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It is certified that the work contained in the thesis titled *A Study of Fuzzy Soft Topologies, Fuzzy Topologies Generated by Fuzzy Relations and Representability of Some Fuzzy Relations* by *Ms. Seema Mishra* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Prof.(Mrs.)Rekha Srivastava

Supervisor
Department of Mathematical Sciences
Indian Institute of Technology
(Banaras Hindu University)
Varanasi-221005

DECLARATION BY THE CANDIDATE

I, *Seema Mishra*, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of *Prof.(Mrs.)Rekha Srivastava* from *September 2012* to *December 2016* at the *Department of Mathematical Sciences*, Indian Institute of Technology (BHU), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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It is certified that the above statement made by the student is correct to the best of my knowledge.

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Supervisor   Head
Department of Mathematical Sciences   Department of Mathematical Sciences
Indian Institute of Technology   Indian Institute of Technology
(Banaras Hindu University)   (Banaras Hindu University)
Varanasi-221005   Varanasi-221005

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Date:

Place: Varanasi   **(Seema Mishra)**

Contents

Preface

The present thesis is concerned with a study of certain separation axioms and compactness in fuzzy soft topological spaces, fuzzy topologies generated by fuzzy relations and representability of fuzzy biorders and fuzzy weak orders.

This thesis is divided into seven chapters.

The first chapter is introductory. It contains a brief survey of the subject related to the thesis, necessary preliminaries and the plan of the thesis.

Next two chapters are devoted to a study of separation axioms, T_0, T_1 and T_2 in fuzzy soft topological spaces. We have introduced these notions and appropriateness of the definitions have been shown by proving several basic desirable results. A comparative study has been done with the earlier existing definitions.

Chapter four is on compactness notion in fuzzy soft topological spaces. In this chapter, a definition of compactness in fuzzy soft topological spaces has been given, as a generalization of the corresponding concept in fuzzy topological spaces, given by Lowen(1976). Counterpart of Alexander's subbase theorem for fuzzy soft topological spaces has been established and using it, we have proved the Tychonoff theorem for fuzzy soft compact topological spaces.

Chapter five and six are on a study of fuzzy topologies generated by fuzzy relations. In chapter five, we have given the notions of fuzzy topologies and fuzzy bitopologies generated by fuzzy relations, as a generalization of the corresponding concepts given by Knoblauch(2009) and Induráin et al.(2013), respectively. Characterizations of fuzzy topologies generated by a fuzzy relation, fuzzy topologies generated by a fuzzy interval order, a preorderable fuzzy topology, an orderable fuzzy topology and fuzzy bitopologies generated by a fuzzy relation, have been obtained. In chapter six, we have introduced fuzzy topologies generated by a fuzzy relation, as a generalization of the corresponding concept, given by Smithson(1969). Sufficient conditions under which these generated fuzzy topologies satisfy separation axioms, fuzzy T_0, fuzzy T_1 and fuzzy T_2, have been obtained. Further, we

have introduced 'finite intersection property' in fuzzy topological spaces and obtained a characterization of Lowen's fuzzy compactness in terms of this property. Using this, we have obtained a sufficient condition under which the fuzzy topology generated by a fuzzy relation, becomes fuzzy compact.

Chapter seven is on the representability of fuzzy biorders and fuzzy weak orders. We have shown that union of a finite family of fuzzy weak orders with respect to a t-norm T is a fuzzy quasi-transitive relation with respect to T. Further, we have obtained a characterization for a T_L-representable fuzzy weak order.