5 Spatially Homogeneous and Anisotropic Cosmological Models with Matter and Cosmic Strings in f(R,T) Gravity Theory *

5.1 Introduction

The evolution of isotropic cosmological models in the presence of perfect fluid has been extensively studied by many cosmologists. It has been certainly of interest to study cosmologies with richer structure both geometrically and physically than the standard perfect fluid models. It is of interest to take into account dissipation processes such as viscosity in cosmological models. Viscous fluid models has been used in an attempt to explain the observed highly isotropic matter distribution on the high entropy per baryon in the present state of the universe. Misner (1967, 1968) suggested that the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of black-body radiation. The viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe. A uniform cosmological model filled with

*The Content of this chapter is published in form of two papers (i)Pramana J Phys 81,67,(2013),Springer, (ii)Electronic Journal of Theoretical Physics 11, 203220 (2014) fluid, which possesses pressure and bulk viscosity, was investigated by Murphy (1973)and has shown that the big-bang type singularity appears in the infinite past. Of late, there have been considerable interests in cosmological models with bulk viscosity, since bulk viscosity leads to the accelerated expansion phase of the early universe, popularly known as the inflationary phase. The possibility of bulk viscosity leading to inflationary -like solutions in general relativistic FRW models has been investigated by several authors viz.Barrow (1986); Padmanabhan and Chitre (1987); Maartens (1995) etc..

In recent years there has been considerable interest in string cosmology as cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big-bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories. Moreover, it is expected that topological defects could have formed naturally during the phase transition followed by spontaneous broken symmetries. Cosmic strings, being linear topological defects, have very interesting properties and might play important roles in the structure formation (Kibble, 1976; Everett, 1981) . Letelier (1983) obtained cosmological solutions of cloud formed by massive strings with particles attached along its extension in Bianchi type-I and Kantowski-Sachs space times. The cloud of strings is the generalization of Takabayasi 's realistic model of string called p-string (Letelier, 1978). Singh and .Ram (1997) obtained exact Bianchi type-III cosmological solutions of massive strings in the presence of magnetic field. Bali and Dave (2003); Bali and Pradhan (2007); Pradhan and Chouhan (2011) etc. have investigated Bianchi type string cosmological models in general relativity. Reddy (2003, 2005); Reddy and Naidu (2007) have studied string cosmological models in different contexts. Several authors viz. Chodos and Detweiler (1980); M.Youshimura (1984); Krori et al. (1994); Reddy (2003); Mohantly and Sahoo (2007) etc. have constructed higher dimensional string cosmological models in certain theories of gravitation.

Recent most remarkable observational discoveries have shown that our current universe is not only expanding but also accelerating. This was first observed from high red shift supernova Ia (Riess et al., 1998, 2004; Perlmutter et al., 1999; Astier et al., 2006), and confirmed later by cross checks from the cosmic microwave background radiation Bennet et al. (2003), Spergel et al. (2003) and large scale structure (Tegmark, et al, 2004). In Einstein,s general relativity, in order to have such acceleration, one needs to introduce a component to the matter distribution of the universe with a large negative pressure. This component is usually referred as dark energy (DE). Astronomical observations indicate that our universe is flat and currently consists of approximately $\frac{2}{3}$ dark energy and $\frac{1}{3}$ dark matter source of dark energy. Many radically different models have been proposed such as a tiny positive cosmological constant, quintessence Caldwell et al. (1998); Liddle and Scherrer (1999); Steinhardt et al. (1999). In view of the late time acceleration of the universe and the existence of dark energy and dark matter, several modified theories of gravity have been developed and studied. Noteworthy amongst them is the f(R) gravity theory (Carroll et al., 2004; Sotiriou and Faraoni, 2010). Bertolami et al. (2007) proposed a generalization of f(R) theory of gravity by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Nojiri and Odintsov (2006a) developed the general scheme for modified f(R) gravity reconstruction from any realistic FRW cosmology. They have shown that modified f(R) gravity indeed represents the realistic alternative to general relativity, being more consistent in dark epoch. Nojiri and Odintsov (2007) developed a general programme for unification of matter-dominated era with acceleration epoch for scalar-tensor theory or dark fluid. Shamir (2010) proposed a physically viable f(R) gravity model, which show the unification of early time inflation and late time acceleration. Very recently, Singh and Singh (2013) investigated functional form of f(R) for a known scale factor in anisotropic locally-rotationally -symmetric (LRS) Bianchi I model with perfect fluid as a source of matter. By assuming the deceleration parameter to be a constant and the shear scalar proportional to the expansion scalar they presented a power -law form of scale factors.

A further generalization of f(R) gravity theory has been proposed by Harko et al. (2011b), is known as f(R,T) theory of gravity where, as usual, R is the Ricci scalar and T is the trace of the energy -momentum tensor. They also argued that due to the coupling of the matter and geometry, this gravity model depends on a source term, which is nothing but the variation of the matter stress-energy tensor. They have also demonstrated that the possible reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function f(T). In this theory the covariant divergence of the stress energy tensor is non-zero. As a result, the motion of test particles is not along geodesic path due to presence of an extra force perpendicular to the four velocity. The cosmic acceleration in the modified f(R,T) theory results not only from geometrical contribution but also from the matter content. Subsequently, Houndjo (2012) has chosen f(R,T) = $f_1(R) + f_2(T)$ and discussed transition of matter dominated era to an accelerated phase. Sharif and Zubair (2012b) have studied thermodynamics in this f(R,T) theory and Azizi (2013) examined the possibility of wormhole geometry in f(R,T) gravity. The f(R,T) gravity models can explain the late time cosmic accelerated expansion of the universe. Several authors viz., Adhav (2012a); Sharif and Zubair (2012a); Reddy et al. (2012b); Chaubey and Shukla (2013); Ram et al. (2013); Reddy et al. (2012a) have investigated spatially homogeneous cosmological models in f(R,T) theory of gravity in the presence of perfect fluid. Ram and Priyanka (2013) presented Kaluza-Klein cosmological models in the presence of perfect fluid in f(R,T) gravity. Reddy et al. (2013) have considered a locally rotationally symmetric (LRS) Bianchi type-II space -time with cosmic strings and bulk viscosity in a modified theory of gravity . Very recently, Ahmed and Pradhan (2013) have studied a cosmological model of Bianchi type V in f(R,T) gravity by assuming

$f(R,T) = f_1(R) + f_2(T)$. Chakraborty (2013) formulated an alternative f(R,T) gravity theory and dark energy model.

In this chapter, we investigate a new class of Bianchi models in the presence of perfect fluid and cosmic strings and bulk viscous fluid within the frame work of f(R,T) gravity theory. By using f(R, T) =R+2λT and f(R,T)=$f_1(R) + f_2(T)$, which was proposed by Harko et al. (2011b), the field equations of the metric in f(R,T) gravity are given in detail. New class of cosmological models on using the special form of the average scale factor derived by Abdussattar and Prajapati (2011), by constraining the deceleration parameter.

5.1.1 Metric and Field Equations

The diagonal form of the metric of general class of Bianchi cosmological models is given by

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2x} dy^2 - a_3^2 e^{-2mx} dz^2 \tag{5.1}$$

where $a_1(t)$, $a_2(t)$ and $a_3(t)$ are the scale factors. The metric (5.1) corresponds to a Bianchi type, III model, for m=0, type V for m=1 and type VI_0 for m=-1.

If we assume

$$f(R,T) = R + 2f(T), \tag{5.2}$$

where f(T) is an arbitrary function of the trace of the stress- energy tensor, the field equations in f(R,T) theory of gravity for (5.2), when the matter source is perfect fluid, are given by (Harko et al., 2011b):

From (1.20), we get the gravitation field equations in this case as

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} - 2f^{'}(T)T_{ij} - 2f^{'}(T)\ominus_{ij} - f(T)g_{ij}, \tag{5.3}$$

where the prime denotes a derivative with respect to the argument. If the matter source is a perfect fluid then the field equations.

$$\ominus_{ij} = -2T_{ij} - pg_{ij} = (\rho, -p, -p, -p), \tag{5.4}$$

where T_{ij} in (1.8).

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f^{'}T_{ij} + [2pf^{'}(T) + f(T)]g_{ij}, \tag{5.5}$$

where the prime indicates the derivative with respect to the argument.
Now, We choose the function f(T) of the trace of energy tensor of the matter so that

$$f(T) = \lambda T, \tag{5.6}$$

where λ is a constant.

Now choosing comoving coordinates, the field Eq.(5.5) , with the help of Eqs. (5.5) and (1.8) for the metric (5.1), can be written as Chaubey and Shukla (2013):

$$\frac{\dot{a_1}\dot{a_2}}{a_1a_2} + \frac{\dot{a_2}\dot{a_3}}{a_2a_3} + \frac{\dot{a_3}\dot{a_1}}{a_3a_1} - \frac{m^2+m+1}{a_1^2} = \lambda p - (8\pi + 3\lambda)\rho, \tag{5.7}$$

$$\frac{\ddot{a_2}}{a_2} + \frac{\ddot{a_3}}{a_3} + \frac{\dot{a_2}\dot{a_3}}{a_2a_3} - \frac{m}{a_1^2} = (8\pi + 3\lambda)p - \lambda\rho, \tag{5.8}$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_3}}{a_3} + \frac{\dot{a_1}\dot{a_3}}{a_2a_3} - \frac{m^2}{a_1} = (8\pi + 3\lambda)p - \lambda\rho, \tag{5.9}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{1}{a^2}=(8\pi+3\lambda)p-\lambda\rho, \tag{5.10}$$

$$(m+1)\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}-m\frac{\dot{a}_3}{a_3}=0. \tag{5.11}$$

Here an overhead overdot denotes ordinary differentiation with respect to time t.

Akarsu and Dereli (2012) proposed the linearly varying deceleration parameter q defined as

$$q=-(kt+n-1) \tag{5.12}$$

and obtained three different forms of the average scale factor a where k and n are positive constants. Chaubey and Shukla (2013) obtained exact solutions of the field Eqs. (5.7)-(5.11) by using the different forms of the law of variation for specific values of k and n. Here we obtain exact solutions of the Eqs. (5.7)-(5.11) by using one of the three different forms of the average scale factor derived by Abdussattar and Prajapati (2011) which leads to a class of non-singular bouncing FRW models obtained by constraining the deceleration parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant.

5.1.2 Cosmological Solution

The two observable parameters q and H are related by the relation

$$q=-1+\frac{d}{dt}(\frac{1}{H}). \tag{5.13}$$

This equation, on integration, gives the scale factor a(t) as

$$a(t) = e^{\delta} exp \int \frac{dt}{\int (1+q)dt + \gamma}. \tag{5.14}$$

where γ and δ are arbitrary constants. For the complete determination of a(t), Abdussattar and Prajapati (2011) proposed the following choice of q as

$$q = -\frac{\alpha}{t^2} + (\beta - 1). \tag{5.15}$$

where $\alpha > 0$ is a parameter having the dimension of square of time and $\beta > 1$ is a dimensionless constant. The different values of α and β give rise to different models. With q given by Eq.(5.15) and $\delta = 0$, r=0, Eq.(5.14)can be integrated to give the scale factor as

$$a(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{1}{2\beta}}. \tag{5.16}$$

Abdussattar and Prajapati (2011) discussed in detail the non-singular bouncing model with a(t) given by (5.16) and have shown the variation of different cosmological parameters graphically for specific values of the parameters of the model.

It is difficult to solve Eqs. (5.7)-(5.11) for five unknowns a_1,a_2,a_3, ρ and p in the exact form. In order to solve the system completely we assume that $a_3 = V^b$, where b any constant number. Then from Eqs. (5.11), (1.23) and (5.16),we obtain the exact expression for the scale factors:

$$a_1(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3+3mb-3b}{2\beta(m+2)}}, \tag{5.17}$$

$$a_2(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3+3m-3b-6mb}{2\beta(m+2)}}, \tag{5.18}$$

$$a_3(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3b}{2\beta}}. \tag{5.19}$$

5.1.3 Physical Properties

For the model presented by the scale factors in Eqs.(5.17)- (5.19), have the directional Hubble parameters and average Hubble parameter are given by

$$H_1 = \frac{3 + 3mb - 3b}{\beta(m+2)} \frac{t}{(t^2 + \frac{\alpha}{\beta})}, \tag{5.20}$$

$$H_2 = \frac{3 + 3m - 3b - 6mb}{\beta(m+2)} \frac{t}{(t^2 + \frac{\alpha}{\beta})}, \tag{5.21}$$

$$H_3 = \frac{3b}{\beta} \frac{t}{(t^2 + \frac{\alpha}{\beta})}, \tag{5.22}$$

$$H = \frac{t}{\beta(t^2 + \frac{\alpha}{\beta})}. \tag{5.23}$$

The expansion scalar, shear scalar and mean anisotropic parameter are found as

$$\theta = 3H = \frac{3t}{\beta(t^2 + \frac{\alpha}{\beta})}, \tag{5.24}$$

$$\sigma^2 = A_1 \frac{t^2}{(t^2 + \frac{\alpha}{\beta})^2}, \tag{5.25}$$

$$A = \frac{2}{3(m+2)^2}\left[(3 + 3mb - 3b)(6 - 3mb - 6b + 3m) + (3 + 3m - 3b - 6mb)(3 + 6m - 3mb)\right] + \frac{2}{3(m+2)^2}\left[(3b(m+2))(6mb + 3b + 3)\right], \tag{5.26}$$

where

$$A_1 = \frac{(3+3mb-3b)^2+(3+3m-3b-6mb)^2+9(m+2)^2(b^2-2)}{2\beta^2(m+2)^2}.$$

Using Eqs. (5.17)- (5.19), we obtain the expressions of pressure and energy density as

$$\begin{aligned} p = & \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{t^2}{(t^2\beta+\alpha)^2(m+2)^2}[3(8\pi+3\lambda)A_3-3\lambda A_2]\right] \\ & -\frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{3(8\pi+3\lambda)E_1}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right] \\ & -\frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{E_2}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right], \end{aligned} \tag{5.27}$$

$$\begin{aligned} \rho = & \frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{t^2}{[t^2\beta+\alpha]^2(m+2)^2}[3\lambda A_2-3(8\pi+3\lambda)A_3]\right] \\ & -\frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{3E_1\lambda}{[t^2+\frac{\alpha}{\beta}]^{\frac{3+3mb-2b}{\beta(m+2)}}}\right] \\ & -\frac{1}{8(8\pi^2+6\pi\lambda+\lambda^2)}\left[\frac{E_3}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right]. \end{aligned} \tag{5.28}$$

where

$$\begin{aligned} A_2 = & (1+mb-b-2mb)(1+mb-b)+b(m+2)(1+m-b-2mb) \\ & +b(1+mb-b)(m+2), \end{aligned}$$

$$A_3 = (3b^2 - b\beta)(m+2) + 3(1+m-b-2mb)(1+m-b-2mb)$$

$$-(3(1+m-b-2mb)(m+2) - \beta(m+2))(3+3m-3b-6mb),$$

$$E_1 = b\alpha(m+2) + \alpha(1+m-b-2mb),$$

$$E_2 = m\lambda - (8\pi + 3\lambda)(m^2+m+1),$$

$$E_3 = m(8\pi + 2\lambda) - (m^2+1)\lambda.$$

For the purpose of reference, we set the origin of the time coordinate at the bounce of this bouncing model and concentrate only on the expanding part of the solutions which represents the observable universe as in Abdussattar and Prajapati (2011).

At the origin of the time coordinate t=0, we find that $a(0) \neq 0$, $\dot{a}(0) = 0$ but $\ddot{a}(0)=$ constant, which indicate that the model is free from initial singularity and start expanding with finite acceleration. From (5.15) we observe that the deceleration parameter $q \to -\infty$ at $t = 0$ and reduces to zero at the time $t = \sqrt{\frac{\alpha}{\beta-1}}$. The period of accelerated expansion also depends on the values of α and β. Afterwards, the model decelerates with deceleration parameter q approaching to $\beta - 1$ for sufficiently large values of t. Obviously we have a constraint on β as $1 \leq \beta \leq 2$. For $\beta = 1$, the universe has an accelerated expansion throughout the evolution. We see that θ, σ^2 H ultimately tend to zero as $t \to \infty$. The energy density and pressure tend to zero for large time provided $b < \frac{1}{1-m}$.

5.1.4 Conclusion

In this section, we have obtained a general class of non-singular cosmological model of the early universe with a perfect fluid as the source of matter in $f(R,T)$ theory of gravity. We have additional classes of Bianchi models from the general class of Bianchi model for different values of m as follows Bianchi type-III corresponds to $m = 0$, Bianchi type-V corresponds to $m = 1$ Bianchi type-VI_0 corresponds to $m = -1$ and all other values of m give Bianchi type-VI. An important aspect of the presented model is that it starts from rest with a finite volume and finite acceleration which gradually decreases and reduce to zero after some time. Thereafter the model decelerates with gradually increasing deceleration approaching to a constant value for large t. It is also observed that all the physical parameters are decreasing functions of time and they approach zero for large t. The model obtained may throw some light on our understanding of $f(R,T)$ cosmology.

5.2 String Cosmological Models with Bulk Viscosity

5.2.1 Models and Field Equations

In this section, we focus to the function

$$f(R,T) = f_1(R) + f_2(T), \tag{5.29}$$

with, $f_1(R) = \mu_1 R$ and $f_2(T) = \mu_2 T$, μ_1 and μ_2 are arbitrary parameters.
We set $\mu_1 = \mu_2 = \mu$, as discussed by Ahmed and Pradhan (2013), so that

$$f(R,T) = \mu(R+T). \tag{5.30}$$

With this choice of f(R,T), the field equations in f(R,T) gravity theory are (Harko et al., 2011b) may be written in the form

$$\mu R_{ij} - \frac{1}{2}\mu(R+T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)\mu = 8\pi T_{ij} - \mu T_{ij} + \mu(2T_{ij} + pg_{ij}). \tag{5.31}$$

Setting $(g_{ij}\Box - \nabla_i\nabla_j)\mu$=0, we obtain

$$\mu G_{ij} = 8\pi T_{ij} + \mu T_{ij} + (\mu p + \frac{1}{2}\mu T)g_{ij}, \tag{5.32}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein's tensor Ahmed and Pradhan (2013). This could be rearranged as

$$G_{ij} - \left(p + \frac{1}{2}T\right)g_{ij} = \left(\frac{8\pi + \mu}{\mu}\right)T_{ij}. \tag{5.33}$$

The diagonal form of the metric of general class of Bianchi cosmological model is given in (5.1)

We consider the energy-momentum tensor of a bulk viscous fluid containing one dimensional cosmic strings as (Reddy et al., 2013):

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \lambda x_i x_j, \tag{5.34}$$

where the bulk viscous pressure $\bar{p}$ is given by

$$\bar{p} = p - 3\zeta H. \tag{5.35}$$

Here ρ is the rest energy density of the system ζ is the coefficient of bulk viscosity $\bar{p}$ is the bulk viscous pressure, H is Hubble parameter and λ is the string tension density. Also u^i is the four velocity vector and x^i is unit space-like vector representing the direction of the strings which satisfy

$$u^i u_i = -x^i x_i = 1, \quad u^i x_i = 0, \tag{5.36}$$

In comoving coordinates, we have

$$u^i = (0,0,0,1), \quad x^i = (\frac{1}{a},0,0,0). \tag{5.37}$$

The proper energy density can be expressed as

$$\rho = \rho_p + \lambda, \tag{5.38}$$

where ρ_p is the rest density of the particles attached to the strings (Letelier, 1983; Khadekar and Tade, 2007; Pradhan, 2007). The tension density λ may be positive or negative. A more general relationship between the proper rest energy density ρ and string tension density λ may be taken of the form

$$\rho = r\lambda, \tag{5.39}$$

where r is an arbitrary constant which can take both positive and negative values. The negative values of r lead to the absence of strings in the universe and the positive values show the presence of one dimensional strings in the cosmic fluid.Therefore, the energy density of the particles attached to the strings is given by

$$\rho_p = \rho - \lambda = (r-1)\lambda. \tag{5.40}$$

For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\bar{p} = p - 3\zeta H = \zeta\rho, \tag{5.41}$$

where

$$\xi = \xi_0 - \tau \quad (0 \leq \tau \leq 1), \quad p = \xi_0 \rho, \tag{5.42}$$

and τ is an arbitrary constant.

For the metric (5.1), the field equations (5.33) together with (5.30) and (5.32), in (5.1) comoving coordinates lead to the following set of highly non-linear equations:

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{m^2+m+1}{a_1^2} = \frac{1}{2\mu}\left[(5\bar{p}\mu + 16\pi\lambda) + (16\pi + 3\mu)\rho\right], \tag{5.43}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{m}{a_1^2} = \frac{1}{2\mu}\left[\bar{p}(5\mu - 16\pi) + (\rho\mu + 8\pi\lambda)\right], \tag{5.44}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_2a_3} - \frac{m^2}{a_1} = \frac{1}{2\mu}\left[\bar{p}(5\mu - 16\pi) + (\rho\mu + 8\pi\lambda)\right], \tag{5.45}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{1}{a^2} = \frac{1}{2\mu}\left[\bar{p}(5\mu - 16\pi) + (\rho\mu + 8\pi\lambda)\right], \tag{5.46}$$

$$(m+1)\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} - m\frac{\dot{a}_3}{a_3} = 0. \tag{5.47}$$

5.2.2 Solution of Field Equations

Equation (5.14) can be integrated to give the time variation of the scale factor as

$$a(t) = e^{\delta} \exp\left[\frac{1}{\beta}\int\left(\frac{tdt}{t^2 + t\frac{\gamma}{\beta} + \frac{\alpha}{\beta}}\right)\right]. \tag{5.48}$$

The integral appearing in (5.48) can not be evaluated for arbitrary values of the constants. Setting $\gamma = 0$ in (5.48) and integrating we obtain the average scale factor a(t) as

$$a(t) = e^{\delta}(t^2 + \frac{\alpha}{\beta})^{\frac{1}{2\beta}}. \tag{5.49}$$

If we take $\alpha = 0$ and e^{δ}=$D^{\frac{1}{\beta}}$ in (5.49), we obtain

$$a(t) = (Dt)^{\frac{1}{\beta}}, \tag{5.50}$$

which corresponds to a with a constant deceleration parameter q=$\beta - 1$ throughout the evolution Berman (1983).

It is difficult to solve equations (5.43)-(5.47) for six unknown $a_1, a_2, a_3, \rho, \lambda$ and $\bar{p}$ in the exact form. In order to solve the system completely we assume that $a_3 = V^b$, where b is any constant number. Now we derive two physically interesting models using the average scale factor given in (5.49) and (5.50).

5.2.3 Model I

From equations (1.23), (5.47) and (5.49), we obtain the expressions for the scale factors:

$$a_1(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3+3mb-3b}{2\beta(m+2)}}, \tag{5.51}$$

$$a_2(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3+3m-3b-6mb}{2\beta(m+2)}}, \tag{5.52}$$

$$a_3(t) = (t^2 + \frac{\alpha}{\beta})^{\frac{3b}{2\beta}}, \tag{5.53}$$

provided $m \neq -2$.

For the model presented by the scale factors in (5.51)-(5.53), the kinematical parameters have values as given below:

$$H_1 = \frac{3 + 3mb - 3b}{\beta(m + 2)} \frac{t}{(t^2 + \frac{\alpha}{\beta})}, \tag{5.54}$$

$$H_2 = \frac{3+3m-3b-6mb}{\beta(m+2)} \frac{t}{(t^2+\frac{\alpha}{\beta})}, \tag{5.55}$$

$$H_3 = \frac{3b}{\beta} \frac{t}{(t^2+\frac{\alpha}{\beta})}, \tag{5.56}$$

$$H = \frac{t}{\beta(t^2+\frac{\alpha}{\beta})}. \tag{5.57}$$

The expansion scalar, shear scalar and mean anisotropic parameter are obtained as

$$\theta = 3H = \frac{3t}{\beta(t^2+\frac{\alpha}{\beta})}, \tag{5.58}$$

$$\sigma^2 = A_1 \frac{t^2}{(t^2+\frac{\alpha}{\beta})^2}, \tag{5.59}$$

$$Am = \frac{2}{3(m+2)^2}\left[(3+3mb-3b)(6-3mb-6b+3m)+(3+3m-3b-6mb)(3+6m-3mb)\right] + \frac{2}{3(m+2)^2}\left[(3b(m+2))(6mb+3b+3)\right], \tag{5.60}$$

where

$$A_1 = \frac{(3+3mb-3b)^2+(3+3m-3b-6mb)^2+9(m+2)^2(b^2-2)}{2\beta^2(m+2)^2}.$$

By using (5.43) and (5.44), we obtain bulk viscous pressure $\bar{p}$, string tension density λ and energy density ρ as r$\neq \pm 1$

$$\bar{p} = \left[\frac{2\mu t^2}{\beta^2(m+2)^2(t^2+\frac{\alpha}{\beta})^2[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right][(8\pi+r\mu\lambda)M_1$$
$$+(16\pi(1+r)+3\mu r)M_2 - \frac{(16\pi(1+r)+3\mu r)\alpha(\beta+3b)(m+2)^2}{t^2}]$$
$$-2\mu\left[\frac{(m^2+m+1)(8\pi+r\lambda\mu)-(16\pi(1+r)+3\mu r)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right]. \tag{5.61}$$

$$\lambda = \left(\frac{2\mu}{8\pi+r\mu}\right)\frac{t^2}{(t^2+\frac{\alpha}{\beta})^2\beta^2(m+2)^2}[M_3 - \frac{(5\mu-16\pi)}{(256\pi^2+(-25-10\mu-17\pi\mu)\mu r)}[(8\pi+r\mu)M_1+$$
$$(16\pi(1+r)+3\mu r)M_2 - \frac{(16\pi(1+r)+3\mu r)\alpha(\beta+3b)(m+2)^2}{t^2} + (m+2)(\alpha+3b)$$
$$(3+3m-3b-6mb)+3b\alpha(m+2)]] - \left(\frac{2\mu}{8\pi+r\mu}\right)\left[\frac{m}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right]$$
$$-\left(\frac{2\mu}{8\pi+r\mu}\right)\left[\frac{(m^2+m+1)(8\pi+r\lambda\mu)-(16\pi(1+r)+3\mu r)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right]. \tag{5.62}$$

Using equation (5.38), we obtain

$$\rho = \left(\frac{2\mu r}{8\pi + r\mu}\right)\frac{t^2}{(t^2+\frac{\alpha}{\beta})^2\beta^2(m+2)^2}[M_3 - \frac{(5\mu-16\pi)}{(256\pi^2+(-25-10\mu-17\pi\mu)\mu r)}[(8\pi+r\mu)M_1 +$$
$$(16\pi(1+r)+3\mu r)M_2 - \frac{(16\pi(1+r)+3\mu r)\alpha(\beta+3b)(m+2)^2}{t^2} + (m+2)(\alpha+3b)$$
$$(3+3m-3b-6mb)+3b\alpha(m+2)]] - \left(\frac{2\mu r}{8\pi+r\mu}\right)\left[\frac{m}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right]$$
$$+\left(\frac{2\mu r}{8\pi+r\mu}\right)\left[\frac{(m^2+m+1)(8\pi+r\lambda\mu)-(16\pi(1+r)+3\mu r)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right]. \tag{5.63}$$

In order to solve Einstein's field equations in the presence of cosmic strings we resort to a tractable assumption concerning a relation between the rest energy density and tension density of the system of strings. Using barotropic equation of state parameter as discussed in detailed (5.39), (5.40) and (5.41),the energy density of the particles attached to the string and bulk viscosity is :

$$\rho_p = (r-1)\left(\frac{2\mu}{8\pi+r\mu}\right)\frac{t^2}{(t^2+\frac{\alpha}{\beta})^2\beta^2(m+2)^2}[M_3 - \frac{(5\mu-16\pi)}{(256\pi^2+(-25-10\mu-17\pi\mu)\mu r)}$$
$$(8\pi+r\mu)M_1+(16\pi(1+r)+3\mu r)M_2 - \frac{(16\pi(1+r)+3\mu r)\alpha(\beta+3b)(m+2)^2}{t^2}$$
$$+(m+2)(\alpha+3b)(3+3m-3b-6mb)+3b\alpha(m+2)] - \left(\frac{2\mu(r-1)}{8\pi+r\mu}\right)*$$
$$\left[\frac{m}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right] + \left(\frac{2\mu(r-1)}{8\pi+r\mu}\right)*$$
$$\left[\frac{(m^2+m+1)(8\pi+r\lambda\mu)-(16\pi(1+r)+3\mu r)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right]. \tag{5.64}$$

$$\zeta=\left(\frac{2\mu}{8\pi+r\mu}\right)\frac{t}{3(t^2+\frac{\alpha}{\beta})\beta(m+2)^2}[M_3-\frac{(5\mu-16\pi)}{(256\pi^2+(-25-10\mu-17\pi\mu)\mu r)}[(8\pi+r\mu)M_1+ (16\pi(1+r)+3\mu r)M_2-\frac{(16\pi(1+r)+3\mu r)\alpha(\beta+3b)(m+2)^2}{t}+(m+2)(\alpha+3b) (3+3m-3b-6mb)+3b\alpha(m+2)]]-\left(\frac{2\mu\beta(t^2+\frac{\alpha}{\beta})}{(8\pi+r\mu)3t}\right)\left[\frac{m}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}\right] +\left(\frac{2\mu(t^2+\frac{\alpha}{\beta})}{(8\pi+r\mu)3t}\right)\left[\frac{(m^2+m+1)(8\pi+r\lambda\mu)-(16\pi(1+r)+3\mu r)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}[256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r]}\right]. \tag{5.65}$$

The scalar curvature R is given by

$$R=2\left[\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_1\dot{a}_3}{a_1a_3}-\frac{(m^2+m+1)}{a_1^2}\right], \tag{5.66}$$

$$=\frac{2t^2M_4}{\beta^2(t^2+\frac{\alpha}{\beta})^2(m+2)^2}+2\left[\frac{(3+3mb-3b)\alpha}{\beta(m+2)}+\frac{\alpha(3+3m-3b-6mb)}{\beta(m+2)^2}+\frac{3b\beta}{\beta^2}\right]\frac{1}{(t^2+\frac{\alpha}{\beta})^2}- \frac{2(m^2+m+1)}{(t^2+\frac{\alpha}{\beta})^{\frac{3+3mb-3b}{\beta(m+2)}}}, \tag{5.67}$$

where

$$M_1=(3+3mb-3b)(3+3m-3b-6mb)+3b(m+2)(b+3m-6b-3mb),$$

$$M_2=(3+3m-3b-6mb)\beta(m+2)-(3+3m-3b-6mb)^2+3b\beta(m+2)^2-9b^2(m+2)^2 -3b(3+3m-3b-6mb)(m+2),$$

$$M_3=\beta(m+2)(3+3m-3b-6mb)+(3+3m-3b-6mb)^2-3b\beta(m+2)+9b^2(m+2),$$

$$M_4 = (3+3mb)-3b)^2-(3+3mb-3b)\beta(m+2)-\beta(m+2)(3+3m-3b-6mb)$$
$$+(3+3m-3b-6mb)^2-3b\beta+9b^2+(3+3mb-3b)(3+3m-3b-6mb)+3b(m+2)$$
$$(3+3m-3b-6mb)+3b(m+2)(3+3mb-3b).$$

The expressions for the Hubble's parameter, scalar expansion θ, magnitude of shear scalar σ are decreasing function of time which essentially tend to zero as t $\rightarrow \infty$. The spatial volume is increasing function of time. However anisotropic parameter is constant. From (5.63), (5.64), (5.65)and (5.66) it is noted that energy density ρ, particle energy density ρ_p , bulk viscosity coefficient ζ and scalar curvature are decreasing function of time and tend to zero large time t. The present model has no singularity . These behaviors of ρ and ρ_p are clearly depicted in fig. (5.5) as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well-known situations. We observe from (5.61) and (5.62) that the bulk viscous pressure $\bar{p}$ and the string tension density λ are increasing function of time and are always negative which approach zero at a large time . Letelier (1983) pointed out that λ may be positive or negative. When $\lambda < 0$, the string phase of the universe disappears, i.e. we have an anisotropic fluid of particles. The behavior of λ is shown in fig.(5.4).

5.2.4 Model II

Making use of $a_3 = V^b$ in (5.47) and (5.50), we obtain

$$a_1 = (Dt)^{\frac{6b+3(m+1)(1-b)}{\beta(m+1)(m+2)}}, \tag{5.68}$$

$$a_2 = (Dt)^{\frac{3(m+1)-3bm-3b(m+1)}{\beta(m+2)}}, \tag{5.69}$$

$$a_3 = (Dt)^{\frac{3b}{\beta}}, \tag{5.70}$$

with condition $m \neq -1, -2$.

For the model given by (5.68),(5.69) and (5.70), the physical and kinematical parameters are given by

$$H = \frac{1}{3(m+1)(m+2)\beta t}[6b + 3b(m+1)(1-b) + (3(m+1) \\ -3bm - 3b(m+1))(m+1) + 3b(m+1)(m+2)], \tag{5.71}$$

$$\theta = \frac{1}{(m+1)(m+2)\beta t}[6b + 3b(m+1)(1-b) + (3(m+1) \\ -3bm - 3b(m+1))(m+1) + 3b(m+1)(m+2)], \tag{5.72}$$

$$\sigma^2 = \frac{A_2}{2\beta^2(m+1)^2(m+2)^2t^2}, \tag{5.73}$$

$$A_m = \frac{3}{[6b+3(m+1)(1-b)+(3(m+1)-3bm-3b(m+1))(m+1)+3b(m+1)(m+2)]^2} * \\ [(6b+3(m+1)(1-b))^2+(3(m+1)-3bm-3b(m+1))^2(m+1)^2+9b^2(m+1)^2(m+2)^2 \\ -\frac{1}{3}[6b+3(m+1)(1-b)+(3(m+1)-3bm-3b(m+1))(m+1)+3b(m+1)(m+2)]^2], \tag{5.74}$$

where

$$A_2 = (6b+3(m+1)(1-b))^2 + (3(m+1)-3bm-3b(m+1)^2+9b^2(m+1)^2(m+2)^2) \\ -2[(6b+3(m+1)(1-b)+(m+1)(3(m+1)-3bm-3b(m+1))+3b(m+1)(m+2))^2].$$

$$\bar{p} = \frac{2\mu}{(256\pi^2(1+r)+(-25-10\mu-17\pi\mu)\mu r)}[\left[\frac{1}{t^2\beta^2(m+1)^2(m+2)^2}\right][N_1(8\pi+r\mu)$$
$$-N_2(16\pi(1+r)+3\mu r)] - \frac{1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}}[(16\pi(1+r)$$
$$+3\mu r)m+(m^2+m+1)(8\pi+r\mu)]], \tag{5.75}$$

$$\lambda = \frac{2\mu}{(16\pi(1+r)+3\mu r)}[\frac{1}{\beta^2t^2(m+2)^2(m+1)^2}[N_1 - \frac{1}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}$$
$$((8\pi+r\mu)N_1-(16\pi(1+r)+3\mu r)N_2)] - \frac{1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}}$$
$$(m^2+m+1) - \frac{(16\pi(1+r)+3\mu r)m+(m^2+m+1)(8\pi+r\mu)}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}], \tag{5.76}$$

$$\rho = \frac{2\mu r}{(16\pi(1+r)+3\mu r)}[\frac{1}{\beta^2t^2(m+2)^2(m+1)^2}[N_1 - \frac{1}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}$$
$$((8\pi+r\mu)N_1-(16\pi(1+r)+3\mu r)N_2)] - \frac{1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}}$$
$$(m^2+m+1) - \frac{(16\pi(1+r)+3\mu r)m+(m^2+m+1)(8\pi+r\mu)}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}], \tag{5.77}$$

$$\rho_p = \frac{2(r-1)\mu}{(16\pi(1+r)+3\mu r)}[\frac{1}{\beta^2 t^2 (m+2)^2 (m+1)^2}[N_1 - \frac{1}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}$$
$$((8\pi + r\mu)N_1 - (16\pi(1+r)+3\mu r)N_2)] - \frac{1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}}$$
$$(m^2+m+1) - \frac{(16\pi(1+r)+3\mu r)m + (m^2+m+1)(8\pi + r\mu)}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}], \tag{5.78}$$

$$\zeta = \left(\frac{2\mu r}{\beta t(m+1)(m+2)[6b+3(m+1)(1-b)+(3(m+1)-3bm-3b(m+1))(m+1)]}\right) *$$
$$[\left(\frac{r}{(16\pi(1+r)+3\mu r)} - 1\right)[N_1 - \frac{1}{256\pi^2(1+r)+(-25-10\mu-17\mu\pi)\mu r}$$
$$(8\pi + r\mu)N_1 - (16\pi(1+r)+3\mu r)N_2]] + \frac{1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}} *$$
$$\left[(16\pi(1+r)+3r\mu)m + (m^2+m+1)(8\pi + r\mu)\right] *$$
$$\left[\frac{1-2r\mu(m+1)(m+2)}{(6b+3(m+1)(1-b)+(3(m+1)-3bm-3b(m+1))(m+1))}\right]$$
$$\left[\frac{2r\mu(m^2+m+1)\beta t(m+1)(m+2)}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}(16\pi(1+r)+3\mu r)}\right] *$$
$$\left[\frac{1}{(6b+3(m+1)(1-b)+(3(m+1)-3bm-3b(m+1))(m+1)}\right], \tag{5.79}$$

$$
\begin{aligned}
R = {} & \frac{2}{\beta^2(m+1)^2(m+2)^2t^2}[(6b+3(m+1)(1-b))(6b+3(m+1)(1-3b) \\
& -\beta(m+1)(m+2)) + (3(m+1)-3bm-3b(m+1))(m+1)[(3(m+1) \\
& -3mb-3b)(m+1)(m+2)-\beta(m+2)] + 3b(m+1)^2(m+2)^2 \\
& (3b-\beta) + (3(m+1)-3bm.3b(m+1))(6b+3(m+1)(1-b))(6b+3(m+1) \\
& (1-b))(m+1)(m+2) + (3(m+1)-3bm-3b(m+1))3b(m+2)(m+1)^2] \\
& \left[-2\frac{m^2+m+1}{(Dt)^{\frac{2(6b+3(m+1)(1-b))}{\beta(m+1)}}}\right],
\end{aligned}
\tag{5.80}
$$

where

$$
\begin{aligned}
N_1 = {} & (m+1)[(3(m+1)-3bm-3b(m+1))(6b+3(m+1)(1-3b)+3b(m+1)) \\
& +3b(6b+3(m+1)(1-3b)+3b(m+1))],
\end{aligned}
$$

$$
\begin{aligned}
N_2 = {} & (6b+3(m+1)(1-3b)+3b(m+1))^2 - (6b+3(m+1)(1-3b)+3b(m+1))\beta(m+1) \\
& (m+2) + (3(m+1)-3bm-3b(m+1))^2(m+1)^2(m+2)^2 - [(3(m+1)-3bm-3b(m+1)) \\
& \beta(m+2)(m+1)^2 + 9b^2(m+2)^2(m+1)^2 - 3b\beta(m+1)^2(m+2)^2.
\end{aligned}
$$

The spatial volume of the model has the expression V=$(Dt)^{\frac{3}{\beta}}$ which is zero at t=0. At this epoch all the physical and kinematical parameters are infinite. Thus, the model starts expanding from a big-bang singularity at t=0. For $0 < t < \infty$, the physical and kinematical parameters are well definite and are decreasing function of time which ultimately tend to zero as $t \to \infty$. The behaviors of the physical and kinematical parameters are shown graphically. The anisotropy parameter is constant for all t.

5.2.5 Conclusion

In this section, we have studied spatially homogeneous and anisotropic cosmological models with bulk viscosity and cosmic strings in f(R,T) gravity theory. We have obtained two types of cosmological solutions of the field equations by setting the average scale factor and one of the scale factors equal to some power of the spatial volume of the model. One class of models is accelerated expanding universe having no finite singularity whereas the other class of models is also accelerating having finite singularity at the initial time t=0. In both types of models the physical and kinematical parameters are decreasing functions of time and tend to zero for large time. The anisotropy in the models is maintained throughout the passage of time. The bulk viscosity contributes negative pressure leading to an repulsive gravity which overcomes the attractive gravity and impetus for rapid expansion of the universe . We have particular classes of Bianchi models from the general class of Bianchi models considered in this chapter for different values of m as follows: Bianchi type-III corresponds to m=0, Bianchi type V corresponds to m=1, Bianchi type VI_0 corresponds to m=-1, and all other values of m give Bianchi type-VI_h.

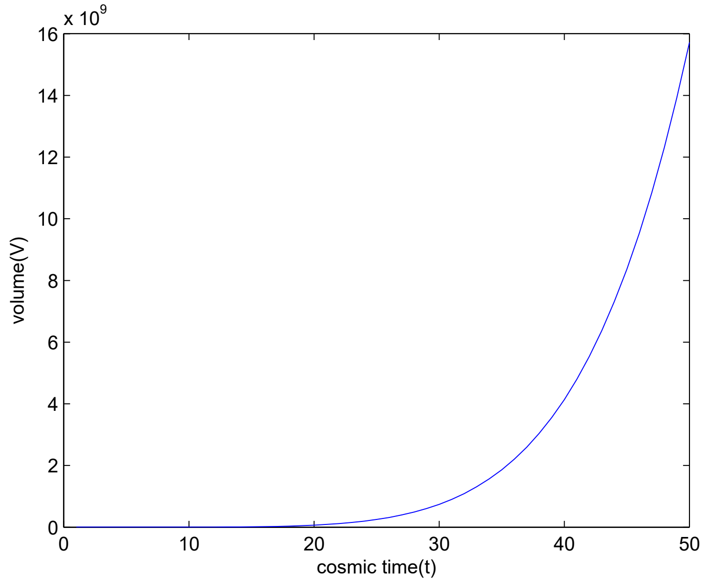

Figure 5.1: The plot of volume V verses cosmic time t,with suitable constant value

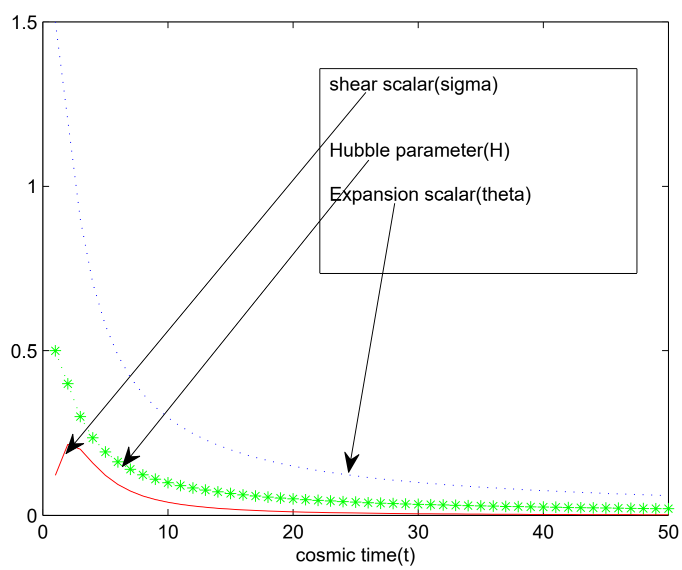

Figure 5.2: The plot of Hubble parameter H, scalar expansion θ and shear scale σ verses cosmic time t

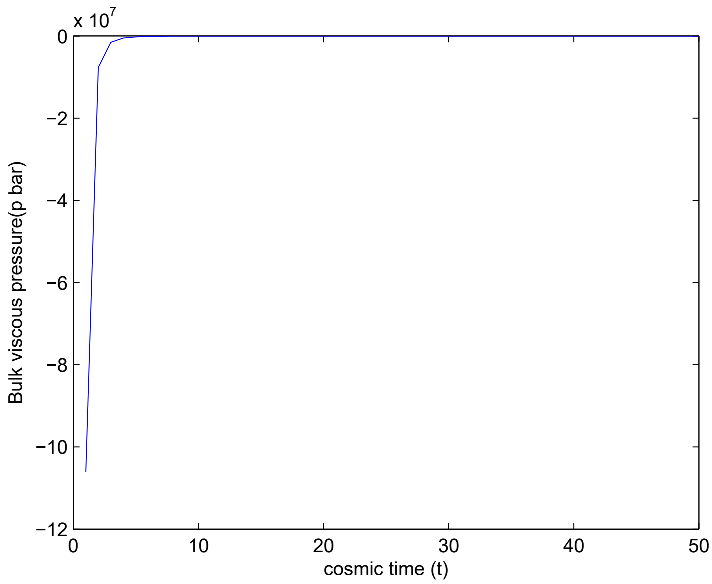

Figure 5.3: The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t

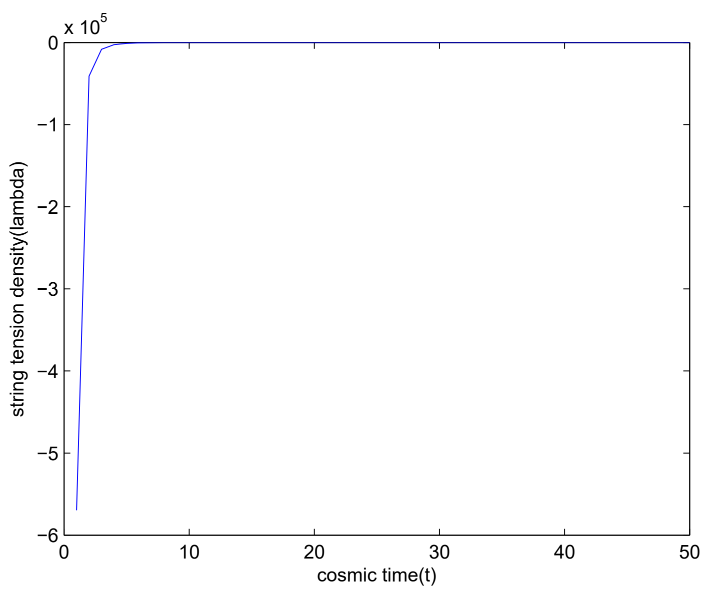

Figure 5.4: The plot of string tension density λ verses cosmic time t

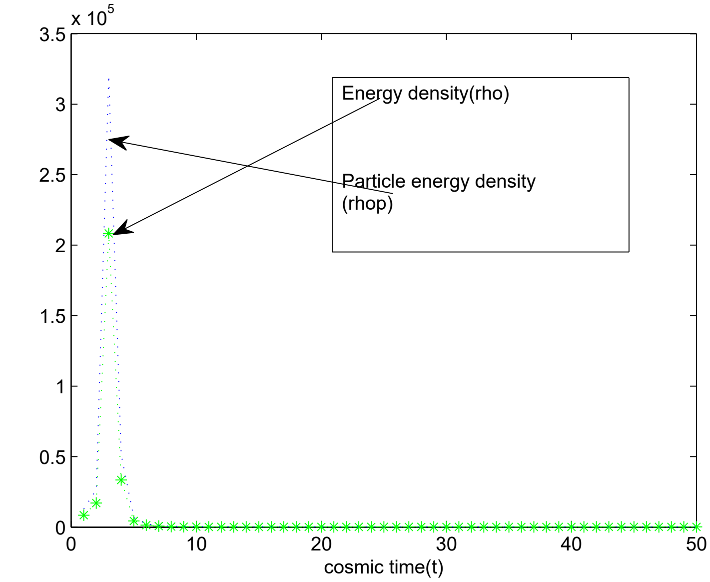

Figure 5.5: The plot of energy density ρ and particle energy density ρ_p verses cosmic time t,with

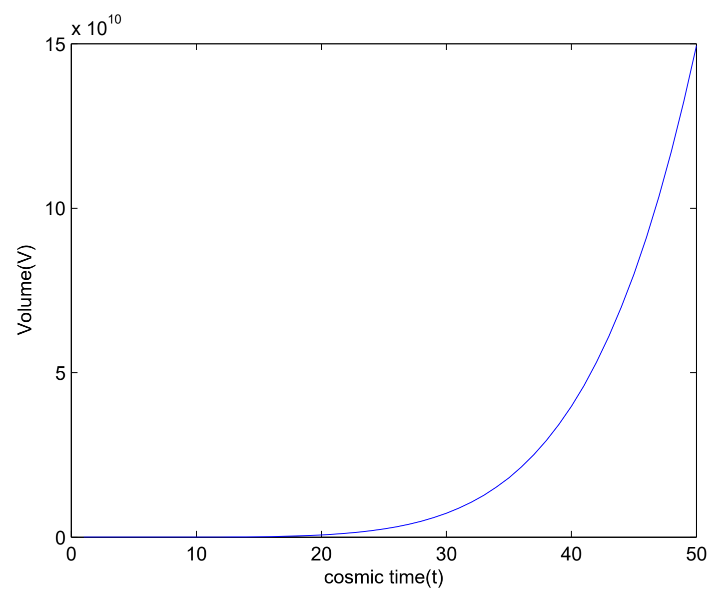

Figure 5.6: The plot of volume V verses cosmic time t,with suitable constant value

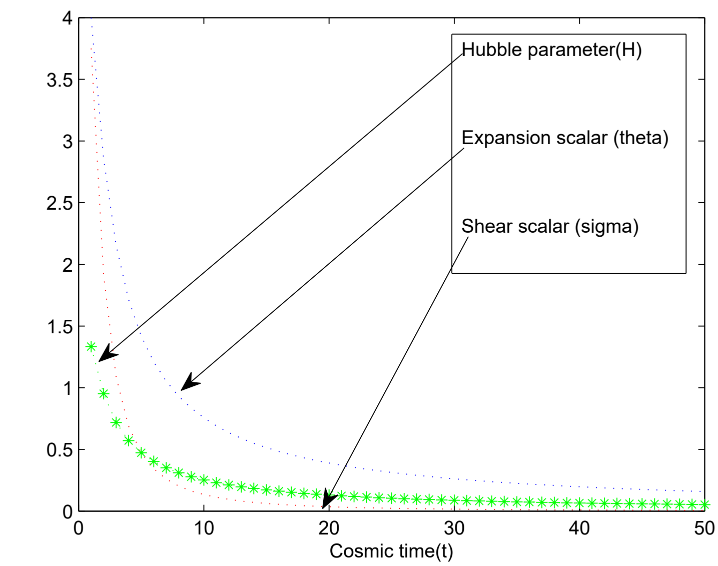

Figure 5.7: The plot of Hubble parameter H, expansion scalar θ and shear scalar σ verses cosmic time t

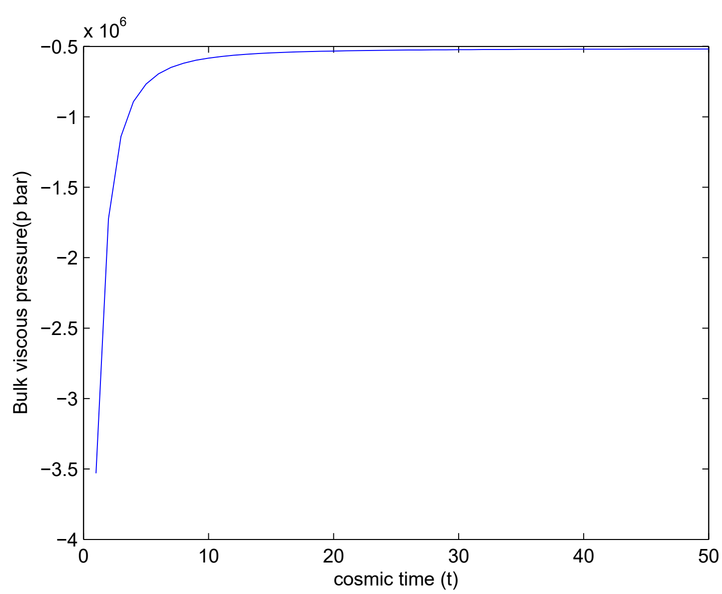

Figure 5.8: The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t,with suitable constant value

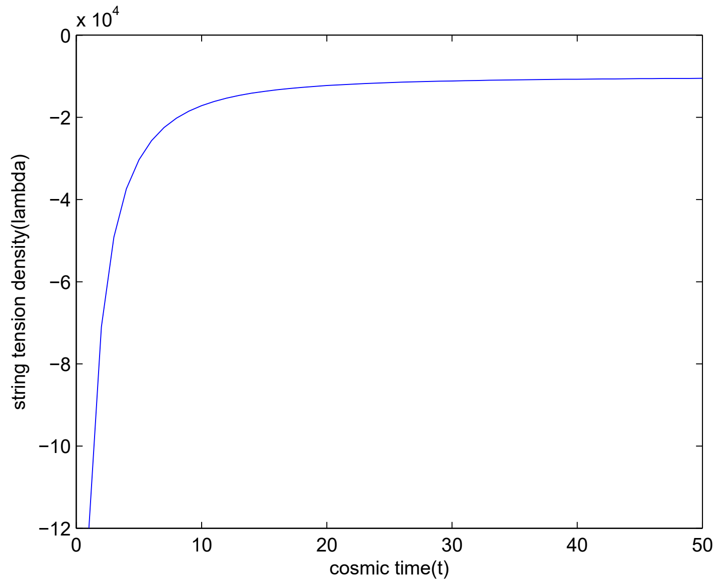

Figure 5.9: The plot of string tension density λ verses cosmic time t

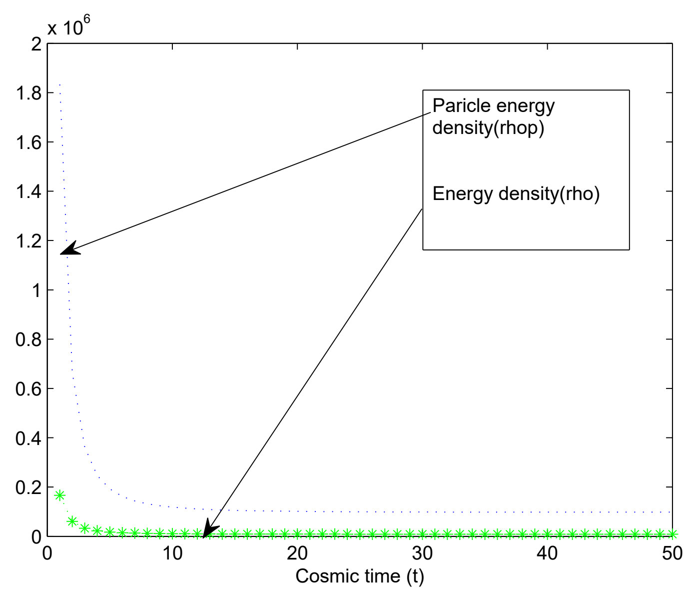

Figure 5.10: The plot of energy density ρ and particle energy density ρ_p verses cosmic time t