4 Bianchi Type-VI_0 Dark Energy Cosmological Models in General Relativity *

4.1 Introduction

Recent observations on expansion history of the universe indicate that the universe is currently experiencing a phase of accelerated expansion. This was first observed from high red shift supernova Ia (Riess et al., 1998, 2004; Perlmutter et al., 1997; Astier et al., 2006; Spergel et al., 2003)etc. and confirmed later by cross checks from the cosmic microwave background radiation (Abazajian et al., 2004, 2005; Hawkins, 2003) etc.. The current accelerating expansion of the universe attributed to the belief that our universe is dominated by an unknown form of energy known as an exotic energy with negative pressure.

The simplest dark energy candidate is the vacuum energy density which is mathematically equivalent to the cosmological constant Λ. As per Copeland et al. (2006) “fine tuning ”and the cosmic “coincidence ”are the two well known difficulties of the cosmological constant problems. There are several alternative theories for the dynamical DE

*Contents of this chapter have been published in Global Journal of Science Frontier Research Mathematics and Decision Sciences 12,0975,(2012) .

scenario which have been proposed by scientists to interpret the present accelerating universe. Wang and Tegmark (2004)have shown that the universe is actually undergoing an acceleration with repulsive gravity of some strange energy-form i.e. DE at work. Dark energy is a mysterious substance with negative pressure and accounts for nearly 70% of total matter-energy of universe, but has no clear explanation. Karami and Abdolmaleki (2009) introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta and Pradhan (2010) proposed a new candidate known as cosmological nuclear-energy as a possible candidate for the dark energy.

Bianchi types I-IX cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi space-times useful in constructing models of spatially homogeneous and anisotropic cosmologies. Considerable works have been done in obtaining various Bianchi type cosmological models and their inhomogeneous generalization. Bianchi type-VI_0 space-time is of special interest in anisotropic cosmology. Barrow (1984) pointed out that Bianchi type-VI_0 models of the universe give a better explanation of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense. Looking to the importance of Bianchi type-VI_0 universes, many authors (Roy and Singh, 1983; Tikekar and L. K. Patel, 1994; Bali et al., 2008a,b; Pradhan and R.Bali, 2008) have studied it in different physical contexts. Ram (1985, 1986) has presented Bianchi type-VI_0 cosmological models filled with dust and perfect fluid in modified Brans-Dicke theory respectively.

Adhav (2011) studied Bianchi type-VI_0 cosmological models with anisotropic dark energy. Abdussattar and Prajapati (2011) obtained a class of bouncing non-singular

FRW models by constraining the deceleration parameter (DP) in the presence of an interacting dark energy represented by a time-varying cosmological constant. They have also discussed the role of deceleration parameter and interacting dark energy in singularity avoidance. Yadav and Saha (2012.) studied DE models with variable equation of state (EoS) parameter. Recently, Saha and Yadav (2012) presented a general relativistic cosmological model with time-dependent DP in LRS Bianchitype-II space-time which can be described by isotopic and variable EoS parameter.

In this chapter, we present general relativistic cosmological models with constant and time-dependent DP in Bianchi type-VI_0 space-time which can be described by isotropic and variable EoS parameters. This chapter is organized as follows: We present the metric and field equations in Sect.4.2. In Sect.4.3, we obtain solutions of the field equations representing Bianchi type-VI0 cosmological models with perfect fluid by imposing the condition that the shear scalar is proportional to expansion scalar. We also discuss the physical behaviors of the cosmological models with dark energy. Concluding remarks are given in Sect.4.4.

4.2 The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI_0 space-time in the form

$$ds^2 = dt^2 - (A^2(t)dx^2 + B^2(t)e^{-2mx}dy^2 + C^2(t)e^{2mx}dz^2) \tag{4.1}$$

where A, B and C are functions of the cosmic time t and m is a constant

In comoving coordinates Einstein's field equation (1.1) together with Eq. (1.8), for the metric (4.1) yield

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{m^2}{A^2}=-\omega\rho, \tag{4.2}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}=-\omega\rho, \tag{4.3}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}=-\omega\rho, \tag{4.4}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}=\rho, \tag{4.5}$$

$$(\frac{\dot{B}}{B}-\frac{\dot{C}}{C})=0 \tag{4.6}$$

where ω is the EoS parameter defined by

$$p=\omega\rho \tag{4.7}$$

and a dot denotes ordinary differentiation with respect to t.

4.3 Solution of Field Equations

Equation (4.6), on integration, gives

$$B=C \tag{4.8}$$

where the constant of integration is absorbed in B or C. Using Eq. (4.8), Eqs. (4.2)-(4.5) reduce to

$$\frac{2\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{m^2}{A^2}=-\omega\rho, \tag{4.9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}=-\omega\rho, \tag{4.10}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{m^2}{A^2} = \rho. \tag{4.11}$$

These are three equations connecting four unknown functions A, B, ρ and ω. In order to solve the above equations we use the physical condition that expansions scalar is proportional to shear scalar, which in our case, leads to

$$A = B^n \tag{4.12}$$

where n is a constant. Here we use the procedure of Saha and Yadav (2012) to find exact solutions of Eqs.(4.9)-(4.11). Combining Eqs. (1.23) and (4.12), we obtain

$$A = V^{\frac{n}{n+2}}, \qquad B = V^{\frac{1}{n+2}}. \tag{4.13}$$

Subtraction of Eq.(4.10) from Eq. (4.9) gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{2m^2}{A^2} = 0. \tag{4.14}$$

Substituting Eq. (4.13) into Eq. (4.14), we obtain

$$\ddot{V} = \frac{2m^2(n+2)}{n-1} V^{\frac{2-n}{n+2}}. \tag{4.15}$$

The first integral of Eq. (4.15) is

$$\int \frac{dV}{\sqrt{(V^{\frac{4}{n+2}} + c)}} = \frac{m(n+2)t}{\sqrt{n-1}} \tag{4.16}$$

where c is an arbitrary constant. Clearly Eq. (4.16) imposes some restriction on the choice of n, namely, $n > 1$. It is not possible to solve Eq. (4.16) in general. So, in order to solve the problem completely, we have to choose either c or n in such a way that Eq.

(4.16) be integrable. Therefore we consider the following cases.

4.3.1 Case 3.1 When c=0

In this case the solution of Eq.(4.16) is

$$V = \left(\frac{mn}{\sqrt{n-1}}\right)^{\frac{n+2}{n}} (t+k_1) \tag{4.17}$$

where k_1 is an arbitrary constant. From (4.13) and (4.17) we obtain the scale factor as

$$A = \frac{mn}{\sqrt{n-1}}(t+k), \tag{4.18}$$

$$B = (\frac{mn}{\sqrt{n-1}})^{\frac{1}{n}}(t+k)^{\frac{1}{n}}. \tag{4.19}$$

With these scale factors, the metric (4.1) can be written in form

$$ds^2 = -dT^2 + (\frac{mn}{\sqrt{n-1}})^2 dx^2 + (\frac{mn}{\sqrt{n-1}})^{\frac{2}{n}} T^{\frac{2}{n}} (e^{-2mx}dy^2 + e^{2mx}dz^2) \tag{4.20}$$

where T=t+k.

The expressions for the energy density ρ and the EoS ω for the model (4.20) are obtained as

$$\rho = \frac{1+n}{n^2T^2}, \tag{4.21}$$

$$\omega = \frac{n-2}{n+1}. \tag{4.22}$$

The other physical and kinematical parameters are given by

$$nH_1 = H_2 = H_3 = \frac{1}{T}, \tag{4.23}$$

$$\theta = 3H = \frac{n+2}{nT}, \tag{4.24}$$

$$\sigma = \frac{1}{\sqrt{3}}\frac{n-1}{nT}, \tag{4.25}$$

$$q = -\frac{2}{n+2}. \tag{4.26}$$

The deceleration parameter q is always negative. The EoS parameter is positive when $n > 2$ and is negative if $1 < n < 2$. Thus, the metric (4.20) represents as ever power-law accelerated expansion universe filled with a perfect fluid. If $1 < n < 2$, $\omega < 0$, we obtain DE cosmological model of Bianchi type-VI_0.

The spatial volume V is zero and all physical parameters diverge at $T = 0$. Therefore, the model has a point-type singularity at $T = 0$. For $0 < T < \infty$, the spatial volume is an increasing function of time. The physical parameters are monotonically decreasing function of time and ultimately tend to zero for large T. The anisotropy in the model is maintained throughout the passage of time. For the physical reality of the model we will have to choose n, greater than 1,in such a way that $\left|\frac{n-2}{n+2}\right| \leq 1$. It deserves mention that we are unable to find n for which ω=$\pm$ 1

4.3.2 Case 3.2 When c$\neq 0$

When c$\neq o$, Eq. (4.16) is not integrable for general values of n. However, for n=2, it becomes

$$\int \frac{dV}{\sqrt{V+c}} = 4mt \tag{4.27}$$

which, after integration, yields

$$V = 4m^2t^2 + 2\beta t + \gamma \tag{4.28}$$

where β and γ are arbitrary constants. The constant c is absorbed in γ. From (4.13) and (4.28), we obtain the scale factors as

$$A = (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{2}}, \tag{4.29}$$

$$B = (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{4}}. \tag{4.30}$$

Therefore, the metric (4.1) of our solutions can be written in the form

$$ds^2 = -dt^2 + (4m^2t^2 + 2\beta t + \gamma)dx^2 + (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{2}}(e^{-2mx}dy^2 + e^{2mx}dz^2) \tag{4.31}$$

The expressions for (H_1, H_2, H_3), H, ρ, θ and σ are obtained as

$$H_1 = \frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}, \tag{4.32}$$

$$H_2 = H_3 = \frac{1}{2}(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{4.33}$$

$$H = \frac{2}{3}(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{4.34}$$

$$\theta = 2(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{4.35}$$

$$\sigma = \frac{1}{\sqrt{3}}(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}). \tag{4.36}$$

The energy density, DP and ω are obtained as

$$\rho = \frac{(8m^2t + 2\beta)^2 - (4m^2\gamma - \beta^2)}{4(4m^2t^2 + 2\beta t + \gamma)^2}, \tag{4.37}$$

$$\omega = -\frac{5(4m^2\gamma - \beta^2)}{(8m^2t + 2\beta)^2 - (4m^2\gamma - \beta^2)}, \tag{4.38}$$

$$q = -\frac{2m^2(4m^2t^2 + 2\beta t + \gamma)}{(4m^2t + \beta)^2}. \tag{4.39}$$

The value of DP is always negative since V is never negative. The EoS parameter ω is negative if $\gamma > \frac{\beta^2}{4m^2}$. If this condition holds, the model (39) corresponds to a Bianchi type-VI_0 dark energy cosmological model with variable q and ω. If $\gamma > \frac{\beta^2}{4m^2}$, the model (4.31) has no finite singularity. The physical and kinematical parameters are all decreasing function of time and ultimately tend to zero for large time. The model essentially gives an empty space-time for large time. The anisotropy in the model never dies out.

4.4 Conclusion

In this chapter, we have presented exact solutions of Einstein's field equations for a Bianchi-type VI_0 space-time filled with perfect fluid satisfying the barotropic equation of state under the assumption that the expansion scalar is proportional to shear scalar. Under some specific choice of problem parameters, the present consideration yields singular and non-singular models of the accelerated expansion universe filled with perfect fluid and dark energy. Models with negative EoS parameter ω may be attributed to the current accelerated expansion of universe. The physical and kinematical parameters are all decreasing function of time and ultimately tend to zero for large time. We can see that the ratio $\frac{\sigma}{\theta}$ does not tend to zero as t$\rightarrow \infty$ which shows that the universe models do not approach to isotropy. The universe models do not approach to isotropy. The models presented in this chapter can be potential tools to describe the present universe as well as the early universe.