameters are decreasing function of time which tend to zero for large time. Since $\frac{\sigma^2}{\theta^2} \neq 0$, the model is anisotropic throughout the evolution of the universe.

6.4 Bianchi type-V Model

The diagonal form of the metric of Bianchi -type V cosmological model is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right] \tag{6.32}$$

where A, B and C are also cosmic scale factors and m is any constant.

Using Eqs.(1.9), (5.2), (6.2) and (6.32) we obtain the following set of equations

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p}, \tag{6.33}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \tag{6.34}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \tag{6.35}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \tag{6.36}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{6.37}$$

Integrating Eq. (6.37),provides A^2 =kBC,where k is integration constant. Without loss of generality, we take k=1.

The same procedure as for the Bianchi type-I solution to solve these equations By making use of Eq. (6.37), we get the constraint equations as follows:

$$p_1 = 1, \quad p_2 = p_3^{-1} = P, \quad q_1 = 0, \quad q_2 = -q_3 = Q. \tag{6.38}$$

Then, From Eqs. (6.33)- (6.38), we readily obtain

$$A = a, \quad B = aP\exp\left[Q\int\frac{dt}{a^3}\right], \quad C = aP^{-1}\exp\left[-Q\int\frac{dt}{a^3}\right] \tag{6.39}$$

Subsituting the value a(t) given in Eq. (6.18) into Eqs (6.33)- (6.36) into the equations in (6.39), we obtain the metric functions A, B and C as follows:

$$A = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}}, \tag{6.40}$$

$$B = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} P\exp\left[Q\tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right], \tag{6.41}$$

$$C = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} P^{-1}\exp\left[-Q\tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right] \tag{6.42}$$

The energy density and bulk viscous pressure for Bianchi type-V space -time model have values give as

$$\rho = \frac{1}{9(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})^2}[(8\pi+3\lambda)(12-36Q^2)t^2 - \lambda(12-t^2(12-t^2(6+34Q^2))) - \frac{3m^2(8\pi+2\lambda)}{(t^2+\frac{2\alpha}{3})^{\frac{1}{3}}}], \tag{6.43}$$

$$\bar{p} = \frac{1}{9(t^2+\frac{2\alpha}{3})^2}[\frac{(8\pi+3\lambda)}{\lambda(8\pi+2\lambda)(8\pi+4\lambda)}[(8\pi+3\lambda)(12-36Q)t^2-\lambda(12-t^2(6+34Q^2))] \\ -(12-36Q^2)t^2]-[\frac{(8\pi+3\lambda)}{\lambda(8\pi+2\lambda)(8\pi+4\lambda)}-1]\frac{m^2}{3(t^2+\frac{2\alpha}{3})^{\frac{2}{3}}}. \tag{6.44}$$

Using equation of state (6.1), we get bulk viscosity coefficient

$$\zeta = \frac{1}{36t(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})} * [\lambda(8\pi+3\lambda)(12-36Q^2)\epsilon+\lambda(8\pi+2\lambda)(8\pi+4\lambda)(12-36Q^2) \\ -(8\pi+3\lambda)(12-36Q^2)] + \frac{1}{36\lambda(8\pi+2\lambda)(8\pi+4\lambda)(t^2+\frac{2\alpha}{3})t} * [\lambda(12-t^2(6+34Q^2))-\lambda^2\epsilon * \\ (12-t^2(6+34Q^2))] - \frac{m^2}{12(8\pi+2\lambda)(8\pi+4\lambda)t\lambda(t^2+\frac{2\alpha}{3})^{\frac{4}{3}}} * [\epsilon(8\pi+2\lambda)+(8\pi+3\lambda)] + \\ \frac{m^2}{12t(t^2+\frac{2\alpha}{3})^{\frac{4}{3}}} \tag{6.45}$$

The directional Hubble parameters H_1, H_2 and H_3 are given as follow:

$$H_1 = \frac{2t}{3(t^2+\frac{2\alpha}{3})}, \tag{6.46}$$

$$H_2 = [3Q+1]\,\frac{2t}{3(t^2+\frac{2\alpha}{3})}, \tag{6.47}$$

$$H_3 = [-3Q+1]\,\frac{2t}{3(t^2+\frac{2\alpha}{3})}. \tag{6.48}$$

The mean anisotropic parameter A_m has the value

$$A_m = 6\left(1 + 72Q^2\right). \tag{6.49}$$

The shear scalar for this model is given by

$$\sigma^2 = \left(\frac{2Qt}{t^2 + \frac{2\alpha}{3}}\right)^2. \tag{6.50}$$

Figs.(6.9)- (6.15) depict the variation of ρ, $\bar{p}$, ζ, $\rho + p$, H, θ and σ with time. From the above results it can be observed that the model has no singularity at t= 0 and the spatial volume increase as t increases giving the accelerated expansion of the universe. In this model, we also note that σ^2, $\bar{p}$, p, ρ, and ζ are finite at t= 0 while they vanish for infinitely large t. However, $\frac{\sigma^2}{\theta^2} \neq 0$, which shows that the model does not approach isotropy for large time t. From Eq. (5.15) we see that q< 0 for t $< \sqrt{(2\alpha)}$ and q> 0 for t $> \sqrt{(2\alpha)}$. It deserve mention that Shamir et al. (2012) have also presented exact solutions of Bianchi type I and V models in f(R, T) gravity theory by applying the law of variation of Hubble,s parameter proposed by Berman (1983); Berman and Gomide (1988) . However, our models are different than those models.

6.5 Conclusion

In this chapter, we have investigated spatially homogeneous and anisotropic cosmological models of Bianchi type I and V filled with bulk viscous fluid in the framework of f(R,T) gravity theory. The absence of an initial time singularity in both models is a significance features of the results. The scale factors admits constant values at early times of the universe (t→ 0) after that scale factors stands increasing with cosmic time without showing any type of initial singularity and finally tends to ∞ as t $\to \infty$. Therefore, the universe represented by both models starts with finite volume in the initial past and expand exponentially approaching to infinite volume.

The expansion scalar θ and shear scalar σ are decreasing functions of time and ultimately become zero for large time. The ratio $\frac{\sigma}{\theta}$ tends to a constant as t$\to \infty$, and therefore the anisotropy in both models are maintained throughout the passage of time. The deceleration parameter q is negative for t$< \sqrt{(2\alpha)}$ and positive for t $> \sqrt{(2\alpha)}$. Therefore, the cosmological models initially accelerate for a certain period of time and thereafter decelerate .

The behavior of the bulk viscosity, is discussed graphically in Figs.(4) and (11). The bulk viscosity decreases with time so that, we get ultimately inflationary models Padmanabhan and Chitre (1987). The matter pressure and energy density are monotonically decreasing functions of time, which ultimately tend to zero for large time. Thus, the models would essentially correspond to empty universe for large time. The conditions (a) $\rho+p \geqslant 0$ (b) $\rho+p \geqslant 0$ are identically satisfied. Models presented in this chapter may be useful to discuss the role of bulk viscosity in explaining the decelerating/ accelerating behaviors and to understand structure formation in universe.

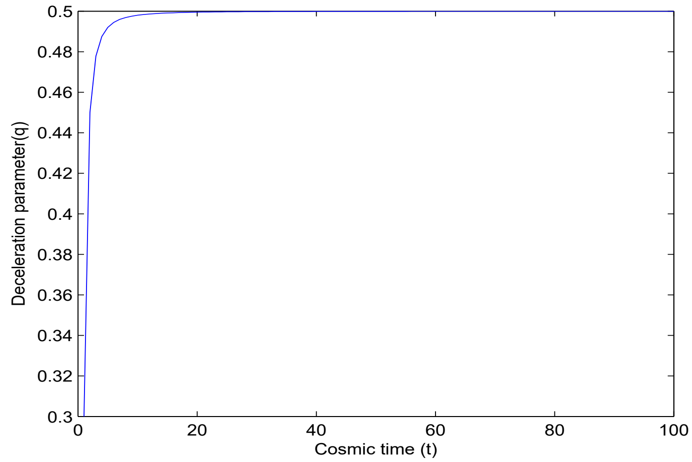

Figure 6.1: The plot of deceleration parameter q verses cosmic time t,β=$\frac{3}{2}$ α=1;

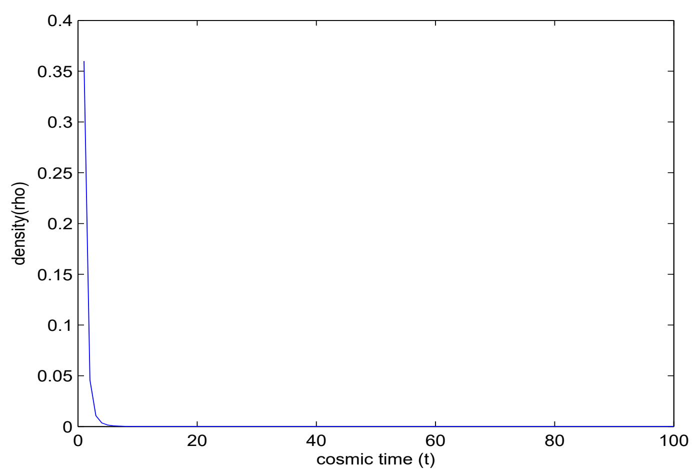

Figure 6.2: The plot of density ρ verses cosmic time t,λ =1, α=1;

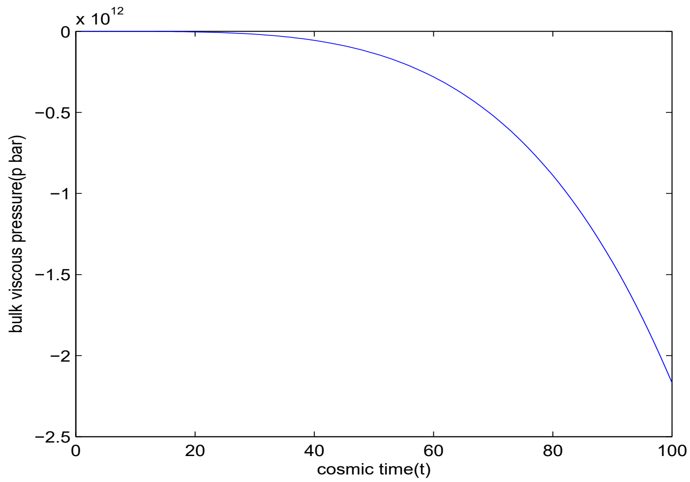

Figure 6.3: The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t, λ =1,α=1;

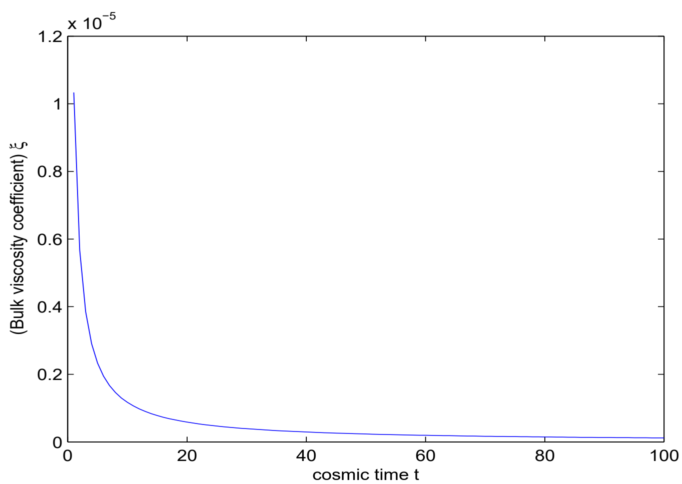

Figure 6.4: The plot of Bulk viscosity coefficient ζ verses cosmic time t, λ =1,α=1;

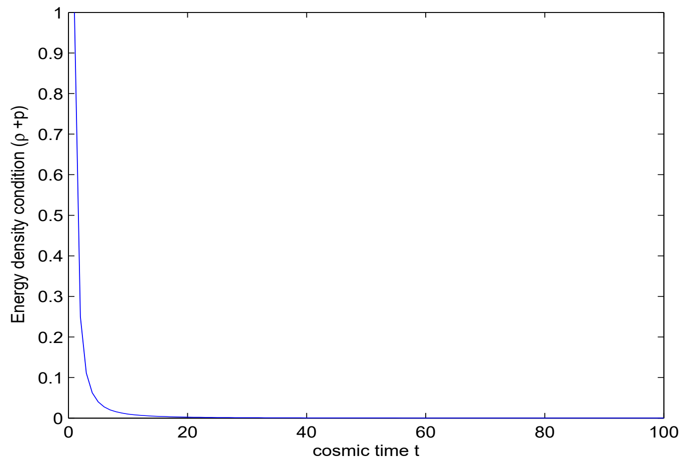

Figure 6.5: The plot of Energy density condition $\rho + p$ verses cosmic time t, λ =1, α=1;

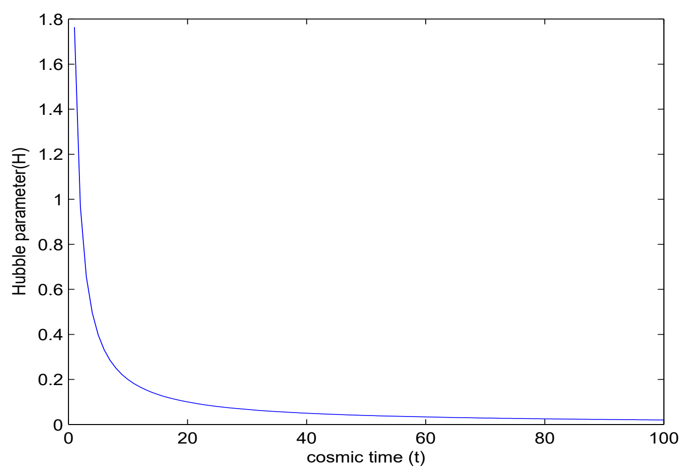

Figure 6.6: The plot of Hubble parameter H verses cosmic time t, α=1;

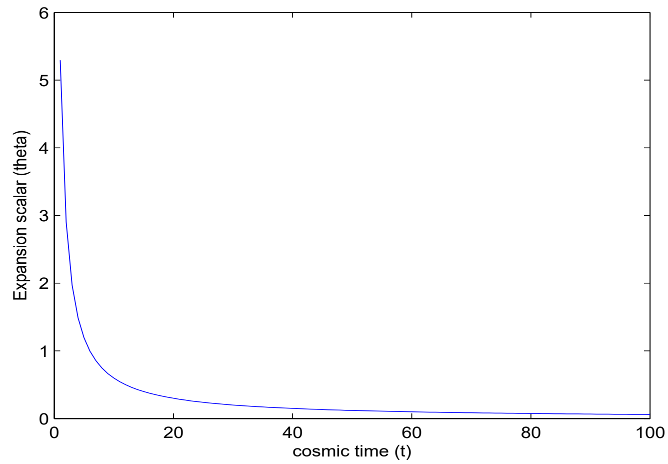

Figure 6.7: The plot of expansion scalar θ verses cosmic time t, α=1;

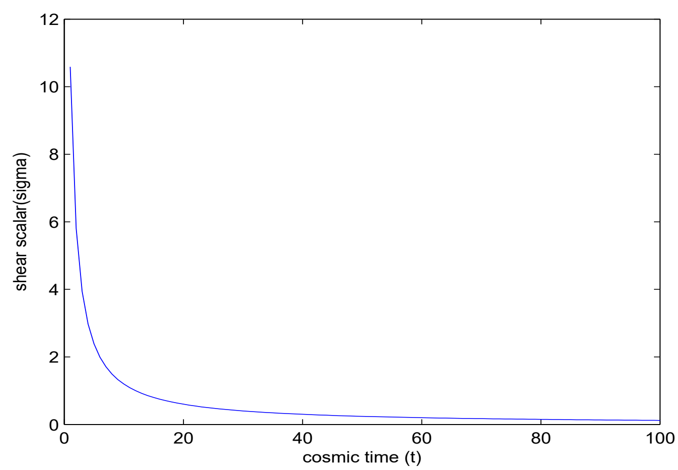

Figure 6.8: The plot of shear scalar σ verses cosmic time t, α=1;

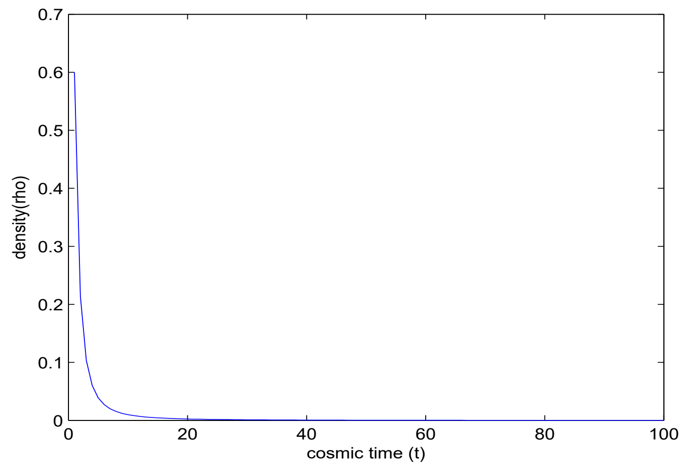

Figure 6.9: The plot of density ρ verses cosmic time t, Q=1, λ=1, m=0.5, α=1;

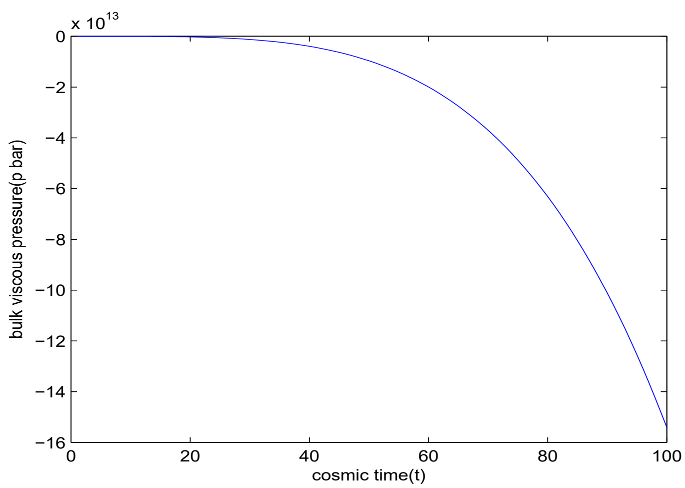

Figure 6.10: The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t,Q=1,λ=1,m=0.5, α=1;

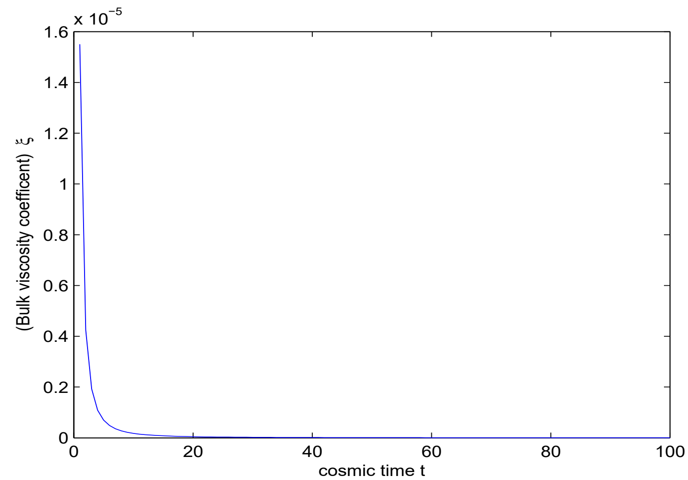

Figure 6.11: The plot of Bulk viscosity coefficient ζ verses cosmic time t,Q=1,λ=1,m=0.5, α=1;

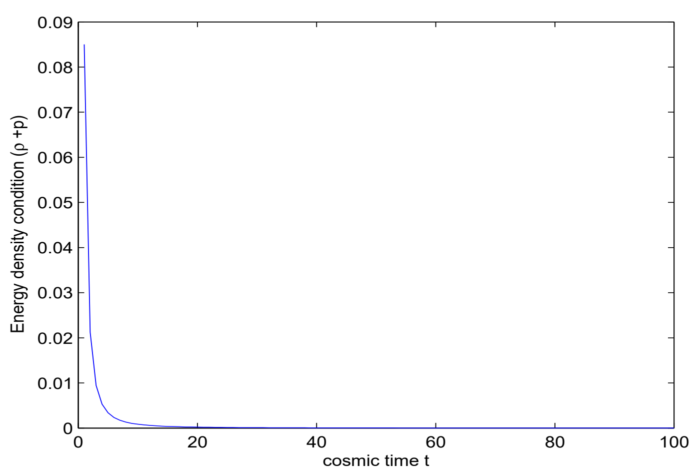

Figure 6.12: The plot of Energy density condition $\rho + p$ verses cosmic time t, Q=1λ=1, m=0.5, α=1;

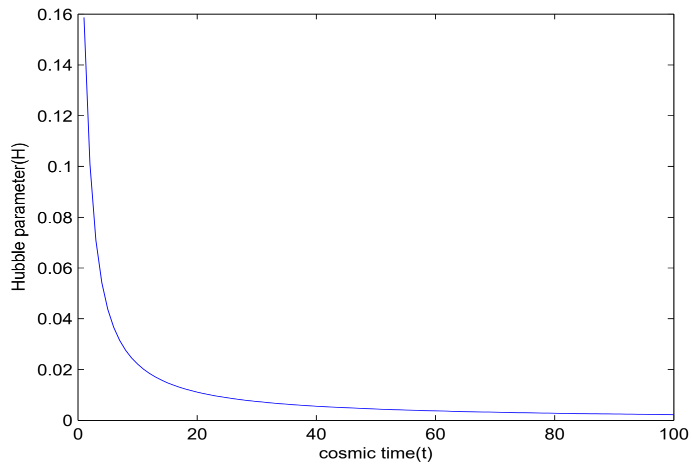

Figure 6.13: The plot of Hubble parameter H (for second model) verses cosmic time t,Q=1, α=1;

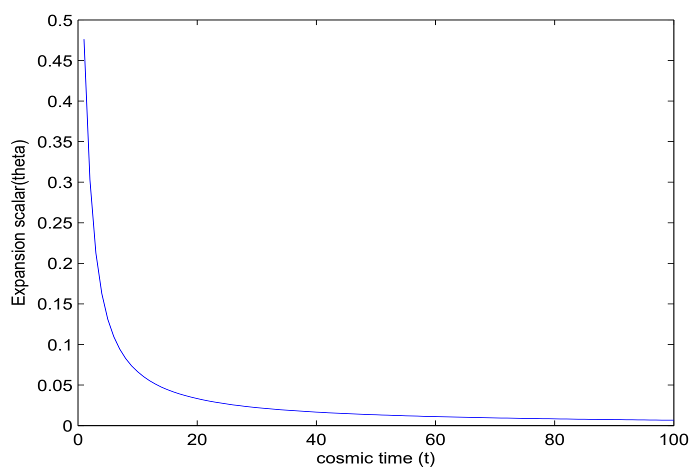

Figure 6.14: The plot of expansion scalar θ (for model second) verses cosmic time t,Q=1, α=1;

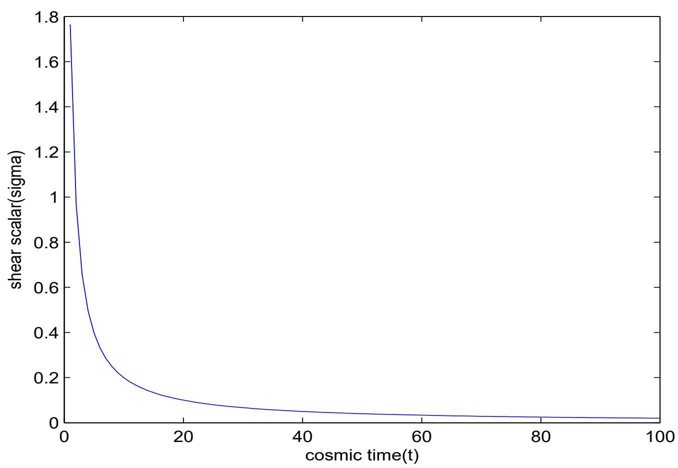

Figure 6.15: The plot of shear scalar σ cosmic time t,Q=1, α=1;

7 Some Kaluza- Klein Cosmological Models in f(R,T) Gravity Theory *

7.1 Introduction

It is widely believed that a consistent unification of all fundamental forces in nature would be possible within the space-time with an extra dimension beyond those four observed so for. Higher dimensional theories of Kaluza-Klein(KK)-type have been considered to study some aspects of early Universe (Chodos and Detweiler, 1980; Freund, 1982; Shafi and Wetterich, 1984; Sahdev, 1984). In such KK theory it has been assumed that the extra dimension form a compact manifold of very small size undetectable at present day energies. Thus, in such higher dimensional theories one would expect that at the grand unification scale the word manifold has more than one dimension. The Kaluza-Klein theory is attractive because it has an elegant presentation interms of geometry. In certain sense, it looks just like ordinary gravity in free space, except that it is phrased in five dimensions instead of four. Kaluza (1921)and (Klein, 1926a.,b) attempted to unify gravitation and electromagnetism. An interesting possibility known

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as the cosmological reduction process is based on the idea that at very early stage all dimensions in the universe are comparable. Later, the scale of the extra dimension becomes so small as to be unobservable by experiencing contraction. Such cosmological models were investigated by Forgacs and Horvath (1979). Guth (1981); Alvarez (1983) observed that during the contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problem, as compared to usual inflationary scenario. Gross and Perry (1983) have shown that the five-dimensional Kaluza-Klein theory of unified gravity and electromagnetism admits soliton solutions. Further, they explained the inequality of the gravitational and inertial masses due to the violation of Birkoffs theorem in Kaluza-Klein theories, which is consistent with the principle of equivalence. Appelquist and Chodos (1983) claimed through solution of the field equations that there is an expansion of four-dimensional space-time while fifth dimension contracts to the unobservable Plankian length scale or remains constant as needed for the real universe.

Recent observations of type Ia Supernovae (SNe Ia)at red shift $z < 1$ provide startling and puzzling evidence that the expansion of the universe at the present time appears to be accelerating behavior, attributed to "Dark Energy" with negative pressure. These observations (Chaterjee, 1992; Frieman and Waga, 1998; Ozer and Taha, 1987; Carvalho et al., 1992; Ratra and Peebles, 1988), strongly favour a significant and positive value of Λ. A number of models for dark energy to explain the late-time cosmic acceleration without the cosmological constant has been proposed, for example, a canonical scalar field, so-called quintessence, a non-canonical scalar field such as phantom, tachyon scalar field motivated by string theories, and a fluid with a special equation of state (EoS) called as Chaplygin gas. Nojiri and Odintsov (2003a,b) have presented a review of various modified gravities which have considered as gravitational alternative for dark energy. Nojiri and Odintsov (2004.) proposed that dark energy may become over standard matter due to universe expansion. Carroll et al. (2004) explained the presence of late time cosmic acceleration of the universe in f(R) gravity and proposed that dark energy model for specific $\frac{1}{R}$ modified gravity . Allemandi et al. (2005) discussed the dark energy dominance cosmic acceleration in first order Palatini formalism. There also exists a proposal of holographic dark energy. One of the most important quantity to describe the features of dark energy models is the equation of state parameter (EoS) ω, which is the ratio of the pressure p to the energy density ρ of dark energy, defined as $\omega=\frac{p}{\rho}$. There are two ways to describe dark energy models. One is a fluid description and the other is to describe the action of a scalar field theory. In both description, we can write the gravitational field equations, so that we can describe various cosmologies, e.g., the Λ CDM model, in which ω is a constant and exactly equal to -1 , quintessence model, where ω is a dynamical quantity and $-1 < \omega < -\frac{1}{3}$, and phantom model, where ω also varies in time and $\omega < -1$. This means that one cosmology may be described equivalently by different model descriptions discussed by Bamba et al. (2012). In view of the late time acceleration of the universe and the existence of dark energy and dark matter, several modified theories of gravitation have been proposed as alternative to Einstein's theory. Noteworthy amongst them is the f(R) gravity theory. Nojiri and Odintsov (2006a) developed the general scheme for modified f(R) gravity reconstruction from any realistic FRW cosmology. They have shown that the modified f(R) gravity indeed represents the realistic alternative to general relativity, being more consistent in dark epoch. Nojiri and Odintsov (2006b) developed a general programme for unification of matter -dominated era with acceleration epoch for scalar -tensor theory or dark fluid. Nojiri and Odintsov (2007) have reviewed various modified gravities considered as gravitational alternative for dark energy.They have considered the version of f (R), f (G) or f (R,G) gravity, model with non-linear gravitational coupling or string inspired model with Gauss-Bonnet-dilaton coupling in the late universe. Nojiri and Odintsov (2011), have studied f (R) gravity in different context. Bertolami et al. (2007)proposed a generalization of f(R) theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Shamir (2010), proposed a physically viable f(R) gravity model, which show the unification of early time inflation and the late time acceleration.

In this chapter, we present some new classes of five dimensional Kaluza-Klein cosmological models in the presence of a perfect fluid source in f(R,T) gravity theory. The chapter is organized as follows: In Sect. 7.2, we revisit the field equations presented by Reddy et al. (2012a). We then derive algorithms for generating new solutions of the field equations in Sect.7.3 . In Sect.7.4, starting with solution of Reddy et al. (2012a), we obtain some solutions of the field equations which represent accelerating cosmological models. The physical and kinematical properties of the models are also discussed.Conclusions are given in Sect.7.5. .

7.2 Metric and Field Equations

We consider a five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\Psi^2 \tag{7.1}$$

where A(t) and B(t) are the scale factors. The fifth coordinate Ψ is taken to be space-like. The field equations in f(R,T)theory of gravity for the function f(R,T), which is given in (5.2), when the matter source is perfect fluid (1.8), are given by Harko et al. (2011b).

The field equations (5.5) for the metric (7.1) in comoving coordinates lead to the following equations

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\lambda)\rho - p\lambda, \tag{7.2}$$

$$\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = -(8\pi + 3\lambda)p + \rho\lambda, \tag{7.3}$$

$$3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right)^2 = -(8\pi + 3\lambda)p + \rho\lambda. \tag{7.4}$$

Here an overhead over dot denotes ordinary differentiation with respect to time t.

For the metric (7.1), the spatial volume V and the average scale factor a are given by

$$V = a^4 = A^3 B \tag{7.5}$$

where a is the scale factor.

The mean Hubble parameter H has the expression

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{7.6}$$

where H_x=H_y= $H_z = \frac{\dot{A}}{A}$ and $H_\Psi = \frac{\dot{B}}{B}$ are directional Hubble parameters.

The scalar expansion θ and shear scalar σ are are given by

$$\theta = \frac{3}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \tag{7.7}$$

$$\sigma^2 = \frac{3}{4}\theta^2. \tag{7.8}$$

In next Sect. we follow Hajj-Bouttros (1986a) to derive algorithms for generating new solutions of the field equations of KK -type perfect fluid cosmological models within the framework of f(R,T) gravity theory .

7.3 Generating Technique

From Eqs. (7.3) and (7.4) we obtain

$$\frac{2\ddot{A}}{A}+2\frac{\dot{A}^2}{A^2}-2\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{B}}{B}=0. \tag{7.9}$$

To treat Eq. (7.9), we introduce new functions R and S given by

$$R=\frac{\dot{A}}{A}, S=\frac{\dot{B}}{B}, \tag{7.10}$$

By use of (7.10), Eq. (7.9), becomes

$$2\dot{R}+4R^2-2RS-\dot{S}-S^2=0. \tag{7.11}$$

The nonlinear equation. (7.11) can be treated as a Riccati equation in R or S.

If we treat Eq. (7.11) as a Riccati equation in R, it can be linearized by means of change of function

$$R=R_0+\frac{1}{X}. \tag{7.12}$$

where R_0 is a particular solution of Eq. (7.11) . Using (7.12) in Eq. (7.11), we obtain

$$\dot{X}+(S-4R_0)X=2 \tag{7.13}$$

Eq. (7.13) is linear first-order differential equation which has the general solution given by

$$X = \frac{A_0^4}{B}\left(\int 2\frac{B}{A_0^4}dt + k_1\right), \tag{7.14}$$

k_1 being an integration constant. From Eqs. (7.12) and (7.14), we obtain after integration

$$A = A_0 k_2 \exp\left[\frac{dt}{\frac{A_0^4}{B}(\int 2\frac{B}{A_0^4}dt + k_1)}\right] \tag{7.15}$$

where k_2 being another constant. Hence, from metric function $[A_0, B]$ we can generate new function [A,B] where (A)is given by Eq.(7.15) and B remains invariable.

If (7.11) is regarded as a Riccati equation in S, we can be linearized it by the change of function

$$S = S_0 + \frac{1}{Y}. \tag{7.16}$$

where S_0 is a particular solution of (7.11).

Introducing (7.16) into Eq. (7.11), we obtain

$$\dot{Y} - (2R + 2S_0)Y = 1 \tag{7.17}$$

Eq. (7.17), on integration, gives

$$Y = A^2 B_0^2 \left(\int \frac{dt}{A^2 B_0^2} + k_3\right) \tag{7.18}$$

where k_3 being a constant . From Eqs. (7.17) and (7.18), we obtain

$$B = B_0 k_4 \exp\left[\int \frac{dt}{A^2 B_0^2 (\int \frac{dt}{A^2 B_0^2} + k_3)}\right] \quad (7.19)$$

where k_4 is another constant of integration. Thus, from the couple $[A, B_0]$ we can generates [A,B] where B is given by Eq.(7.19) and A remains invariable

Reddy et al.(2012a) have presented the solutions of the field equations (7.2)-(7.4) in f(R,T) gravity theories has given by the metric

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - [kt]^{\frac{2m}{k}} d\Psi^2 \quad (7.20)$$

where $k=\frac{m^2+2m-3}{m-1}$, $m \neq 1$. Starting with this metric, we now generate new solutions of the field equations (7.2)-(7.4) by applying the generating techniques (7.15) and (7.19)

7.4 Model I

To apply our generation technique (7.19) to the metric (7.20), we take

$$A = (kt)^{\frac{1}{k}}, B_0 = (kt)^{\frac{m}{k}}$$

Then, performing the integration in (7.19) , the new metric function B is obtained as

$$B = k_4 (k)^{\frac{m}{k}} [t]^{\frac{k^2 - 2m^2 - 2m + km}{k(k-2m-2)}} \quad (7.21)$$

by putting k_3=0. Hence the metric of our new solution can be written in the form

$$ds^2 = dt^2 - [kt]^{\frac{2}{k}}(dx^2 + dy^2 + dz^2) - \left\{k^{\frac{m}{k}} t^{\frac{k^2-2m^2-2m+mk}{k(k-2m-2)}}\right\}^2 d\Psi^2 \quad (7.22)$$

For the model (7.22) the physical and kinematical parameters are given by

$$H = \frac{1}{4mkt(k-2m-2)}[k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \tag{7.23}$$

$$\theta = 3H = \frac{3}{4mkt(k-2m-2)}[k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k], \tag{7.24}$$

$$\sigma^2 = \frac{3}{4}\theta^2 = \frac{3}{4}\left(\frac{3}{4mkt(k-2m-2)}[k^2(1-m) - 2mk - 2k - 2m^3 - 2m^2 + m^2k]\right)^2. \tag{7.25}$$

$$V = k^{\frac{m+3}{k}} t^{\frac{k^2-2m-2m+mk}{k(k-2m-2)}+\frac{3}{k}}. \tag{7.26}$$

The deceleration parameter q is defined in (1.31) which has the value given by

$$q = \frac{4k}{3(k-2m-2) + k^2 - 2m^2 - 2m + mk} - 1. \tag{7.27}$$

The pressure and energy density are obtained as

$$p = \left[\frac{\lambda[k(k-2m-2)E_1 - (8\pi+3\lambda)E_2m^2]}{(8\pi+4\lambda)(8\pi+2\lambda)m^2k^2(k-2m-2)t^2}\right], \tag{7.28}$$

$$\rho = \left[\frac{3(1+\frac{k^2-2m^2-2m+mk}{k(k-2m-2)})}{m^2t^2(8\pi+3\lambda)} + \frac{\lambda^2(k(k-2m-2) - \frac{(8\pi+3\lambda)}{\lambda}E_2m^2)}{(8\pi+2\lambda)(8\pi+3\lambda)(8\pi+4\lambda)m^2k^2t^2}\right] \tag{7.29}$$

where

$$E_1 = k(k-2m-2) + 3m(k^2 - 2m^2 - 2m + mk),$$

$$E_2 = \left[2 - m + (2m-1)(k^2 - 2m^2 - 2m + mk))(k(k-2m-2)\right] + (k^2 - 2m^2 - 2m + mk)$$

From the above results we observed that the model has initial singularity at t=0 if $k > 2(m+1)$ which leads to $m < 1$. We see that θ, σ ,H, p and ρ have infinite value at the initial singularity t=0. These parameters are decreasing function of time which tend to zero for large time. Since $\frac{\sigma^2}{\theta^2} \neq 0$, the model is anisotropic throughout the evolution of the universe. We also find that the deceleration parameter q is negative , which corresponds to an accelerating model of the universe in five-dimensional Kaluza-Klein theory.

7.5 Model II

We apply formula (7.15) for the metric (7.20) to generate the new function A by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (k)^{\frac{m}{k}} [t]^{\frac{k^2-2m^2-2m+km}{k(k-2m-2)}}$$

Then, after integration, we obtain

$$A = k_2 (k)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(mk-4k)(k-2m-2))} + \frac{1}{k}} \tag{7.30}$$

assuming k_1 =0. The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left\{ (k)^{\frac{1}{k}} [t]^{\frac{mk(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(km-4k)(k-2m-2))}+\frac{1}{k}} \right\}^2 (dx^2+dy^2+dz^2) - \left(k^{\frac{m}{k}} t^{\frac{k^2-2m^2-2m+mk}{k(k-2m-2)}} \right)^2 d\Psi^2. \tag{7.31}$$

The metric (7.31) represents the five-dimensional Kaluza-Klein cosmological model in f(R,T) gravity theory with the following physical and kinematical parameters.

$$V = k^{\frac{m+3}{k}} [t]^{\frac{3k(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+(m-4)k(k-2m-2))}+\frac{k^2-2m^2-2m+mk}{k(k-2k-2)}}, \tag{7.32}$$

$$H = \left[\frac{1}{8t(m(k^2-2m^2-2m+mk)+km(k-2m-2)-4k^2(k-2m-2)^2)} \right] . \left[((m-4)k(k-2m-2)(1+(k-2m-2)((m-4)k+3mk^2)) \right] + \left[(k^2-2m^2-2m+km)(2km(k-2m-2)+2m^2(k^2-2m^2-2m+km))) \right], \tag{7.33}$$

$$\theta = \left[\frac{3}{8t(m(k^2-2m^2-2m+mk)+km(k-2m-2)-4k^2(k-2m-2)^2)} \right] . \left[((m-4)k(k-2m-2)(1+(k-2m-2)((m-4)k+3mk^2)) \right] + \left[(k^2-2m^2-2m+km)(2km(k-2m-2)+2m^2(k^2-2m^2-2m+km))) \right], \tag{7.34}$$

$$\sigma^2 = \frac{3}{4}\theta^2, \tag{7.35}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -[1+(E_3^3 E_4)^{\frac{1}{2}}], \tag{7.36}$$

$$p = -\left[\frac{(16\pi+3\lambda)(E_3^2+E_3E_4)+(8\pi+3\lambda)(E_4^2-(E_3+E_4))}{t^2(8\pi+4\lambda)(8\pi+2\lambda)} \right], \tag{7.37}$$

$$\rho = \left[(3E_3E_4) - \frac{\lambda}{(8\pi+4\lambda)(8\pi+2\lambda)}(16\pi+3\lambda)(E_3^2+E_3E_4) + (8\pi+3\lambda)(E_4^2-(E_3+E_4))\right] \cdot \left[\frac{1}{t^2(8\pi+3\lambda)}\right] \tag{7.38}$$

where

$$E_3 = \left[\frac{k^2(k-2m-2)}{2(m(k^2-2m^2-2m+mk)+mk(k-2m-2)-4k(k-2m-2))} + \frac{1}{k}\right],$$

$$E_4 = \frac{k^2-2m^2-2m+mk}{k(k-2m-2)}$$

From the Fig.(7.5), it is clear that the model (7.31) represents a five-dimensional Kaluza-Klein accelerating cosmological model . The other physical and kinematical behaviors of the model are same as model I.

7.6 Model III

We now use formula (7.15) to generate new metric function A by taking

$$A_0 = (kt)^{\frac{1}{k}}, B = (kt)^{\frac{m}{k}}$$

Then performing integration in (7.15) ,we obtain

$$A = k_2 k^{\frac{1}{k}} [t]^{\frac{km}{2(m^2-4k+m)}+\frac{1}{k}}, \tag{7.39}$$

assuming k_1=0 Then the metric (7.1) can be written in the form (7.40) where k_5 is integration constant.

The metric can be written as

$$ds^2 = dt^2 - \left(k^{\frac{1}{k}}[t]^{\frac{km}{2(m^2-4k+m)}+\frac{1}{k}}\right)^2 (dx^2 + dy^2 + dz^2) - (kt)^{\frac{2m}{k}} d\Psi^2 \tag{7.40}$$

The model (7.40) represents the five-dimensional Kaluza-Klein cosmological with perfect fluid in f(R,T) gravity theory . The physical and the kinematical parameters of the model (7.40) are given as follows:

$$V = (k)^{\frac{m+3}{k}}[t]^{\frac{3+m(2m^2-5k+2m)}{2k(m^2-4k+m)}}, \tag{7.41}$$

$$H = \frac{1}{4tk}(3kE_5 + m), \tag{7.42}$$

$$\theta = 3H = \frac{3}{4kt}(3kE_5 + m), \tag{7.43}$$

$$\sigma^2 = \frac{27}{64t^2k^2}(3kE_5 + m)^2, \tag{7.44}$$

$$q = -1 + \left[\frac{8(m^2 - 4k - m)k}{t(3k^2m + 2m(m^2 - 4k + m))}\right], \tag{7.45}$$

$$p = \left[\frac{E_5((8\pi + 3\lambda)k^2 + 3\lambda km^2) - (8\pi + 3\lambda)(E_5(2mk - E_5) + m(m-k)) - (8\pi + 2\lambda)k^2E_5^2}{(8\pi + 4\lambda)(8\pi + 2\lambda)k^2t^2}\right], \tag{7.46}$$

$$\rho = \left[\frac{1}{(8\pi+4\lambda)(8\pi+3\lambda)(8\pi+2\lambda)k^2t^2}\right].\left[((3E_5^2(E_5-1)^2k^2+3E_5m)(8\pi+4\lambda)(8\pi+2\lambda)\right] + \left[\lambda E_5(8\pi+3\lambda)k^2+3\lambda km^2-\lambda(8\pi+3\lambda)(m-k)m-(8\pi+2\lambda)k^2E_5)\right] \tag{7.47}$$

where

$$E_5 = \frac{k}{2(m^2-4k+m)} + \frac{1}{k}.$$

For the metric (7.40) the spatial volume is zero at t=0 if $k < \frac{m(m+1)}{4}$ .The physical and kinematical properties same as perfect fluid Model I

7.7 Model IV

We use the formula (7.19) for the metric (7.40) to generate the new function B by setting

$$A = k^{\frac{1}{k}}[t]^{\frac{km}{2(m^2-4k+m)}+\frac{1}{k}}, B = (kt)^{\frac{m}{k}}.$$

Then, from Eq.(7.19) , the new function B is obtained as:

$$B = k_4 k^{\frac{m}{k}}[t]^{\frac{mk(m^2-4k+m)}{km^2+2(m^2-4k+m)+(3k+2m)(m^2-4k+m)+mk(m^2-4k+m)}} \tag{7.48}$$

taking k_3=0. The metric of the solution can be written in the form

$$ds^2 = dt^2 - \left[k^{\frac{1}{k}}[t]^{\frac{km}{2(m^2-4k-m)}+\frac{1}{k}}\right]^2(dx^2+dy^2+dz^2) - \left[(k)^{\frac{m}{k}}[t]^{\frac{km(m^2-4k+m)}{km^2+2(m^2-4k+m)(1+m+3km)}}\right]^2 d\Psi^2 \tag{7.49}$$

The metric (7.49) represents five-dimensional Kaluza-Klein cosmological model in f(R,T) gravity with the following physical and kinematic parameters in the model.

$$V = k^{\frac{m+3}{k}} \left[t^{\frac{3km}{2(m^2-4k+m)}+\frac{3}{k}+\frac{m}{k}+\frac{(km(m^2-4k+m))}{km^2+(2+m(4+k))(m^2-4k+m)}} \right], \tag{7.50}$$

$$H = \frac{(3(km^2+1)+2m(m^2-4k+m)E_6)}{8tm(m^2-4k+m)}, \tag{7.51}$$

$$\theta = 3H = 3\left[\frac{(3(km^2+1)+2m(m^2-4k+m)E_6)}{8tm(m^2-4k+m)}\right], \tag{7.52}$$

$$\sigma^2 = \frac{27}{256}\left[\frac{(3(km^2+1)+2m(m^2-4k+m)E_6)}{8tm(m^2-4k+m)}\right]^2, \tag{7.53}$$

$$q = -(1-\frac{1}{E_7}), \tag{7.54}$$

$$\begin{aligned} p = -\left[\frac{1}{4m^2(m^2-4k+m)^2t^2(8\pi+4\lambda)(8\pi+2\lambda)}\right] . \big[((km^2+1)^2(16\pi+3\lambda)+((32\pi+6\lambda)E_6\big] \\ + \big[(km^2+1)m(m^2-4k+m))\big] + \big[(8\pi+3\lambda)m(m^2-4k+m)(km^2+1)\big] \\ + \big[(8\pi+3\lambda)E_6(E_6-1)m(m^2-4k+m))\big], \end{aligned} \tag{7.55}$$

$$\rho = \left[\frac{3(km^2+1)^2E_6(km^2+1)}{4m^2(m^2-4k+m)^2t^2(8\pi+3\lambda)}\right] - \left[\frac{\lambda}{(8\pi+4\lambda)(8\pi+3\lambda)(8\pi+2\lambda)t^2 4m^2(m^2-4k+m)^2}\right]$$
$$.\left[((km^2+1)^2(16\pi+3\lambda)+(32\pi+6\lambda)E_6(km^2+1)m(m^2-4k+m)\right]$$
$$+\left[(8\pi+3\lambda)E_6(E_6-1)m(m^2-4m+m))\right] \tag{7.56}$$

where

$$E_6 = \frac{m}{k} + \frac{m^2-4k+m}{(km^2+(2+4km+m)(m^2-4k+m)},$$
$$E_7 = \left[\frac{3km}{2(m^2-4k+m)} + \frac{m}{k} + \frac{km(m^2-4k+m)}{km^2+(m^2-4k+m)(2+4m+km)}\right].$$

We observe that the spatial volume of the model (7.49) is zero at t=0 and increases with time if $k < \frac{m(m+1)}{4}$.Therfore the model has a point type singularity at t=0 where θ, σ^2, H, p and ρ diverge . These parameters are decreasing function of time and ultimately tend to zero for large time. The negative pressure, as shown by Fig.(7.11), indicates that the model is accelerating .

7.8 Conclusions

The higher dimensional cosmological models are of considerable importance because of the underlying idea that cosmos in early stages of evolution might have had a higher dimensional era. The extra space reduces to a volume with the passage of time, which is beyond the ablity of experimental observation at the moment Reddy (2009). It is well known that Kaluza-Klein models represent the cosmos in its early stages of evolution. In the present work, we have derived algorithms for generating new solutions of the field equations with a perfect fluid for a five dimension Kaluza-Klein space-time within the framework of f(R,T) gravity theory proposed by Harko et al. (2011b). Starting from the model obtained by Reddy et al. (2012a), we have presented new cosmological models of the present-day accelerating universe. These models are expanding, shearing and accelerating which have point-type singularity at t=0. All the physical and kinematical parameters, being infinite at the initial singularity , are decreasing functions of time which ultimately tend to zero for large time . The anisotropy in the cosmological models are maintained throughout the passage of time.

Nojiri and Odintsov (2003a) studied a modify theory of gravity where the universe interns inflates, decelerates and then accelerates in early times, radiation dominated era. Our models are similar to the case of five dimensional f(R) gravity except the decelerating behavior in the presence of a perfect fluid source discussed by Huang et al. (2010) and Agmohammadi, et al. (2009). It has been observed that in the five dimensional f(R) and f(R,T) gravity theories, the expansion and contraction of the extra dimension could result in the present accelerated expansion of other spatial dimensions. This is possible by cosmic re-collapse of the universe in the finite future. It follows that the the present accelerating models of the universe are consistent with the recent observation of type-Ia supernovae (Perlmutter et al., 1999; Riess et al., 1998).

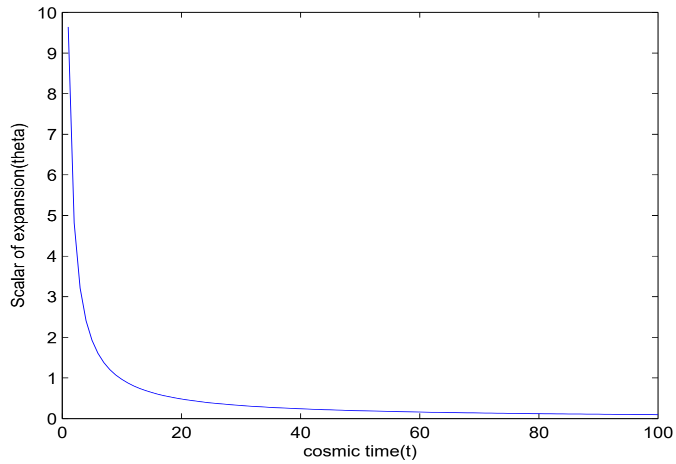

Figure 7.1: The plot of scalar expansion θ verses cosmic time t, m=0.5;λ=1;k=1;

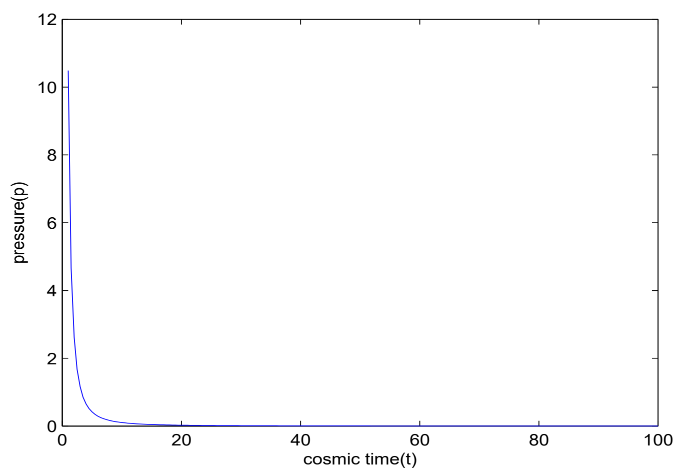

Figure 7.2: The plot of pressure p verses cosmic time t, m=0.5;λ=1;k=1;

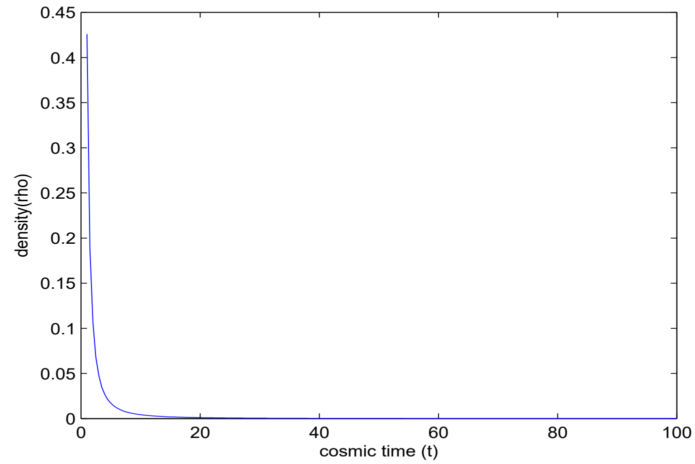

Figure 7.3: The plot of density ρ verses cosmic time t, m=0.5;λ=1;k=1;

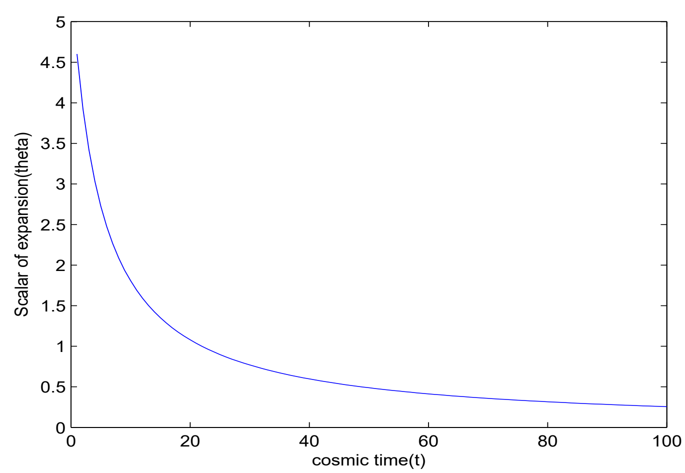

Figure 7.4: The plot of θ verses cosmic time t, m=0.5;λ=1;k=1;

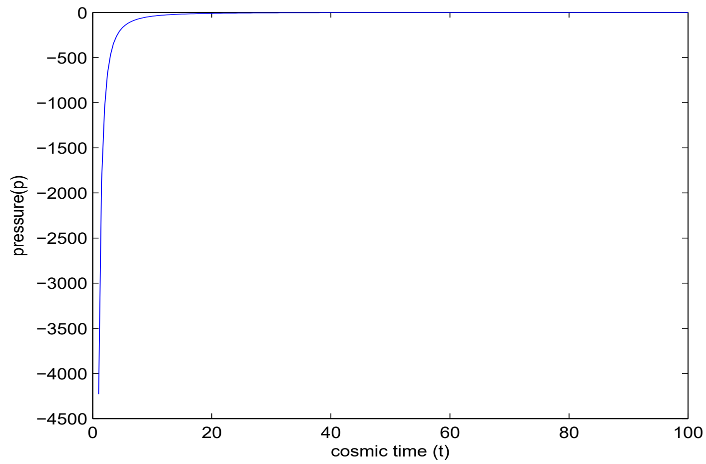

Figure 7.5: The plot of pressure p verses cosmic time t, m=0.5;λ=1;k=1;

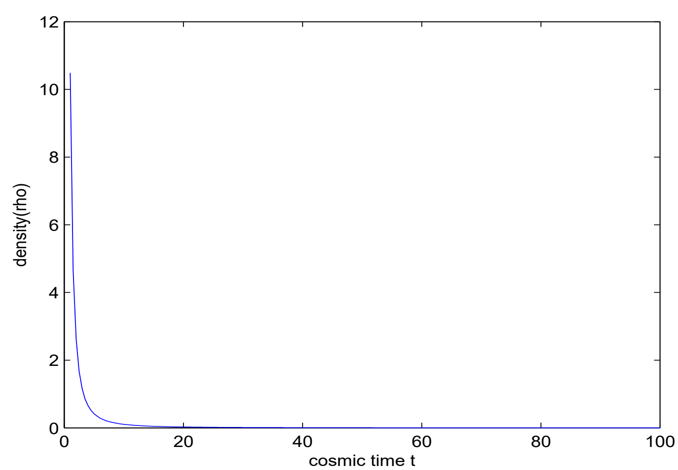

Figure 7.6: The plot of density ρ verses cosmic time t, m=0.5;λ =1;k=1;

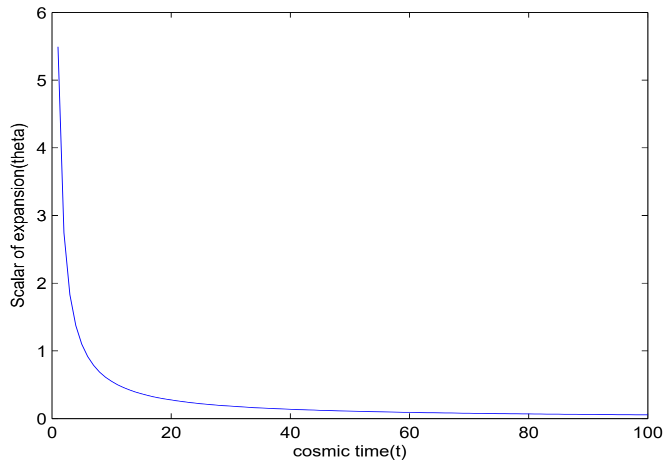

Figure 7.7: The plot of scalar expansion θ verses cosmic time t, m=0.5;λ=1;k=1;

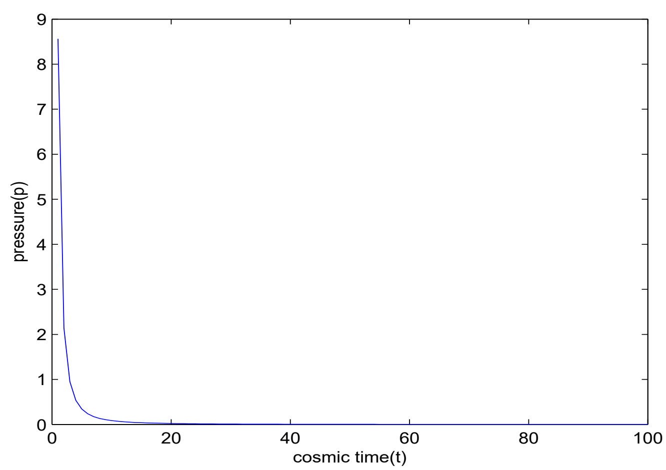

Figure 7.8: The plot of pressure p verses cosmic time t, m=0.5;λ=1;k=1;

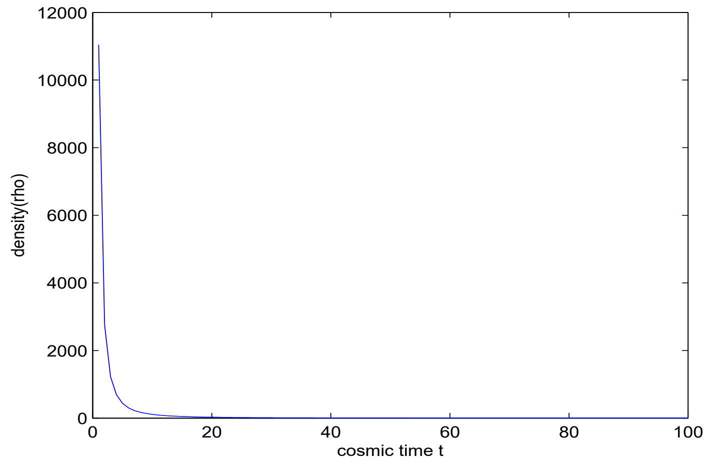

Figure 7.9: The plot of density ρ verses cosmic time t, m=0.5;λ=1; k=1;

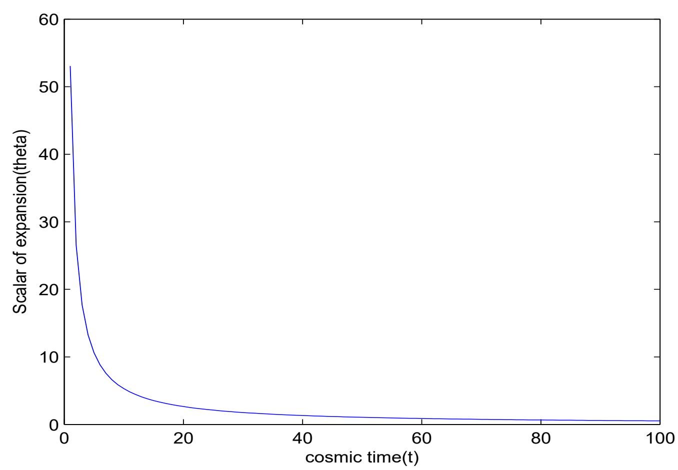

Figure 7.10: The plot of Scalar of expansion θ verses cosmic time t, m=0.5;λ=1;k=1;

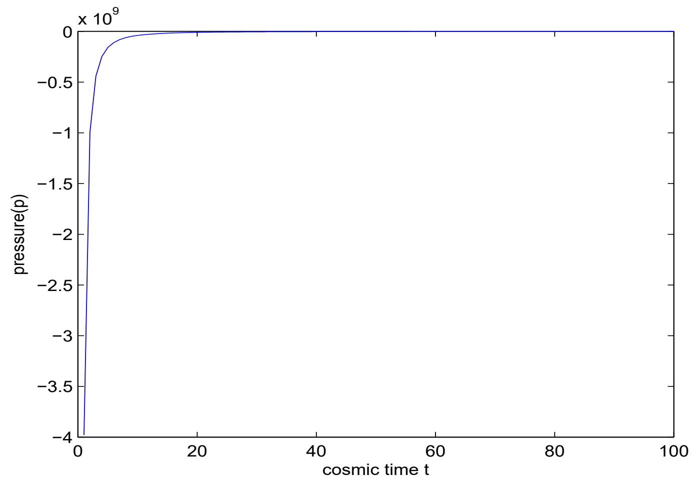

Figure 7.11: The plot of pressure p verses cosmic time t, m=0.5;λ=1;k=1;

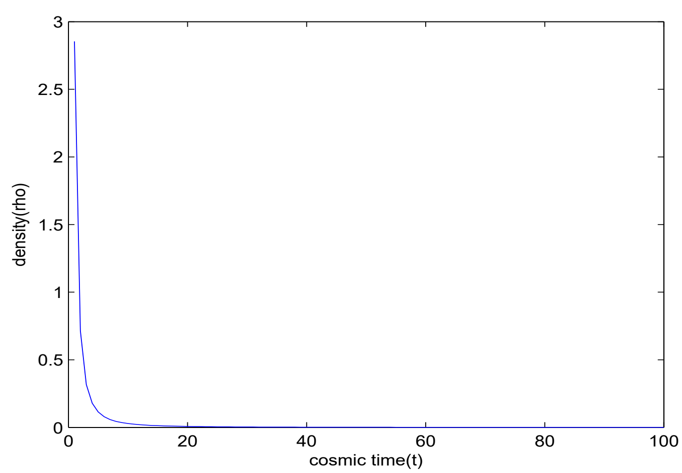

Figure 7.12: The plot of density ρ verses cosmic time t, m=0.5;λ=1;k=1;

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List of Publications

1 Shri Ram , **Priyanka**. Bianchi types I and V bulk viscous fluid cosmological models in $f(R,T)$ gravity theory" ***Cent. Eur. J. Phys. 12, 744-754 (2014) Springer*** .

2 Shri Ram, **Priyanka** Spatially Homogeneous String Cosmological Models with Bulk Viscosity in $f(R,T)$ Gravity Theory" ***Electronic Journal of Theoretical Physics 11,203 - 220 , (2014)***.

3 Shri Ram **Priyanka** Bianchi Type -II Inflationary Models with Stiff Matter and Decaying Cosmological Term ***CHIN. PHYS. LETT 31, 070401 (2014), IOP***.

4 Shri Ram, **Priyanka**. Some Kaluza- Klein cosmological models in $f(R,T)$ gravity theory ***Astrophys Space Sci 347, 389, (2013), Springer*** .

5 Shri Ram **Priyanka**, M. K. Singh Anisotropic cosmological models in f(R, T) theory of gravitation ***Pramana J. Phys. 81 , 67, (2013), Springer***

6 **Priyanka**, M. K. Singh, Shri Ram. Anisotropic Bianchi type-III Bulk Viscous Fluid Universe in Lyra Geometry ***Advances in Mathematical Physics Volume 2013, Article ID 416294, 5 pages, Hindawi***,

7 **Priyanka**, S. Chandel, M.K. Singh and Shri Ram . Bianchi Type-VI_0 Dark Energy Cosmological Models in General Relativity , ***Global Journal of Science Frontier Research 12, 0975 (2012), Mathematics and Decision Sciences*** .

8 **Priyanka** , S. Chandel , M.K. Singh and Shri Ram. AN Isotropic Dark Energy Cosmological Model of Bianchi Type-III ***International Journal of Theoretical and Applied Physics 2 , 147 (2012),ASCENT***

9 **Priyanka** , Shri Ram Anisotropic perfect fluid cosmological models in f(R) theory of gravity (**Communicated**).