1 INTRODUCTION

This chapter presents a brief introduction to the General Relativity and Cosmology. It gives a short review of our current understanding about the universe, its history and present status, and the important equations that govern its evolution. Some relevant cosmological solutions with perfect fluid and viscous fluid sources in general relativity and f(R,T) gravity theory have been introduced. Various cosmological parameters, which describe physical and geometrical properties of the expanding universe, have been presented. The motivation and plan of the work have also been discussed in detail.

1.1 Basic Cosmology

Cosmology deals with the large-scale structure of universe as a whole extending to distances of billions of light years,its history and extrapolation of its future course of evolution. Cosmologists construct theoretical models within the framework of general relativity and study the physical behaviour of the universe. Concentrating on its large-scale features, they compare models with the universe as observed by astronomers. Prior to general relativity, cosmologists had tried to understand the structure of universe through Newton's theory of gravitation. The attempt to construct Newtonian cosmological models led to a number of problems for dealing with the dynamics of the universe as a whole. This theory involves instantaneous propagation of gravitational disturbance, which is a doubtful concept especially when applied over large distances. Due to this reason, Newtonian theory could not progress. Later on, Newtonian theory was replaced by Einstein's general theory of relativity. Modern cosmology found its greatest inspiration in Einstein's general theory of relativity . This theory provides, for the first time a concept of physical and mathematical frame work of general relativity for dealing with the problems on cosmic scale.

Matter in the universe is found to be distributed in agglomerations of stars, galaxies and cluster of galaxies. Cosmology treats this distribution as a fine structure and the universe is described in the continuum approximation i.e. via a cosmological fluid. The volume occupied by the galaxies is less than or equal to 10^{-6} times the total volume of the universe. This explains why galaxies are considered as points when cosmological models are constructed. The physical viability of a cosmological models that describe our universe and its history appropriately, is tested by cosmological parameters such as Hubble's parameter, density parameter,deceleration parameter, curvature etc. There have been strong agreements between theoretical predictions and experimental observations before accepting any cosmological model.

Traditionally cosmology has excited the imagination of religious thinkers, philosophers and poets besides its roots in science and astronomy. As the cosmology is the study of large scale structure of the universe, but our tools of observations and the knowledge of the laws of nature are not good enough and sufficiently advanced to interpret scientific information about the large-scale structure of the universe. The answers to these questions, are obvious in the remark passed by Einstein: incomprehensible thing about the universe is that it is incomprehensible. Einstein (1915) gave the complete theory of gravitation, general theory of relativity, which is basically a geometrical phenomena in which the basic interactions and trajectories of any particle are dealt with a single field equation. Schwarzchild (1916) was the first to be obtain physically significant solution of Einstein's field equations. It showed how space-time is curved around a spherically symmetric distribution of matter. The three crucial tests make us indeed confident that relativity provides a real advancement over the Newtonian theory of gravitation, and that it furnishes an acceptable treatment for the field in the empty space surrounding a star out to distances of the order of the dimensions of the solar system. Thus, general relativity certainly provides at the present time the only possible theory of gravitation that could be applied to study the behavior of large portions of the universe, and hence we are forced to make use of this theory if we are to carry out cosmological speculations at all.

It is very difficult to approve any kind of cosmological theory for there are real limitations in our observational knowledge as to the actual nature of the universe and its contents. So there are serious gaps in the information which we could desire. Although, we can make observations in our immediate neighborhood to greater distances, we have no real justification for assuming that the whole universe has the same properties as that portion which we have already seen. In general, we shall actually make great use of homogeneous models in our studies, but we shall have to realize that we do this primarily in order to secure a definite and relatively simple mathematical problem, rather than to secure a correspondence with known reality. Also we have very little information as to the density of other forms of matter and radiations in the enormous extragalactic spaces lying between the observed nebulae. It is only due to the works of Hubble that we could know the density of matter in the form of extragalactic dust as thousands times great as the averaged out density of the nebulae. This is a very serious limitation on our knowledge, So only on the basis of dubious metaphysical predictions, we can choose between universes which are finite or infinite in spatial extent. Due to these uncertainties in observational knowledge, much of our actual works must necessarily consist in a study of cosmological models, constructed in accordance with the general theory of relativity.

1.1.1 Einstein's Field Equations

In 1915, Einstein's postulated a link between the geometry of space and the source of gravitational field by introducing field equations for gravity within the framework of general theory of relativity. The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi G}{c^4} T_{ij}, \tag{1.1}$$

where G is gravitational constant; c is speed of light; T_{ij} is stress energy tensor of the matter field; $R=g^{ij}R_{ij}$ is the Ricci scalar corresponding to the space-time metric g_{ij} of signature (+,-,-,-) and R_{ij} is the Ricci tensor corresponding to space -time metric given by

$$ds^2 = g_{ij} dx^i dx^j \quad (i, j = 1, 2, 3, 4) \tag{1.2}$$

$$R_{ij} = \frac{\partial}{\partial x^j} \Gamma^l_{il} - \frac{\partial}{\partial x^l} \Gamma^l_{ij} + \Gamma^m_{il} \Gamma^l_{mj} - \Gamma^m_{ij} \Gamma^l_{ml} \tag{1.3}$$

with

$$\Gamma^l_{ij} = \frac{1}{2} g^{lh} \left(\frac{\partial g_{ij}}{\partial x^j} + \frac{\partial g_{jh}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^h} \right) \tag{1.4}$$

the Christofell's symbols of second kind

The tensor G_{ij} defined as

$$R_{ij} - \frac{1}{2}g_{ij}R = G_{ij} \tag{1.5}$$

is called Einstein's tensor. As a result of the symmetry of G_{ij} and T_{ij}, actual number of the field equations reduces to 10, although there are additional four differential identities (the Bianchi identities), given by

$$G^{ij}_{;j} = 0 \tag{1.6}$$

where a semicolon denotes the covariant differentiation. So covariant divergence of the Einstein tensor vanishes, which in turn implies

$$T^{ij}_{;j} = 0, \tag{1.7}$$

known as energy conservation equation.

Einstein field equations (1.1) provide a complicated differential equations, which can, in general, be solved if one makes simplifying assumptions or uses numerical techniques. The non-linearity of the Einstein's field equations steams from the fact that masses effect the very geometry of the space. In fact, the fundamental insight of (1.1) is that mass curves the geometry of space-time, and the geometry of space-time, in turn tells masses how to move.

1.1.2 Perfect Fluid Medium

The energy-momentum tensor or stress energy tensor T_{ij} describes the source of matter or gravitational field. It plays an important role during different epochs in the history of the universe. The list of matter fields includes vacuum, matter fluids(perfect/imperfect), scalar fields, electromagnetic fields etc.. For instance,if ρ is the energy density of matter, p is the thermodynamical pressure and $u^i = \frac{dx^i}{ds}$ refers to the 4-velocity vector of the cosmic fluid satisfying $g_{ij}u^iu^j = 1$, the energy momentum tensor for a perfect fluid, is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{1.8}$$

1.1.3 Viscous Fluid Medium

The adequacy of cosmological models with perfect fluid is no basis for expecting that it is equally suitable for describing its early stages of evolution. At the early stages of evolution of the universe, when radiation in the form photon as well as neutrinos decoupled, the matter behaved like a viscous fluid. The energy momentum tensor of a fluid is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p} g_{ij} \tag{1.9}$$

where

$$\bar{p} = p - \xi u^i_{;i}. \tag{1.10}$$

Here $\bar{p}$ is the effective pressure and ξ (> 0) is the bulk viscosity coefficient.

1.2 Homogeneous and Isotropic Cosmological Models

The origin of modern cosmology is the Einstein's general theory of relativity which opens new avenues of approach to the solution of problems related to the universe on cosmic scale. Einstein (1917) himself constructed the static cosmological models filled with a continuous distribution of perfect fluid. However, the model is unsatisfactory for several reasons. It contradicts the actual universe where, according to Hubble and

L.Humason (1934), a definite redshift is observed in the light from the nebulae which (redshift) increases atleast very closely in linear propagation with distance. Very shortly after the presentation of Einstein's static model, de sitter (1917) described the Einstein static universe including a pressure term and gave the field equations for this case. The de-sitter universe is completely empty containing neither matter nor radiation. In the de-sitter universe we get an explanation of the actual redshift observed by Hubble and Humason (1934). Through the de-sitter universe is completely empty, it predicts the observed recession of nebulae. On the other hand, Einstein universe is full of matter, but it does not predict the observed recession of nebulae . Thus, neither Einstein's universe nor de-sitter's universe represent true model of the actual universe. In order to construct a model in which advantages of the two static models of Einstein's and de-sitter are combined, one has to take recourse to non-static models in which the metric tensor in intrinsically time-dependent.

1.2.1 Standard Model and Cosmological Constant

Using the Cosmological Principle, Friedmann (1922, 1924) solved the Einstein's field equations, and obtained non-static cosmological solutions, representing an expanding universe. Therefore, the most suitable line element describing a non-static, and homogeneous model of the universe, is the Friedmann-Robertson-Walker (FRW) metric. In standard spherical coordinates (x^i)= (t, r, θ, ϕ), a spatially homogeneous and isotropic FRW line element has the form (in units c=1)

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.11}$$

where a(t) is cosmic scale factor, which describes how the distances (scales) change in an expanding or contracting universe, and is related to the red shift of the 3-space; k is the curvature parameter, which describes the geometry of the spatial section of space-time with closed, flat and open universes corresponding to k=-1,0,1, respectively. The FRW models have been remarkably successful in describing the observed nature of the universe satisfactorly.

The Einstein's field equations (1.1),for the metric (1.11), in case of the energy-momentum tensor (1.7), reduce to the following equations:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1.12}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{1.13}$$

where an over dot denotes derivative with respect to the cosmic time t.

For the FRW space-time (1.11) and the perfect fluid energy-momentum tensor (1.8), equation (1.7) yields a single conservation equation

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0 \tag{1.14}$$

This equation is actually not independent of Friedmann equations but is required for consistency . It implies that the universe (as specified by Hubble parametr H= $\frac{\dot{a}}{a}$) can lead to local changes in the energy density. Note that there is no notion of conservation of " total energy" since energy can be interchanged between matter and the space-time geometry.

In cosmology, the Friedmann-Robertson-Walker (FRW) models play an essential role. These models truly represent the universe, but in some sense they are good global approximations of the present universe. FRW models are characterized by (i) the universe being the same at all points in space (spatially homogeneous) and (ii) all spatial directions at a point being equivalent (isotropy).

In recent years, experimental studies of the cosmic microwave radiation and speculation about the amount of helium formed at early stages and many other effects have stimulated theoretical interest in anisotropic cosmological models. The spatially homogeneous and anisotropic Bianchi models present a medium way between FRW models and completely inhomogeneous and anisotropic universe and thus play an important role in current modern cosmology. A spatially homogeneous Bianchi model necessarily has a three-dimensional groups, which acts simply transitively on space like three-dimensional orbits.

Assumption concerning homogeneity and isotropy of the universe helps in the sense that all spatial directions are equivalent and no part of the universe can be distinguished from any other. The distribution of galaxies in sky along their apparent magnitudes and red shifts, the distribution of radio sources, cosmic X-ray background, cosmic microwave background all offer at least some circumstantial evidence that distribution of these materials on large-scale exhibit to be isotropy.

1.3 Spatially Homogeneous and Anisotropic Models

Even through the universe, on a large scale, appears isotropic and homogeneous at the present time, there are no observational data that guarantee in an epoch prior to the recombination. The sorts of matter fields in the early universe are uncertain. In the early stages of evolution, the universe could not have had such a smoothed out picture because near the big bang singularity neither the assumption of spherical symmetry nor of isotropy can be strictly valid.Therefore, anisotropy at early times is a very natural phenomenon to investigate in order to sort out problems like the local anisotropies that we observe today in galaxies, clusters and superclusters. These anisotropies may have many possible sources; they could be associated with cosmological magnetic or electric fields, long wavelength gravitational waves, Yang-Mills fields etc. (Barrow, 1997). Moreover, the experimental studies of the isotropy of the CMBR and speculation about the amount of helium formed at the early stages of the evolution of universe have stimulated theoretical interest in the cosmological models with anisotropic background. Therefore, to describe the early evolution of universe, it appears appropriate to suppose a geometry that is more general than just the isotropy and homogeneous FRW geometry. Anisotropic cosmological models play significant role in understanding the behavior of the universe at its early stages of evolution. Modern cosmology is concerned with the through understanding and explanation of the past history, the present state and the future evolution of the universe. Recent cosmological observations support the existence of an anisotropic phase that approaches to isotropic one for large time.

1.3.1 Bianchi Models

The Bianchi cosmologies are spatially homogeneous space-times with a three- parameter isometry group acting on spatial slices, which are described dynamically by the dynamical system whose state space variables are independent of spatial coordinates. Bianchi cosmologies are anisotropic generalizations of the homogeneous FRW -cosmologies, and orthonormal frame methods have been very useful in the study of them because of the close connection between the orthonormal frame variables and the structure constant of the Lie algebra of the killing vector fields of the isometry group. The different group types correspond to different invariant of the state space and arranges the different Bianchi types into a hierarchy of increasing complexity.

The classification of the three dimensional Lie algebra is called the Bianchi classification, and each Lie algebra is labeled by numbers I-IX. By using one of these Lie algebras, we can construct spatially homogeneous cosmological models. The corresponding cosmological models are called Bianchi models. Here it is possible to take the surface of symmetry $\sum$ to be given by t=Const. and choose a frame of vector e_a with component E_a^i dependent only on spatial variables the line element of such space times is given by

$$ds^2 = dt^2 - g_{ij}(t)(E^i_\mu dx^\mu)(E^j_\nu dx^\nu) \tag{1.15}$$

Here E^i_μ is the matrix inverse of E^μ_i . The evolution of the universe is then represented by the time dependence of the six independent frame components g_{ij} of the metric. The basis vectors of E_i of the Bianchi models satisfy $[E_i,E_j]=C^k_{ij}\ E_k$, in which C^k_{ij} are the structure constants of the relevant symmetric group, and it is possible to classify this according to the scheme given by Bianchi in 1897. Complex transformations formally relate type VIII to type VI to VII.

The Bianchi models have been studied extensively since the late 60's as example of exact solutions (Stephani et al., 2003), and from a dynamical systems perspective at least since 1971 by Collins (1971) and developed further by Bogoyavlensky (1985),Rosquist and Jantzen (1988) and others. The book by Coley (2003) and the collaborative work, edited by **?** give detailed accounts of the uses of the dynamical system in Bianchi cosmologies . Worth mentioning is the proof by Ringstrom (2001) that the past asymptotic states of Bianchi type IX models with an orthogonal perfect fluid, obeying the strong and weak energy condition, generically are contained on the closure of the union of the vacuum Bianchi type I and II subsets.

In connection with data from Wilkinson microwave probe(Jafe et al., 2005; Hinshaw et al., 2009), it has been discovered that the standard cosmological model requires positive and dynamic cosmological parameters, a case which resemble Bianchi morphology. According to this result, the universe should achieve the following features: (i) a slightly anisotropic geometry in spite of inflation, and (ii) a non trivial isotropization history of the universe due to the presence of an anisotropic energy source. The advantage of these anisotropic models are that they have a significant role in the description of evolution of early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models.

A spatially homogeneous and anisotropic Bianchi type-I model is considered as the simplest generalization of the FRW flat model. It is described by the line element

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \tag{1.16}$$

where A, B and C are the metric functions or directional scale factors of cosmic time t. If any two of the directional scale factors are equal and third one is different (say,A $\neq$ B=C), the space- time is said to be axially symmetric or locally rotationally symmetric (LRS. In case A $\neq$ B$\neq$ C, the space-time is totally anisotropic.

1.3.2 Big-Bang Theory

This theory was first proposed by Lemaitre (1917). He put forward the idea that the universe was once condensed into a single huge mass that became unstable and exploded. He called it the cosmic egg and suggested the fact that galaxies can now be observed receding from each other was the direct result of the original explosion. The fragments of the cosmic egg flung apart by the upheaval, have evolved into the galaxies that now exist. It is observed that no fragment of the primeval egg are flying from the site of an ancient explosion, but the space itself is expanding. On the process of expansion, it carried with it the stars, galaxies and dust clouds that make up the observable universe. All these ideas may be verified by Einstein's theory of a space-time that can twist, bend stretch and even expand. The space-time embedded in the fireball as it expands, the space itself expands and the space outside the fireball is known as hyper space or super space. This means that the picture of the expanding universe reveals the galaxies as stationary objects apparently moving apart as the space between them expands. This theory was strongly supported by the existence of Cosmic Microwave Background Radiation (CMBR) which was discovered and identified as a faint background noise in the electromagnetic range coming from all directions of the outer space.

A thousand millionth of a second after the big-bang its temperature was about 2 Lack million degrees Fahrenheit. A mass equivalent to that of earth would have been squeezed into a volume no greater than an average size bucket. Such extreme conditions meant that the only form in which matter could exist was in the form of tiny elementary particles, the building block of the bigger particles such as proton and neutrons. In the beginning the universe was not dominated by matter but by intense radiations ranging over all the wavelengths of the electromagnetic spectrum. Light will only have had time to travel the distances across an atomic nucleus in the first 100000 million, millionth of the second. Mass equivalent to our entire galaxy crushed into volume less than four thousandth of an inch across. Such kind of condition is called quantum era in which the very concept of space-time cease to have any meaning. There are no words to describe events under such conditions, and scientists trying to understand the ultimate mysteries of big-bang have to rely on the language of mathematics alone.

1.3.3 Non-Singular Models

It is widely believed that only cosmological models having an extremely hot, dense initial state can explain the observed features of our universe. In large part, this is due to the discovery of the microwave back-ground radiation. Many attempts have been made to describe the early evolution of the universe and to deal with such problems as the fireball spectrum, helium synthesis and so forth (Weinberg 1972). As there is considerable uncertainty in the Hubble parameter, the deceleration parameter and the hidden energy of the universe in neutrino, gravitons, so a wide range of models can fit the data. Wheeler (1961) has pointed out that to allow singularities in a field theory is really to allow anything at all. Singularities make a theory unsatisfactory. The problem here is precisely the same as that of the final state in gravitational collapse. It is possible to formulate a mathematically simple and attractive model avoiding initial singularity, while still possessing hot dense state. This type of model may be possible by inclusion of the second viscosity of the fluid filling the model. This type of model is beautifully explained by Murphy(1973)

1.3.4 Steady State Theory

Three English astronomers, namely Hermann Bondi, Thomas Gold and Fred Hoyle proposed a radical alternative to the big-bang theory known as the steady state theory. They argued that despite appearances, the universe was unchanging in overall terms. This theory says that if people could travel either backward or forward in time they would still see the universe looking very much as it does today. The difficulty of having to account for the origin of the universe was solved by rejecting the idea of beginning altogether. Bondi, Gold and Hoyle realized that the universe was not static and even accepted that the galaxies are receding. But they believed that the observable change is only a local phenomena and if the universe is viewed on a large enough scale, no overall change would be taking place. They have also proved that the omnipresent creation field never lose or gain energy, but merely shuffle it between them. Ultimately, this theory failed because of its central idea that the universe looks the same from whatever moment in time it is viewed. After the detection of CMBR in 1965, the steady state theory became no longer acceptable.

1.4 Lyra's Geometry

Einstein while discussing cosmological solutions had to introduce the cosmological constant Λ into the field equations because the large scale recession of the galaxies i.e.' the expansion of the universe had not been discovered at the time; this was discovered later by Hubble. The theory has been successful in describing not only the gravitational phenomena but has served as a basis for cosmological models of the universe. Gravitation, however, is not the only force described by classical physics, Electromagnetic forces are also important and they are not explained by general relativity as a geometric phenomena. Subsequently, there have been many attempts to unify electromagnetism and gravitation. Weyl (1918) proposed a more general theory in which electromagnetism is also described geometrically. Lyra(1951) suggested a modification of the Riemannian geometry, which may also be considered as a modification of Weyl geometry, by introducing a gauge-function into the structure-less manifold as a result of which a displacement field arises naturally. Halford (1972) pointed out that the constant displacement vector field ϕ in Lyra's geometry plays the role of cosmological constant in the normal general relativistic treatment. He has also shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as Einstein's theory. Sen(1957), Sen and Dunn(1971) proposed a scalar-tensor theory of gravitation and constructed analogue of Einstein field equations based on Lyra's geometry. The field equations, in normal gauge, for Lyra's manifold are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_\alpha\phi^\alpha) = -8\pi GT_{ij} \tag{1.17}$$

where ϕ is a time-like displacement vector field defined as $\phi_i = (0, 0, 0, \beta(t))$, and other symbols have their usual meaning as in Riemannian geometry.

Several authors have studied cosmological models based on Lyra's geometry with a constant displacement vector field (Ram 1985,1986; and Adhav et al. 2007). However, the restriction of the constant displacement vector field is a coincidence and there is no a prior reason for it. Beesham (1988) has investigated Friedmann-Robertson Walker (FRW) cosmological models in Lyra's geometry with time dependent displacement vector field.

1.5 Kaluza Klein Universe

A potential way to geometrize the physics of gravity and electromagnetism was suggested by Kaluza's (1920), who added a fifth dimension to Einstein's general relativity. Kaluza showed in essence that the apparently empty 5D field equations R$_{ij}$ =0 (i,j=0,1,2,3,4) in terms of the Ricci tensor, contain Einstein's equations for gravity and Maxwell's equations for electromagnetism. Einstein's, after some thought, endorsed this step. However in the 1920s, quantum mechanics was gaining a foothold in theoretical physics, and in the 1930s there was a vast expansion of interest in this area, at the expense of general relativity. This explains why there was such a high degree of attention to the proposal of Klein, who in 1926 suggested that the fifth dimension of Kaluza ought to have a closed topology (i.e., a circle), in order to explain the fundamental quantum of electric charge (l). Klein's argument actually related this gravity to the momentum in the extra dimension, but in so doing introduced the fundamental unit of action (h) which is now known as Planck's constant. However, despite the appeal of Klein's idea, it was destined for failure. There are several technical reasons for this, but it is sufficient to note here that the crude 5D gravity/quantum theory of Kaluza/Klein implied a basic role for the mass quantum $\left(\frac{hc}{G}\right)^{\frac{1}{2}}$. This is of order 10^{-5}g, and does not play a dominant role in the spectrum of masses observed in the real universe. (In more modern terms, the so-called hierarcy problem is centered on the fact that observed particle masses are far less than the Planck mass, or any other mass derivable from a tower of states where this is a basic unit.) Thus, we have a dead-end. This does not though imply that, there is anything wrong with the basic proposition, which follows from the work of Einstein and Kaluza, that matter can be geometrized with the aid of the fundamental constants. As a simple example, an astrophysicist presented with a problem involving a gravitationally-dominated cloud of density ρ will automatically note that the free-fall or dynamical timescale is the inverse square root of G_ρ. This tells him immediately about the expected evolution of the cloud. Alternatively instead of taking the density as the relevant physical quantity, we can form the length $\left(\frac{c^2}{G_\rho}\right)^{\frac{1}{2}}$ and obtain an equivalent description of the physics in terms of a geometrical quantity.

1.6 Dark Energy

In 1993, when Baade and Zwicky (1934) estimated the total amount of mass in a cluster of galaxies, known as the Coma cluster based on the motion of the galaxies near the edge of the cluster and compared it to one based on the number of galaxies and total brightness of the cluster, he found that $\frac{9}{10}$ of the matter in the Coma cluster was not luminous and therefore could not be seen. Also the gravity of the visible galaxies in the cluster should be far too small for such fast orbits, so something extra was required. Zwicky inferred that there must be some other form of matter existent in the cluster which provides enough of mass and gravity to hold the cluster together, Zwicky called this as "dark matter. Rubin and Ford (1970) played a major role in establishing the existence of dark matter in spiral galaxies were moving much more quickly than one would have predicted. Without dark matter, spiral galaxies could fly apart, but the dark matter stabilizes these galaxies.

At the time while the prospect of a universe filled with dark matter has itself challenged our understanding of the physical world; an even more starting cosmological discovery has come to light in 1998. Until the late 1990s ,cosmologists took it for granted that the expansion of the universe was slowing down under the influence of gravitation. A dramatic breakthrough happened in 1998 when two independent teams of astronomers, one led by Perlmutter et al.(1997) and the other by Riess et al.(1998), were searching for distant supernovae hoping to measure the rate at which the expansion of the universe was slowing down. They traced the expansion of the universe over the past five billion years and were in a shock to find that the cosmic expansion is not slowing down but speeding up. This discovery has created a confusing situation among the cosmologists because although the standard cosmological models have been confirmed by data from

Wilkinson Microwave Anisotropic Probe (WAMP) and by other telescope surveys of the large-scale structure of the universe it was not known why the cosmic expansion is accelerating. To unveil the truth, intensive search is going on both at theoretical and observational level. Many researchers suggested modifications and changes to Einstein's general theory of relativity.

Some other expected a conventional explanation for the accelerating expansion of the universe based on astrophysics, e.g. the effect of dust on difference between young and the old supernovae. But to the cosmologists around the world, a kind of repulsive force which acts as anti-gravity is responsible for gearing up the universe some five billion years ago (Capozziello et al., 2006a) This hitherto unknown physical entity is dubbed as dark energy which has negative pressure and makes up about three quarters of the total present cosmic density. Many cosmologists like to select the cosmological constant Λ , introduced by Einstein in his field equations, as a suitable candidate for dark energy because of its weired repulsive gravity. The cosmological constant provides a pretty good explanation to the expansion of the universe being accelerated. But selection of the cosmological constant as dark energy faces some serious problems.

Einstein's general relativity (1916) is widely accepted as a fundamental theory to describe the geometric properties of space-time. In a homogeneous and isotropic space-time the Einstein field equations give rise to the Friedmann equations that describe the evolution of the universe. In fact, the standard big-bang cosmology based on radiation and matter dominated epoch can be well described within the framework of general relativity. However, the rapid development of observational cosmology which started from 1990s shown that universe has undergone two phases of cosmic acceleration. The first one is called inflation (Starobinsky, 2007; Guth, 1981) , which is believed to have occurred prior to radiation domination (Bassett, et al, 2006). As per Smoot (1992), this phase is required not only to solve the flatness and horizon problems plagued in the big-bang cosmology, but also to explain a nearly flat spectrum of temperature anisotropies observed in (CMBR) cosmic microwave background. The second accelerating phase has started after the matter domination. As per Huterer and Turner (1999), the unknown component giving rise to this late -time cosmic acceleration is called dark energy (Carroll, 2001; Padmanabhan, 2003; Sahni and Starobinsky, 2000; Peebles and Ratra, 2003). The existence of dark energy has been confirmed by a number of observations, such as supernovae Ia(Perlmutter et al., 1999; Riess et al., 1998, 2004), large scale structure (Tegmark, et al, 2004), baryon acoustic oscillations (Eisenstein et al., 2005; Percival et al., 2007)and cosmic microwave background (CMB) (Spergel et al., 2003).

1.7 f(R) and f(R,T) Theories of Gravitation

The recent observational data on the late-time acceleration of the universe and the existence of dark matter and dark energy have posed a fundamental theoretical challenge to gravitational theories. One possibility in explaining the observations is by assuming that at large scales the Einstein gravity model of general relativity breaks down, and a more general action describes the gravitational field. Theoretical models, in which the standard Einstein-Hilbert action is replaced by an arbitrary function of Ricci scalar R (Nojiri and Odintsov, 2007) have been extensively investigated lately. Carroll et al.(2004) proved that the presence of a late-time cosmic acceleration of the universe can indeed be explained by f(R) gravity. The conditions of the existence of viable cosmological models have been found by many researchers (Capozziello et al., 2006b; Koivisto, 2007; Carloni et al., 2008; Ananda and Carloni, 2008; Guarnizo et al., 2011) and severe weak field constraints obtained from the classical tests of general relativity for the solar system regime seem to rule out most of the models proposed so far (Chiba et al., 2007; Nojiri and Odintsov, 2008; Capozziello et al., 2008). However,Faraoni et al. (2006) , has proved that viable models, passing solar system tests, can be constructed.Cognola et al. (2006),Nojiri and Odintsov (2010) considered f(R) models that satisfy local tests and unify inflation with dark energy. In the framework of f(R) gravity models the possibility that the galactic dynamic of massive test particles can be understood without the need for dark matter was considered by Capozziello et al. (2006c, 2007), Martins and Salicci (2007), Boehmer et al.(2008). For reviews of f(R) generalized gravity models see Capozziello and Faraoni (2010); Nojiri and Odintsov (2011).

Bertolami et al. (2007) proposed a generalization of f(R) modified theories gravity by including in the theory an explicit coupling of an arbitrary function of Ricci scalar R with matter Lagrangian density L_m. As a result of the coupling the motion of the massive particles is non-geodesic and extra force orthogonal to four velocity arises. The connections with modified Newtonian dynamics (MOND) and pioneer anomaly were also explored. Harko (2008) extended this model to the case of the arbitrary couplings in both geometry and matter. Harko and Lobo (2010) and Harko et al. (2011a) investigated astrophysical and cosmological implications of the non-minimal matter -geometry coupling and Palatini formulation. Poplawski (2011) proposed specific application of the $f(R,L_m)$ gravity which has considered as a relativistic covariant model of interacting dark energy based on the principle of least action. This model was known as $\Lambda(T)$ gravity model. This $\Lambda(T)$ gravity is more general than Palatini f(R)gravity and reduces to later when we neglect pressure of matter. Sharif and Shamir (2009, 2010) have studied the solutions of Bianchi type-I and V space-times in the framework of f (R) gravity. Shamir (2010) studied the exact vacuum solutions of Bianchi type I, III and Kantowski-Sachs space-times in the metric version of f (R) gravity.

Harko et al. (2011b) considered another extension of standard general relativity which is known as f(R,T) modified theories of gravity. In this f(R,T) gravity theory the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor. Note that the dependence from T may be induced by exotic imperfect fluids or quantum effects. The field equations of f(R,T) gravity model are derived from Hilbert-Einstein type variational principle. The f(R,T) gravity model depends on a source term representing the variation of the matter stress-energy tensor with respect to the metric. A general expression for this source term is obtained as a function of the matter Lagrangian L_m which would generate a specific set of field equations.

In f(R,T) gravity theory models, the field equations are obtained from the Hilbert-Einstein type variational principle.

$$S = \int \sqrt{-g} dx^4 \left(\frac{1}{16\pi} f(R,T) + L_m \right). \tag{1.18}$$

Here f(R,T) is an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor of the matter T_{ij}. The stress-energy tensor of matter is

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{ij}} \tag{1.19}$$

The corresponding field equations of the f(R,T) gravity are found by varying the action with respect to the metric g_{ij} Harko et al. (2011b):

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\ominus_{ij}, \tag{1.20}$$

where

$$f_R = \frac{\delta f(R,T)}{\delta R},\ f_T = \frac{\delta f(R,T)}{\delta T},\ \Box \equiv \nabla^i\nabla_j,$$

∇_i is the covariant derivative and T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

By contracting (1.20), we get

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)T - f_(T)(R,T)\ominus. \tag{1.21}$$

Generally, the field equations depend through the tensor θ_{ij} on the physical nature of the matter field. Hence in the case of f(R,T) gravity theoretical models corresponding to different matter contributions for f(R,T) gravity are possible.

Harko et al. (2011b) gave three classes of these models:

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T). \end{cases} \tag{1.22}$$

Harko et al. (2011b) have presented some particular models corresponding to the choices of the f(R,T)=R+2f(T). Subsequently Adhav (2012), Reddy et al. (2012a), Chaubey and Shukala (2013) Chandel and Ram (2013), presented Bianchi types cosmological models in the presence of bulk viscous fluid within the framework of f(R,T) gravity theory. Recently, Reddy et al. (2012b), investigated a five dimensional Kaluza-Klein space-time in the presence of a perfect fluid source in f (R,T) theory of gravitation with negative constant deceleration parameter.

1.8 Cosmological Parameters

In this section, we discuss some observational parameters which are of great importance in cosmology. A cosmological model may have many versions, each of which could be correct. Observations must be used to determine, which one is correct. This is attempted by defining a few measurable cosmological parameters, which are closely related to the thesis work.

1.8.1 Average Scale Factor(a) and Volume Scalar (V)

The volume scalar V, which deals the volume element of the universe, can be expressed in terms of the average scale factor and metric functions (1.16) which is given by the relation as

$$a^3 = V = ABC. \tag{1.23}$$

1.8.2 Expansion Scalar (θ)

The expression for the expansion scalar θ which deals with the expansion of the universe is given in tensor from as,

$$\theta = u^i_{;j}. \tag{1.24}$$

For the Bianchi type -I space-times the expansion scalar θ has the expression, given by

$$\theta = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{1.25}$$

1.8.3 Hubble's Parameter (H)

Hubble (1929) proposed a law relating to apparent luminosities of distant galaxies to their redshift as

$$V = HD, \tag{1.26}$$

where V is the speed of recession of galaxy at a distance D from us and H is proportionality constant, called Hubble parameter is given by

$$H = \frac{\dot{a}}{a}, \tag{1.27}$$

which measures the rate of expansion of the universe.

1.8.4 Shear Scalar (σ^2)

The shear scalar is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{1.28}$$

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}\, p^{\alpha}_{j} + u_{j;\alpha}\, p^{\alpha}_{i}\right) - \frac{1}{3}\theta p_{ij}. \tag{1.29}$$

The expression of σ^2 for Bianchi type-I space-times is given as

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}. \tag{1.30}$$

1.8.5 Deceleration Parameter (q)

An important observational quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{1.31}$$

This parameter measures the rate at which the expansion of the universe is changing with time in terms of the scale factor, and its sign characterizes accelerating or decelerating nature of the universe. In case q$>$ 0, the universe decelerates whereas q$<$ 0 describes an accelerating universe and q=0 corresponds to expansion of universe with constant velocity.

1.8.6 Anisotropic Parameter (A_m)

The expression for the anisotropy parameter, which explains the isotropy of the evolution of the universe, is given by

$$A_m = \frac{1}{3}\Sigma_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{1.32}$$

where $\Delta H_i = H_i - H$, (i=1,2,3) are the directional Hubble parameters in directions of x, y, z, axes respectively. These directional Hubble's parameter for (1.16)is given by

$$H_1 = \frac{\dot{A}}{A},\ H_2 = \frac{\dot{B}}{B},\ H_3 = \frac{\dot{C}}{C}, \tag{1.33}$$

The generalized Hubble's parameter in term of directional derivatives for (1.16) is given by

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right). \tag{1.34}$$

1.9 Phases of Universe

Today's universe (redshift z=0) is dominated by DE but it did undergo three transition, from inflationary phase to radiation-dominated, radiation-dominated to matter-dominated and from matter-dominated to dark energy-dominated. These different phases of the universe can be described by an equation of state (EoS), where pressure and energy density are related via

$$p = (\gamma - 1)\rho \tag{1.35}$$

where γ is an EoS parameter lying in the range $0 \leq \gamma \leq 2$. The different phases of evolution of the universe depend on the values of γ given as the values of $\gamma = 1$, 2 and $\frac{4}{3}$ correspond to dust (p=0),stiff-matter (p=ρ) and radiation-phase (p=$\frac{\rho}{3}$) respectively.

A constant γ leads to a great simplification in solving the cosmological equations. In

particular, using (1.35)into (1.14), we find that the energy density evolves with the scale factor according to

$$\rho \propto a^{-3\gamma} \tag{1.36}$$

In case of flat spatial sections (k=0) and a constant EoS parameter γ, we may exactly solve the Friedmann equation(1.12) to obtain

$$a(t) = a_0 \frac{t}{t_0}^{\frac{2}{3\gamma}}, \tag{1.37}$$

where a_0 is the present value of scale factor, and $\gamma \neq 0$.

In case of γ=0, we obtain

$$a(t) \propto e^{Ht} \tag{1.38}$$

1.10 Inflationary Phase

There is yet another phase known as the inflationary phase. The present paradigm is that our universe has undergone rapid phase of expansion, called inflation, possibly prior to the radiation era. This rapid expansion during the inflation period is manifested in the evolution of the scale factor a(t). In case of inflation, a(t)=t^n, where n>1, i.e., power-law inflation or a(t)$\propto e^{Ht}$ (exponential expansion).

In the early 1980's, Alan Guth explained the physical mechanism for inflation, and discussed the corresponding phase transition in the early universe. This idea was conceived from particle physics. This resolves the horizon problem since causal regions in the early universe are stretched to regions much larger than the Hubble distance. Inflation is capable of solving many of the initial value or 'fine-tuning'problems of the hot big-bang model.